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**Form Factor and Mean-Square Radius for the Scalar Current of Composite System
of Two Particles with Arbitrary Masses and Spin 1/2**

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Plan of Talk

- **1. Introduction**
- **2. Equation for the wave function**
- **3. Form factor of the relativistic two-particle system**
- **4. Form factor and root-mean-square radius for the Coulomb interaction**
- **5. Conclusions**

1. Introduction

For description of the behavior of form factors the different approaches can be used [J. G. Korner, M. Kuroda, *Phys. Rev. D* **16**, 2165 (1977); S. Dubnicka, *Nuovo Cimento A* **103**, 1417 (1991)]. The using of three-dimensional relativistic covariant two-particle quasipotential (RQP) equation of Logunov-Tavkhelidze [A. A. Logunov and A. N. Tavkhelidze, *Nuovo Cimento* **29**, 380 (1963)] for description of the form factors of composite systems were executed in [V. A. Matveev, R. M. Muradyan, and A. N. Tavkhelidze, Preprint No. E2-3498, JINR (Dubna, 1967); No. P2-3900, JINR (Dubna, 1968); R. N. Faustov, *Ann. Phys. (N.Y.)* **78**, 176 (1973)]. However, use of the equation Logunov-Tavkhelidze for wave function in the momentum representation has not allowed to research the behavior of the form factor in broad interval of importances of the momentum transfer of the relativistic two-particle bound system.

The other model was considered in [N. B. Skachkov and I. L. Solovtsov, Preprint No. E2-11727, JINR (Dubna, 1978); Sov. J. Nucl. Phys. **30**, 562 (1979); N. B. Skachkov and I. L. Solovtsov, Theor. Math. Phys. **43**, 494 (1980)]. This model has used the RQP approach [V. G. Kadyshevsky, Nucl. Phys. B **6**, 125 (1968); V. G. Kadyshevsky and M. D. Mateev, Nuovo Cimento A **55**, 275 (1968)] on the basis of covariant Hamiltonian formulation of quantum field theory [V. G. Kadyshevsky, Sov. Phys. JETP **19**, 443, 597 (1964); Sov. Phys. Dokl. **10**, 46 (1965)] in which the contribution of small distances in the proton form factor takes into account by means of transition to the three-dimensional relativistic configurational representation (**r-representation**) in the case of two relativistic spinless particles with equal masses m [V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, Nuovo Cimento A **55**, 233 (1968)].

In the RQP approach [V. G. Kadyshevsky, Nucl. Phys. B **6**, 125 (1968)] for a bound system of two relativistic spinless particles of arbitrary masses developed in [V. G. Kadyshevsky, M. D. Mateev, and R. M. Mir-Kasimov, Yad. Fiz. **11**, 692 (1970); V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, Fiz. Elem. Chastits At. Yadra **2**, 635 (1972)], new covariant expressions of the components of elastic form factor for the cases of a scalar and vector currents as functions of the invariant variable $\Delta_{\mathcal{P}, \mathcal{Q}}^2$, which there is the square of the momentum-transfer vector in the Lobachevsky space, have been found in [Yu. D. Chernichenko, Phys. At. Nucl. **77**, 229 (2014); Yu. D. Chernichenko, Phys. At. Nucl. **78**, 201 (2015)].

The aim of present study, considered as continuation in [Yu. D. Chernichenko, *Phys. At. Nucl.* **77**, 229 (2014); **78**, 201 (2015); *Phys. At. Nucl.* **81**, 360 (2018); *Phys. At. Nucl.* **82**, 334 (2019)], are to obtain the new expressions for the elastic form factor and mean-square radius for the scalar current of composite system of two relativistic particles with arbitrary masses and spin $1/2$.

Consideration is conducted within the framework of RQP approach [V. G. Kadyshevsky, *Nucl. Phys. B* **6**, 125 (1968); V. G. Kadyshevsky and M. D. Mateev, *Nuovo Cimento A* **55**, 275 (1968)] by transition to the three-dimensional relativistic configurational representation in the case of two relativistic spin particles with arbitrary masses [V. G. Kadyshevsky, M. D. Mateev, and R. M. Mir-Kasimov, *Yad. Fiz.* **11**, 692 (1970); V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, *Fiz. Elem. Chastits At. Yadra* **2**, 635 (1972)].

2. Equation for the wave function

Within the framework of RQP approach [V. G. Kadyshevsky, Nucl. Phys. B **6**, 125 (1968); V. G. Kadyshevsky and M. D. Mateev, Nuovo Cimento A **55**, 275 (1968)] for spherically symmetric potentials the equation in the **r-representation** [V. G. Kadyshevsky, M. D. Mateev, and R. M. Mir-Kasimov, Yad. Fiz. **11**, 692 (1970); V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, Fiz. Elem. Chastits At. Yadra **2**, 635 (1972)] for $\varphi_\ell(r, \chi')$ with $\ell \geq 0$ consisting of two relativistic fermions with m_1, m_2 , has the form [Yu. D. Chernichenko, Phys. At. Nucl. **84**, 339 (2021)]

$$\left(\hat{H}_{0,\ell}^{\text{rad}} - \cosh \chi' \right) \varphi_\ell(r, \chi') = -V(r; M_{\mathcal{Q}}) \hat{A} \left(\hat{H}_{0,\ell}^{\text{rad}} \right) \varphi_\ell(r, \chi'). \quad (1)$$

Here $M_{\mathcal{Q}}^2 = s_q = \mathcal{Q}^2 = (q_1 + q_2)^2 = \mathcal{Q}_0^2 - \mathcal{Q}^2$, $q_i, i = 1, 2$ are 4-momentas of composite particle,

the operator

$$\hat{H}_{0,\ell}^{\text{rad}} = \cosh \left(i\lambda' \frac{d}{dr} \right) + \frac{\lambda'^2 \ell(\ell + 1)}{2r(r + i\lambda')} \exp \left(i\lambda' \frac{d}{dr} \right)$$

is the radial part of free Hamiltonian operator ($\hbar = c = 1$)

$$\hat{H}_0 = 2m' \left[\cosh \left(i\lambda' \frac{\partial}{\partial r} \right) + \frac{i\lambda'}{r} \sinh \left(i\lambda' \frac{\partial}{\partial r} \right) - \frac{\lambda'^2}{2r^2} \Delta_{\theta,\varphi} \exp \left(i\lambda' \frac{\partial}{\partial r} \right) \right],$$

where $\Delta_{\theta,\varphi}$ is its the angular part, $\lambda' = 1/m'$ is the Compton wavelength associated with an effective relativistic particle playing the role of the two-particle system, having the mass $m' = \sqrt{m_1 m_2}$, the relative momentum $\Delta_{q',m'\lambda_Q}$ and carrying the total energy of interacting particles M_Q , the rapidity χ' is used to parametrize the momentum and energy

$$\begin{aligned} \Delta_{q',m'\lambda_Q} &= m' \sinh \chi' \mathbf{n}_{\Delta_{q',m'\lambda_Q}}, \quad |\mathbf{n}_{\Delta_{q',m'\lambda_Q}}| = 1, \quad M_Q = 2m' g' \Delta_{q',m'\lambda_Q}^0, \\ \Delta_{q',m'\lambda_Q}^0 &= m' \cosh \chi', \end{aligned} \quad (2)$$

factor g' gives by expression

$$g' = \frac{m'}{2\mu} = \frac{m_1 + m_2}{2\sqrt{m_1 m_2}}, \quad (3)$$

$\mu = m_1 m_2 / (m_1 + m_2)$, $\Delta_{q', m' \lambda_Q}^0$ and $\Delta_{q', m' \lambda_Q}$ are, respectively, the time and spatial components of the 4-vector $\Lambda_{\lambda_Q}^{-1} q' = \Delta_{q', m' \lambda_Q}$ from the Lobachevsky space associated by the pure Lorentz transformations $\Lambda_{\lambda_Q}^{-1}$ with the velocity 4-vector of the composite particle $\lambda_Q = (\lambda_Q^0; \boldsymbol{\lambda}_Q) = Q/\sqrt{Q^2}$ by the formulas

$$\Lambda_{\lambda_Q}^{-1} \mathbf{q}' = \Delta_{q', m' \lambda_Q} = \mathbf{q}' (-) m' \boldsymbol{\lambda}_Q = \mathbf{q}' - \boldsymbol{\lambda}_Q \left(q'_0 - \frac{\mathbf{q}' \cdot \boldsymbol{\lambda}_Q}{1 + \lambda_Q^0} \right), \quad (4)$$

$$(\Lambda_{\lambda_Q}^{-1} q')^0 = \Delta_{q', m' \lambda_Q}^0 = q'_0 \lambda_Q^0 - \mathbf{q}' \cdot \boldsymbol{\lambda}_Q = \sqrt{m'^2 + \Delta_{q', m' \lambda_Q}^2},$$

all 4-momenta belong to the upper sheet of the mass hyperboloid

$$\Delta_{q', m' \lambda_Q}^2 = \Delta_{q', m' \lambda_Q}^{02} - \Delta_{q', m' \lambda_Q}^2 = m'^2. \quad (5)$$

Quasipotential $V(r; M_Q)$ is local in the sense of Lobachevsky geometry, the group parameter r plays the role of modulus of relativistic relative coordinate \mathbf{r} ($\mathbf{r} = r\mathbf{n}$, $|\mathbf{n}| = 1$), and \hat{A} is

$$\hat{A} \left(\hat{H}_{0,\ell}^{\text{rad}} \right) = \frac{1}{4} \left[a' \left(\hat{H}_{0,\ell}^{\text{rad}} \right)^2 + b' \right], \quad (6)$$

where spin parameters a' and b' are connected with factor (3) as

$$a' = \begin{cases} g'^2, \hat{O} = \gamma_5 \text{ (pseudoscalar);} \\ \frac{1}{2}g'^2, \hat{O} = \gamma_\mu \text{ (vector);} \\ -\frac{1}{2}g'^2, \hat{O} = \gamma_5\gamma_\mu \text{ (pseudovector);} \end{cases} \quad b' = \begin{cases} 1 - g'^2, \hat{O} = \gamma_5; \\ \frac{3}{4} - \frac{1}{2}g'^2, \hat{O} = \gamma_\mu; \\ \frac{1}{4} + \frac{1}{2}g'^2, \hat{O} = \gamma_5\gamma_\mu. \end{cases} \quad (7)$$

The value of spin parameters a' and b' for $m_1 = m_2 = m$ coincides with the values of their analogs a and b obtained in [Yu. D. Chernichenko, Phys. At. Nucl. **80**, 707 (2017)].

3. Form factor of the relativistic two-particle system

In Refs. [N. B. Skachkov, I. L. Solovtsov, Preprint No. E2-11727, JINR (Dubna, 1978); *Yad. Fiz.* **30**, 1079 (1979)], founded on Refs. [V. A. Matveev, R. M. Muradyan, A. N. Tavkhelidze, Preprint No. E2-3498, JINR (Dubna, 1967); No. P2-3900, JINR (Dubna, 1968); V. R. Garsevanishvili, A. N. Kvinikhidze, V. A. Matveev, A. N. Tavkhelidze, and R. N. Faustov, *Theor. Math. Phys.* **23**, 533 (1975); R. N. Faustov, *Theor. Math. Phys.* **3**, 478 (1970); *Ann. Phys.* **78**, 176 (1973)], the form factor of two-particle system was defined as the matrix element of the local current operator between bound states with the 4-momentum Q and P through the covariant wave RQP-functions satisfying RQP-equation in the momentum representation. The invariant expression for the matrix element in the momentum representation near the poles of the bound states with the 4-momentum Q and P of the local scalar-current operator of the two relativistic with arbitrary masses fermions bound state has the form

$$\begin{aligned}
& \langle \mathcal{P} | J(0) | \mathcal{Q} \rangle = \tag{8} \\
& = - \frac{z_1}{\sqrt{M_{\mathcal{P}} M_{\mathcal{Q}}} (4\pi)^3} \int \frac{d\tau_{\mathcal{P}} d\tau_{\mathcal{Q}} d\mathbf{k}_2 d\mathbf{k}'_1 d\mathbf{k}_1}{\sqrt{m_2^2 + \mathbf{k}_2^2} \sqrt{m_1^2 + \mathbf{k}'_1{}^2} \sqrt{m_1^2 + \mathbf{k}_1^2}} \frac{\Gamma_{M_{\mathcal{P}}}^*(\mathcal{P}k_2)}{\tau_{\mathcal{P}} + i\varepsilon} \times \\
& \quad \times \text{tr}[\hat{O}^+(\hat{k}_1 + m_1)(\hat{k}'_1 + m_1)\hat{O}(\hat{k}_2 - m_2)] \frac{\Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2)}{\tau_{\mathcal{Q}} - i\varepsilon} \times \\
& \quad \times \delta^{(4)}(-\mathcal{Q} + k'_1 + k_2 - \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}}) \delta^{(4)}(\mathcal{P} - k_1 - k_2 + \lambda_{\mathcal{P}}\tau_{\mathcal{P}}) + (1 \leftrightarrow 2).
\end{aligned}$$

Here, the functions $\Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2)$ and $\Gamma_{M_{\mathcal{P}}}(\mathcal{P}k_2)$ are the scalar parts of the vertex functions which depends each only on one the Lorentz-invariant scalar parameter $\mathcal{Q}k_2$ and $\mathcal{P}k_2$, respectively; for \hat{O} , we take the Dirac matrices γ_5, γ_μ , and $\gamma_5\gamma_\mu$ ($\mu = 0, 1, 2, 3$), $\hat{k}_i = k_i^\mu \gamma_\mu$, m_i is the mass of the i th component ($i = 1, 2$) carrying the 4-momentum k_i and k'_i , and all 4-momenta belong to the upper sheets of the masses hyperboloids

$$k_i^2 = k_{i0}^2 - \mathbf{k}_i^2 = m_i^2, i = 1, 2. \tag{9}$$

The expression in (8) corresponds diagram on Fig. 1, where because of the transition to different proper times of the system before (τ_Q) and after (τ_P) the interaction event, the 4-velocities of composite particle before (λ_Q) and after (λ_P) the interaction event, are also different.

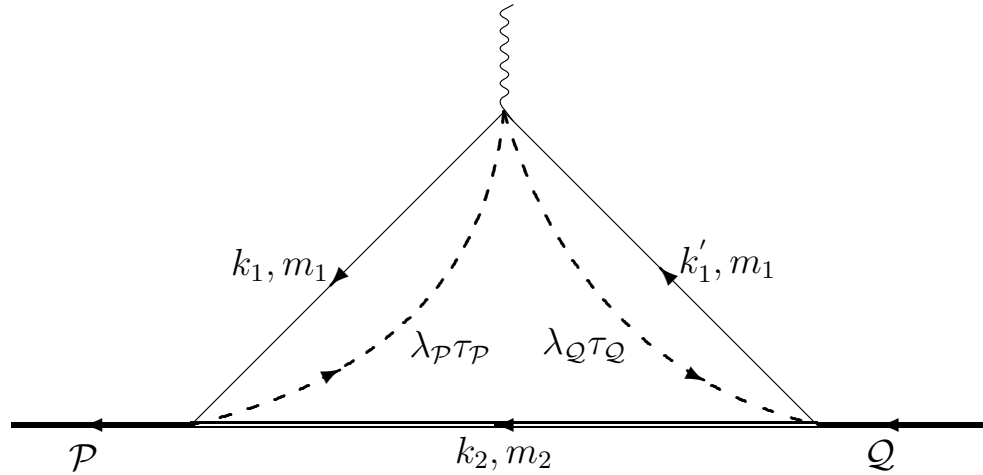


Fig. 1: The diagram of matrix element of the local current operator in the case of two relativistic spinor particles with arbitrary masses

On Fig. 1 the solid lines correspond to the constituents that carry the 4-momenta $k'_1, k_i, i = 1, 2$, while the dashed lines represent spurious quasiparticles. The composite-particle 4-velocities were chosen to be $\lambda_Q = (\lambda_Q^0; \boldsymbol{\lambda}_Q) = Q/\sqrt{Q^2} = Q/M_Q$, $\lambda_P = (\lambda_P^0; \boldsymbol{\lambda}_P) = P/\sqrt{P^2} = P/M_P$, where $M_Q^2 = s_q = Q^2 = (q_1 + q_2)^2$, $M_P^2 = s_p = P^2 = (p_1 + p_2)^2$.

Values of the trace in (8) with matrix $\hat{O} = \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu$ gives by expressions

$$\begin{aligned} & \text{tr}[\hat{O}^+ (\hat{k}_1 + m_1)(\hat{k}'_1 + m_1)\hat{O}(\hat{k}_2 - m_2)] = \\ & = -4(\tilde{a}m_2k_1k'_1 + \tilde{b}m_1k_1k_2 + \tilde{b}m_1k'_1k_2 + \tilde{a}m_2m_1^2), \end{aligned} \quad (10)$$

$$\tilde{a} = \begin{cases} 1, & \hat{O} = \gamma_5 \text{ (pseudoscalar);} \\ 4, & \hat{O} = \gamma_\mu \text{ (vector);} \\ 4, & \hat{O} = \gamma_5 \gamma_\mu \text{ (pseudovector);} \end{cases} \quad \tilde{b} = \begin{cases} 1, & \hat{O} = \gamma_5; \\ 2, & \hat{O} = \gamma_\mu; \\ -2, & \hat{O} = \gamma_5 \gamma_\mu, \end{cases} \quad (11)$$

which for $m_1 = m_2 = m$ coincides with the values of their analogs obtained in [Yu. D. Chernichenko, *Phys. At. Nucl.* **81**, 360 (2018); *Phys. At. Nucl.* **82**, 334 (2019)].

In order to perform integration with respect to $\mathbf{k}'_1, \mathbf{k}_1, \tau_Q$ and τ_P in expression (8), we make, in the integrals with respect to $\mathbf{k}'_1, \mathbf{k}_2$ and \mathbf{k}_1 , the pure Lorentz transformations (4) $L = \Lambda_{\lambda_Q}^{-1}$ and $L = \Lambda_{\lambda_P}^{-1}$, respectively – $\Lambda_{\lambda_Q}^{-1} Q = (M_Q; \mathbf{0})$ and $\Lambda_{\lambda_P}^{-1} P = (M_P; \mathbf{0})$ – and consider that Qk_2 and Pk_2 are Lorentz scalars – $Qk_2 = \Lambda_{\lambda_Q}^{-1}(Qk_2) = (\Lambda_{\lambda_Q}^{-1} Q)(\Lambda_{\lambda_Q}^{-1} k_2) = M_Q \Delta_{k_2, m_2 \lambda_Q}^0$ and $Pk_2 = M_P \Delta_{k_2, m_2 \lambda_P}^0$ – and that the integration measures, the total energy of interacting particles M_Q and the delta functions in (8) on the mass hyperboloid (9) are invariant under the Lorentz transformations $\Lambda_{\lambda_{Q(P)}}^{-1}$ [Yu. D. Chernichenko, Phys. Atom. Nucl. **77**, 229 (2014); **78**, 201 (2015)]:

$$d\Omega_{\mathbf{k}_i} = m_i d\mathbf{k}_i / k_{i0} = d\Omega_{\Delta_{k_i, m_i \lambda_{Q(P)}}} = m_i d\Delta_{k_i, m_i \lambda_{Q(P)}} / \Delta_{k_i, m_i \lambda_{Q(P)}}^0, i = 1, 2,$$

$$\sqrt{s_k} = \Lambda_{\lambda_{Q(P)}}^{-1} \sqrt{s_k} = \sqrt{s_{\Delta_{k_2, m_2 \lambda_{Q(P)}}}} = \sqrt{m_1^2 + \Delta_{k_2, m_2 \lambda_{Q(P)}}^2} + \sqrt{m_2^2 + \Delta_{k_2, m_2 \lambda_{Q(P)}}^2}.$$

We thus recast expression in (8) into the form

$$\begin{aligned}
& \langle \mathcal{P} | J(0) | \mathcal{Q} \rangle = \tag{12} \\
& = \frac{z_1}{\sqrt{M_{\mathcal{P}} M_{\mathcal{Q}}} (4\pi)^3} \int \frac{d\Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}}{\sqrt{m_2^2 + \Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}^2} \sqrt{m_1^2 + \Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}^2}} \times \\
& \quad \times \frac{\Gamma_{M_{\mathcal{P}}}^*(\Delta_{k_2, m_2 \lambda_{\mathcal{P}}})}{\sqrt{m_1^2 + \Delta_{k_2, m_2 \lambda_{\mathcal{P}}}^2} \left(M_{\mathcal{P}} - \sqrt{s \Delta_{k_2, m_2 \lambda_{\mathcal{P}}}^2} - i\varepsilon \right)} \times \\
& \quad \times \hat{B}(\Delta_{k_2, m_2 \lambda_{\mathcal{P}}}, \Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}, t) \frac{\Gamma_{M_{\mathcal{Q}}}(\Delta_{k_2, m_2 \lambda_{\mathcal{Q}}})}{\left(M_{\mathcal{Q}} - \sqrt{s \Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}^2} + i\varepsilon \right)} + (1 \leftrightarrow 2).
\end{aligned}$$

Here we introduced the notations

$$\Gamma_{M_{\mathcal{P}}}(\mathcal{P}k_2) = \Gamma_{M_{\mathcal{P}}}(\Delta_{k_2, m_2 \lambda_{\mathcal{P}}}), \quad \Gamma_{M_{\mathcal{Q}}}(\mathcal{Q}k_2) = \Gamma_{M_{\mathcal{Q}}}(\Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}),$$

and $\hat{B}(\Delta_{k_2, m_2 \lambda_{\mathcal{P}}}, \Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}, t)$ gives by expression ($M_{\mathcal{Q}} = M_{\mathcal{P}} = M$)

$$\begin{aligned}
\hat{B}(\Delta_{k_2, m_2 \lambda_{\mathcal{P}}}, \Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}, t) = & 2 \left[4\tilde{a}m_2m_1^2 - 2\tilde{b}m_1 (m_1^2 + m_2^2) + \right. \\
& + \left(\tilde{b}m_1 - \tilde{a}m_2 \right) \left(s_{\Delta_{k_2, m_2 \lambda_{\mathcal{P}}}} + s_{\Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}} \right) + \\
& \left. + 2\tilde{a}m_2 \left(1 - \frac{t}{2M^2} \right) \sqrt{s_{\Delta_{k_2, m_2 \lambda_{\mathcal{P}}}} s_{\Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}}} \right], \tag{13}
\end{aligned}$$

where $t = (\mathcal{P} - \mathcal{Q})^2 = -Q^2$ is the square of the 4-momentum transfer and we used correlations

$$\begin{aligned}
2k_1k_2 &= s_{\Delta_{k_2, m_2 \lambda_{\mathcal{P}}}} - m_1^2 - m_2^2, \quad 2k'_1k_2 = s_{\Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}} - m_1^2 - m_2^2, \\
2k_1k'_1 &= 2m_1^2 - s_{\Delta_{k_2, m_2 \lambda_{\mathcal{P}}}} - s_{\Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}} + \\
& + 2 \left(1 - \frac{t}{2M^2} \right) \sqrt{s_{\Delta_{k_2, m_2 \lambda_{\mathcal{P}}}} s_{\Delta_{k_2, m_2 \lambda_{\mathcal{Q}}}}},
\end{aligned}$$

which follow from laws of the conservation

$$-Q + k'_1 + k_2 - \lambda_{\mathcal{Q}}\tau_{\mathcal{Q}} = 0, \quad \mathcal{P} - k_1 - k_2 + \lambda_{\mathcal{P}}\tau_{\mathcal{P}} = 0.$$

Farther, by following Refs. [Yu. D. Chernichenko, *Phys. Atom. Nucl.* **77**, 229 (2014); **78**, 201 (2015)], in expression (12) we shall perform the change of variables in the form

$$\Delta_{k_2, m_2 \lambda_Q} = g' \Delta_{k', m' \lambda_Q} \sqrt{\frac{4\mu^2 + \Delta_{k', m' \lambda_Q}^2}{m'^2 + \Delta_{k', m' \lambda_Q}^2}}, \quad (14)$$

and shall take into account correlations

$$\sqrt{m_1^2 + \Delta_{k_2, m_2 \lambda_{Q(\mathcal{P})}}^2} = g' f_+^{-1}(\Delta_{k', m' \lambda_{Q(\mathcal{P})}}), \quad \sqrt{m_2^2 + \Delta_{k_2, m_2 \lambda_{Q(\mathcal{P})}}^2} = g' f_-^{-1}(\Delta_{k', m' \lambda_{Q(\mathcal{P})}}),$$

$$m_1 = m' (g' + \sqrt{g'^2 - 1}) > m_2 = m' (g' - \sqrt{g'^2 - 1});$$

$$\sqrt{m_1^2 + \Delta_{k_2, m_2 \lambda_{Q(\mathcal{P})}}^2} = g' f_-^{-1}(\Delta_{k', m' \lambda_{Q(\mathcal{P})}}), \quad \sqrt{m_2^2 + \Delta_{k_2, m_2 \lambda_{Q(\mathcal{P})}}^2} = g' f_+^{-1}(\Delta_{k', m' \lambda_{Q(\mathcal{P})}}),$$

$$m_1 = m' (g' - \sqrt{g'^2 - 1}) < m_2 = m' (g' + \sqrt{g'^2 - 1});$$

$$M_Q = 2g' \Delta_{q', m' \lambda_Q}^0, \quad M_{\mathcal{P}} = 2g' \Delta_{p', m' \lambda_{\mathcal{P}}}^0, \quad \sqrt{s \Delta_{k_2, m_2 \lambda_{Q(\mathcal{P})}}^2} = 2g' \Delta_{k', m' \lambda_{Q(\mathcal{P})}}^0,$$

$$\frac{d\Delta_{k_2, m_2 \lambda_Q}}{\sqrt{m_2^2 + \Delta_{k_2, m_2 \lambda_Q}^2} \sqrt{m_1^2 + \Delta_{k_2, m_2 \lambda_Q}^2}} = g' \frac{d\Delta_{k', m' \lambda_Q}}{\Delta_{k', m' \lambda_Q}^0} f(\Delta_{k', m' \lambda_Q}),$$

where

$$f_{\pm}(\Delta_{k',m'}\lambda_{\mathcal{Q}(\mathcal{P})}) = \frac{\sqrt{m'^2 + \Delta_{k',m'}^2\lambda_{\mathcal{Q}(\mathcal{P})}}}{m'^2 + \Delta_{k',m'}^2\lambda_{\mathcal{Q}(\mathcal{P})} \pm m'\sqrt{m'^2 - 4\mu^2}},$$
$$f(\Delta_{k',m'}\lambda_{\mathcal{Q}(\mathcal{P})}) = \frac{\sqrt{4\mu^2 + \Delta_{k',m'}^2\lambda_{\mathcal{Q}(\mathcal{P})}}}{m'^2 + \Delta_{k',m'}^2\lambda_{\mathcal{Q}(\mathcal{P})}}.$$

Then expression in (12) takes the form ($M_{\mathcal{Q}} = M_{\mathcal{P}} = M$)

$$\begin{aligned}
\langle \mathcal{P} | J(0) | \mathcal{Q} \rangle = & \frac{8m'^3(z_1 + z_2)}{g' M (2\pi)^3} \int d\Omega_{\Delta_{k', m' \lambda_{\mathcal{Q}}}} \Psi_M^*(\Delta_{k', m' \lambda_{\mathcal{P}}}) \times \\
& \times \left\{ \frac{f_+(\Delta_{k', m' \lambda_{\mathcal{P}}}) + f_-(\Delta_{k', m' \lambda_{\mathcal{P}}})}{2f(\Delta_{k', m' \lambda_{\mathcal{P}}})} \left[\tilde{a} + \tilde{b} - 2\tilde{b}g'^2 + \right. \right. \\
& \quad + g'^2 (\tilde{b} - \tilde{a}) \frac{\Delta_{k', m' \lambda_{\mathcal{P}}}^{02} + \Delta_{k', m' \lambda_{\mathcal{Q}}}^{02}}{m'^2} + \\
& \quad \left. \left. + 2\tilde{a}g'^2 \left(1 - \frac{t}{2M^2} \right) \frac{\Delta_{k', m' \lambda_{\mathcal{P}}}^0 \Delta_{k', m' \lambda_{\mathcal{Q}}}^0}{m'^2} \right] + \right. \\
& \quad + \frac{\sqrt{g'^2 - 1} \left(f_+(\Delta_{k', m' \lambda_{\mathcal{P}}}) - f_-(\Delta_{k', m' \lambda_{\mathcal{P}}}) \right)}{2g' f(\Delta_{k', m' \lambda_{\mathcal{P}}})} \times \\
& \quad \times \left[\tilde{a} + \tilde{b} - 2\tilde{b}g'^2 + g'^2 (\tilde{b} + \tilde{a}) \frac{\Delta_{k', m' \lambda_{\mathcal{P}}}^{02} + \Delta_{k', m' \lambda_{\mathcal{Q}}}^{02}}{m'^2} - \right. \\
& \quad \left. \left. - 2\tilde{a}g'^2 \left(1 - \frac{t}{2M^2} \right) \frac{\Delta_{k', m' \lambda_{\mathcal{P}}}^0 \Delta_{k', m' \lambda_{\mathcal{Q}}}^0}{m'^2} \right] \right\} \Psi_M(\Delta_{k', m' \lambda_{\mathcal{Q}}}), \tag{15}
\end{aligned}$$

where we have defined the wave function of system in momentum space as

$$\Psi_M(\Delta_{k',m'\lambda_Q}) = \frac{f(\Delta_{k',m'\lambda_Q})\Gamma_{M_Q}(\Delta_{k',m'\lambda_Q})}{2^{3/2}\sqrt{m'}(2\Delta_{q',m'\lambda_Q}^0 - 2\Delta_{k',m'\lambda_Q}^0 + i\varepsilon)},$$

and $d\Omega_{\Delta_{k',m'\lambda_Q}} = m'd\Delta_{k',m'\lambda_Q}/\Delta_{k',m'\lambda_Q}^0$ is the relativistic three-dimensional volume element in Lobachevsky space. All 4-momenta now belong to the upper sheet of the mass hyperboloid (5). This sheet is embedded in 4-dimensional momentum space. It serves as a model of the Lobachevsky space momentum. On the surface of the mass hyperboloid (5) the Lorentz group is its motion group. The Lorentz transformations proper mean translations in Lobachevsky space. The role of plane waves corresponding to these translations is played by relativistic plane waves

$$\xi(\Delta_{k',m'\lambda_Q}, \mathbf{r}) = \left(\frac{\Delta_{k',m'\lambda_Q}^0 - \Delta_{k',m'\lambda_Q} \cdot \mathbf{n}}{m'} \right)^{-1-ir/\lambda'}. \quad (16)$$

The functions in (16) correspond to the main series of unitary irreducible representations of the Lorentz group and satisfy the conditions of completeness and orthogonality

$$\begin{aligned} \frac{1}{(2\pi)^3} \int d\Omega_{\Delta_{k',m'\lambda_Q}} \xi(\Delta_{k',m'\lambda_Q}, \mathbf{r}) \xi^*(\Delta_{k',m'\lambda_Q}, \mathbf{r}') &= \delta(\mathbf{r}' - \mathbf{r}), \\ \frac{1}{(2\pi)^3} \int d\mathbf{r} \xi(\Delta_{p',m'\lambda_Q}, \mathbf{r}) \xi^*(\Delta_{k',m'\lambda_Q}, \mathbf{r}) &= \\ &= \frac{\Delta_{p',m'\lambda_Q}^0}{m'} \delta(\Delta_{k',m'\lambda_Q} - \Delta_{p',m'\lambda_Q}), \end{aligned} \quad (17)$$

and the finite-difference equation

$$\left(\hat{H}_0 - 2\Delta_{k',m'\lambda_Q}^0 \right) \xi(\Delta_{k',m'\lambda_Q}, \mathbf{r}) = 0. \quad (18)$$

The vector $\Delta_{k',m'\lambda_{\mathcal{P}}}$ from the Lobachevsky space

$$\begin{aligned} \Delta_{k',m'\lambda_{\mathcal{P}}} &= \mathbf{k}'(-)m'\lambda_{\mathcal{P}} = \Lambda_{\lambda_{\mathcal{P}}}^{-1} \mathbf{k}' = \Lambda_{\lambda_{\mathcal{P}}}^{-1} \Lambda_{\lambda_{\mathcal{Q}}} \Delta_{k',m'\lambda_{\mathcal{Q}}} = \\ &= V(\Lambda_{\lambda_{\mathcal{Q}}}, \mathcal{P}) \left(\Lambda_{\Delta_{\mathcal{P},\mathcal{Q}}}^{-1} \Delta_{k',m'\lambda_{\mathcal{Q}}} \right) = V(\Lambda_{\lambda_{\mathcal{Q}}}, \mathcal{P}) \Delta_{k',m'\lambda_{\mathcal{Q}}}(-) \frac{m'}{M} \Delta_{\mathcal{P},\mathcal{Q}}, \end{aligned} \quad (19)$$

where $V(\Lambda_{\lambda_{\mathcal{Q}}}, \mathcal{P}) = \Lambda_{\lambda_{\mathcal{P}}}^{-1} \Lambda_{\lambda_{\mathcal{Q}}} \Lambda_{\Delta_{\mathcal{P},\mathcal{Q}}}$ is Wigner's rotation matrix, $\Delta_{\mathcal{P},\mathcal{Q}} = \Lambda_{\lambda_{\mathcal{Q}}}^{-1} \mathcal{P}$ is the 4-momentum transfer:

$$\begin{aligned} \Delta_{\mathcal{P},\mathcal{Q}} &= \Lambda_{\mathcal{Q}}^{-1} \mathcal{P} = \mathcal{P} - \frac{\mathcal{Q}}{M} \left(\mathcal{P}_0 - \frac{\mathcal{P} \cdot \mathcal{Q}}{Q_0 + M} \right) = M \sinh \chi_{\Delta} \mathbf{n}_{\Delta}, \\ \Delta_{\mathcal{P},\mathcal{Q}}^0 &= (\Lambda_{\mathcal{Q}}^{-1} \mathcal{P})^0 = \frac{\mathcal{P}_0 Q_0 - \mathcal{P} \cdot \mathcal{Q}}{M} = \frac{\mathcal{P} Q}{M} = M \cosh \chi_{\Delta}, \\ \mathcal{P} &= M \sinh \chi_{\mathcal{P}} \mathbf{n}_{\mathcal{P}}, \quad \mathcal{Q} = M \sinh \chi_{\mathcal{Q}} \mathbf{n}_{\mathcal{Q}}, \quad \mathcal{P}_0 = M \cosh \chi_{\mathcal{P}}, \\ Q_0 &= M \cosh \chi_{\mathcal{Q}}, \quad |\mathbf{n}_{\mathcal{P}}| = |\mathbf{n}_{\mathcal{Q}}| = |\mathbf{n}_{\Delta}| = 1, \quad \Delta_{\mathcal{P},\mathcal{Q}}^{02} - \Delta_{\mathcal{P},\mathcal{Q}}^2 = M^2, \end{aligned} \quad (20)$$

where $\chi_{\Delta}, \chi_{\mathcal{P}}, \chi_{\mathcal{Q}}$ are the respective rapidities, and

$$Q^2 = -t = -2M^2 + 2M \sqrt{M^2 + \Delta_{\mathcal{P},\mathcal{Q}}^2} = 2M^2 (\cosh \chi_{\Delta} - 1). \quad (21)$$

From Eqs. (19)–(21) we have

$$\Delta_{k',m'\lambda_{\mathcal{P}}}^0 \approx \frac{m'^2 \Delta_{\mathcal{P},\mathcal{Q}}^0}{2M \Delta_{k',m'\lambda_{\mathcal{Q}}}^0}, \quad (22)$$

and the factors

$$\frac{f_+(\Delta_{k',m'\lambda_{\mathcal{P}}}) \pm f_-(\Delta_{k',m'\lambda_{\mathcal{P}}})}{2f(\Delta_{k',m'\lambda_{\mathcal{P}}})},$$
$$\frac{m' \sqrt{m'^2 - 4\mu^2}}{m'^2 + \Delta_{k',m'\lambda_{\mathcal{P}}}^2} < 1, \quad \frac{m'^2 - 4\mu^2}{4\mu^2 + \Delta_{k',m'\lambda_{\mathcal{P}}}^2} < 1,$$

can be simplified to the form

$$\begin{aligned}
& \frac{f_+(\Delta_{k',m'}\lambda_{\mathcal{P}}) + f_-(\Delta_{k',m'}\lambda_{\mathcal{P}})}{2f(\Delta_{k',m'}\lambda_{\mathcal{P}})} \approx \\
& \approx 1 + \frac{8M^4 (g'^2 - 1)}{g'^2 (2M^2 - t)^2} \frac{\Delta_{k',m'}^0 \Delta_{k',m'}^0}{m'^2}, \\
& \frac{f_+(\Delta_{k',m'}\lambda_{\mathcal{P}}) - f_-(\Delta_{k',m'}\lambda_{\mathcal{P}})}{2f(\Delta_{k',m'}\lambda_{\mathcal{P}})} \approx -\frac{16M^4 \sqrt{g'^2 - 1}}{g' (2M^2 - t)^2} \frac{\Delta_{k',m'}^0 \Delta_{k',m'}^0}{m'^2}.
\end{aligned} \tag{23}$$

The elastic form factor $F(t)$ in the case of scalar current we define

$$F(t) = G(\chi_\Delta) \langle \mathcal{P} | J(0) | \mathcal{Q} \rangle, \quad (24)$$

where

$$G(\chi_\Delta) = \frac{\chi_\Delta}{\sinh \chi_\Delta} \quad (25)$$

is the relativistic geometric factor in the RQP approach [V. G. Kadyshevsky, *Nucl. Phys. B* **6**, 125 (1968)]. The factor in (25) serves the measure of the contribution of relativistic effects caused by the dynamics of quarks [N. B. Skachkov, *Theor. Math. Phys.* **25**, 1154 (1975)]. Thereby, the elastic form factor $F(t)$ can consider as a function of the invariant variable $\Delta_{\mathcal{P}, \mathcal{Q}}^2$, which is the square of the 3-momentum transfer in the Lobachevsky space. Taking into consideration expressions (15), (19)–(25), the form factor $F(t)$ then represents a convolution of covariant wave RQP-functions in this space:

$$\begin{aligned}
F(t) = & \frac{8m'^3(z_1 + z_2)G(\chi_\Delta)}{g'M(2\pi)^3} \int d\Omega_{\Delta_{k',m'\lambda_{\mathcal{Q}}}} \Psi_M^*(\Delta_{k',m'\lambda_{\mathcal{P}}}) \times \\
& \times \left\{ \tilde{a} + \tilde{b} - 2\tilde{b}g'^2 + g'^2 (\tilde{b} - \tilde{a}) \frac{\Delta_{k',m'\lambda_{\mathcal{P}}}^{02} + \Delta_{k',m'\lambda_{\mathcal{Q}}}^{02}}{m'^2} + \right. \\
& \quad + 2\tilde{a}g'^2 \left(1 - \frac{t}{2M^2} \right) \frac{\Delta_{k',m'\lambda_{\mathcal{P}}}^0 \Delta_{k',m'\lambda_{\mathcal{Q}}}^0}{m'^2} + \\
& \quad + \frac{8M^4 (g'^2 - 1)}{g'^2 (2M^2 - t)^2} \left[- (\tilde{a} + \tilde{b} - 2\tilde{b}g'^2) \frac{\Delta_{k',m'\lambda_{\mathcal{P}}}^0 \Delta_{k',m'\lambda_{\mathcal{Q}}}^0}{m'^2} + \right. \\
& \quad \left. + 6\tilde{a}g'^2 \left(1 - \frac{t}{2M^2} \right) \frac{\Delta_{k',m'\lambda_{\mathcal{P}}}^{02} \Delta_{k',m'\lambda_{\mathcal{Q}}}^{02}}{m'^4} - \right. \\
& \quad \left. \left. - g'^2 (3\tilde{a} + \tilde{b}) \frac{\Delta_{k',m'\lambda_{\mathcal{P}}}^{03} \Delta_{k',m'\lambda_{\mathcal{Q}}}^0 + \Delta_{k',m'\lambda_{\mathcal{P}}}^0 \Delta_{k',m'\lambda_{\mathcal{Q}}}^{03}}{m'^4} \right] \right\} \Psi_M(\Delta_{k',m'\lambda_{\mathcal{Q}}}).
\end{aligned} \tag{26}$$

Consequently, by using the Shapiro transformations [I. S. Shapiro, *Sov. Phys. Dokl.* **1**, 91 (1956); *Sov. Phys. JETP* **16**, 1219 (1963)]

$$\psi_M(\mathbf{r}) = \frac{1}{(2\pi)^3} \int d\Omega_{\Delta_{k',m'\lambda_Q}} \xi(\Delta_{k',m'\lambda_Q}, \mathbf{r}) \Psi_M(\Delta_{k',m'\lambda_Q}),$$

$$\Psi_M(\Delta_{k',m'\lambda_Q}) = \int d\mathbf{r} \xi^*(\Delta_{k',m'\lambda_Q}, \mathbf{r}) \psi_M(\mathbf{r}),$$

the addition theorem for relativistic plane waves (16) [V. G. Kadyshevsky, R. M. Mir-Kasimov, and N. B. Skachkov, *Fiz. Elem. Chastits At. Yadra* **2**, 635 (1972)],

$$\int d\omega_n \xi(\Delta_{p',m'\lambda_Q}(-)\Delta_{k',m'\lambda_Q}, \mathbf{r}) = \int d\omega_n \xi(\Delta_{p',m'\lambda_Q}, \mathbf{r}) \xi^*(\Delta_{k',m'\lambda_Q}, \mathbf{r}),$$

the completeness condition in (17), the equation (18), and the fact that the free-Hamiltonian operator (8) is Hermitian, we can represent the form factor (26) as the relativistic Fourier transform of the covariant wave RQP-functions in the \mathbf{r} -representation:

$$\begin{aligned}
F(t) = & \frac{8m'^3(z_1 + z_2)G(\chi_\Delta)}{g'M} \int d\mathbf{r}\xi^* \left(\frac{m'}{M} \Delta_{\mathcal{P}, \mathcal{Q}, \mathbf{r}} \right) \times \\
& \times \left\{ \left(\tilde{a} + \tilde{b} - 2\tilde{b}g'^2 \right) |\psi_M(\mathbf{r})|^2 + \right. \\
& + 2g'^2 \left(\tilde{b} - \tilde{a} \right) \operatorname{Re} \left[\psi_M^*(\mathbf{r}) + \left(\frac{\hat{H}_0}{2m'} \right)^2 \psi_M(\mathbf{r}) \right] + \\
& + 2\tilde{a}g'^2 \left(1 - \frac{t}{2M^2} \right) \left| \frac{\hat{H}_0}{2m'} \psi_M(\mathbf{r}) \right|^2 + \\
& + \frac{8M^4 (g'^2 - 1)}{g'^2 (2M^2 - t)^2} \left[- \left(\tilde{a} + \tilde{b} - 2\tilde{b}g'^2 \right) \left| \frac{\hat{H}_0}{2m'} \psi_M(\mathbf{r}) \right|^2 + \right. \\
& + 6\tilde{a}g'^2 \left(1 - \frac{t}{2M^2} \right) \left| \left(\frac{\hat{H}_0}{2m'} \right)^2 \psi_M(\mathbf{r}) \right|^2 - \\
& \left. \left. - 2g'^2 \left(3\tilde{a} + \tilde{b} \right) \operatorname{Re} \left[\left(\frac{\hat{H}_0}{2m'} \psi_M(\mathbf{r}) \right)^* \left(\frac{\hat{H}_0}{2m'} \right)^3 \psi_M(\mathbf{r}) \right] \right] \right\}.
\end{aligned} \tag{27}$$

Similar approach was used to get the expressions of form factors for scalar and vector currents as for the system of two spinless particles with arbitrary masses [Yu. D. Chernichenko, *Phys. At. Nucl.* **77**, 229 (2014); **78**, 201 (2015)], so and for the system of spinor particles with equal masses [Yu. D. Chernichenko, *Phys. At. Nucl.* **81**, 360 (2018); *Phys. At. Nucl.* **82**, 334 (2019)].

For s -state ($\ell = 0$) of the composite system from Eq. (27) we have

$$F_{\ell=0}(t) = \frac{32\pi m'^2(z_1 + z_2)G(\chi_\Delta)}{g'M} \int_0^\infty d\rho \frac{\sin(\rho\chi_\Delta)}{\rho \sinh \chi_\Delta} \left\{ \tilde{R}_1(\rho, \chi_n) - \right. \quad (28)$$

$$\left. - \frac{t}{2M^2} \tilde{R}_0(\rho, \chi_n) + \frac{8M^4(g'^2 - 1)}{g'^2(2M^2 - t)^2} \left[\tilde{R}_2(\rho, \chi_n) - \frac{t}{2M^2} \tilde{R}_3(\rho, \chi_n) \right] \right\},$$

where $\rho = rm'$, $M = M_n = 2m'g' \cosh \chi_n$, and were introduced the notations:

$$\begin{aligned}
\tilde{R}_0(\rho, \chi_n) &= 2\tilde{a}g'^2 |\hat{H}_{0,\ell=0}^{\text{rad}} \varphi_0(\rho, \chi_n)|^2, \\
\tilde{R}_1(\rho, \chi_n) &= (\tilde{a} + \tilde{b} - 2\tilde{b}g'^2) |\varphi_0(\rho, \chi_n)|^2 + 2\tilde{a}g'^2 |\hat{H}_{0,\ell=0}^{\text{rad}} \varphi_0(\rho, \chi_n)|^2 + \\
&\quad + 2g'^2 (\tilde{b} - \tilde{a}) \text{Re} \left[\varphi_0^*(\rho, \chi_n) (\hat{H}_{0,\ell=0}^{\text{rad}})^2 \varphi_0(\rho, \chi_n) \right], \\
\tilde{R}_2(\rho, \chi_n) &= -(\tilde{a} + \tilde{b} - 2\tilde{b}g'^2) |\hat{H}_{0,\ell=0}^{\text{rad}} \varphi_0(\rho, \chi_n)|^2 + \\
&\quad + 6\tilde{a}g'^2 |(\hat{H}_{0,\ell=0}^{\text{rad}})^2 \varphi_0(\rho, \chi_n)|^2 - \\
&\quad - 2g'^2 (3\tilde{a} + \tilde{b}) \text{Re} \left[(\hat{H}_{0,\ell=0}^{\text{rad}} \varphi_0(\rho, \chi_n))^* (\hat{H}_{0,\ell=0}^{\text{rad}})^3 \varphi_0(\rho, \chi_n) \right], \\
\tilde{R}_3(\rho, \chi_n) &= 6\tilde{a}g'^2 |(\hat{H}_{0,\ell=0}^{\text{rad}})^2 \varphi_0(\rho, \chi_n)|^2.
\end{aligned} \tag{29}$$

4. Form factor and root-mean-square radius for Coulomb interaction

The expression for the invariant root-mean-square radius (r.m.s.) of a composite system in terms of the wave function *s*-state according to (28) and (29) has the form

$$\begin{aligned} \langle r_0^2 \rangle &= \frac{6\partial F_{\ell=0}(t)/\partial t|_{t=0}}{F_{\ell=0}(0)} = & (30) \\ &= \frac{1}{M_n^2} \left\{ 2 + \frac{\int_0^\infty d\rho \left[\tilde{R}_{01}(\rho, \chi_n) + \frac{2(g'^2 - 1)}{g'^2} \tilde{R}_{23}(\rho, \chi_n) \right]}{\int_0^\infty d\rho \left[\tilde{R}_1(\rho, \chi_n) + \frac{2(g'^2 - 1)}{g'^2} \tilde{R}_2(\rho, \chi_n) \right]} \right\}, \end{aligned}$$

$$\begin{aligned} \tilde{R}_{01}(\rho, \chi_n) &= \rho^2 \tilde{R}_1(\rho, \chi_n) - 3\tilde{R}_0(\rho, \chi_n), & (31) \\ \tilde{R}_{23}(\rho, \chi_n) &= (\rho^2 + 6) \tilde{R}_2(\rho, \chi_n) - 3\tilde{R}_3(\rho, \chi_n). \end{aligned}$$

As example, we consider the form factor (28) and the r.m.s. (30) of meson in the case of the Coulomb field

$$V_{\text{Coul}} = -\frac{\alpha_s}{r}, \alpha_s > 0. \quad (32)$$

The expression for the radial wave RQP function of the s -state of composite system of two relativistic particles with arbitrary masses, spin 1/2 and Coulomb interaction (32), which does not contain i -periodic constants, can be obtained directly from the solution [Yu. D. Chernichenko, Phys. At. Nucl. **84**, 339 (2021)]

$$\begin{aligned} \varphi_0(\rho, \chi') &= 2 C_0(\chi') e^{iB'\chi'} \sinh [\pi (\rho - \tilde{\rho}')] \times \\ &\times \int_{-\infty}^{\infty} dx \frac{\exp \left\{ \frac{i\tilde{\alpha}'_s a'}{4} \sinh x + [1 + i(\rho - \tilde{\rho}')] x \right\}}{(e^x + e^{x'})^2} \left[\frac{e^x + e^{-x'}}{e^x + e^{x'}} \right]^{-1+iB'} \end{aligned} \quad (33)$$

at $\chi' = i\kappa$ ($0 < \kappa < \pi/2$), where the rapidity κ is determined from

$$\begin{aligned} \frac{\tilde{\alpha}'_s (a' \cos^2 \kappa + b')}{4 \sin \kappa} &= n, \quad n = 1, 2, \dots, \\ \tilde{\rho}' &= \frac{\tilde{\alpha}'_s a' \cosh \chi'}{4}, \quad B' = \frac{\tilde{\alpha}'_s (a' \cosh^2 \chi' + b')}{4 \sinh \chi'}, \quad \tilde{\alpha}'_s = m' \alpha_s. \end{aligned} \quad (34)$$

Expressions (33) and (34) were found in [Yu. D. Chernichenko, *Phys. At. Nucl.* **84**, 339 (2021)] when solving the scattering problem based on the integral form of RQP equation in the \mathbf{r} -representation for the radial wave RQP function of the s -state of a composite system of two fermions of arbitrary masses with Coulomb interaction (32).

It is obvious that the integral function in the solution (33) at $\chi' = i\kappa$ and (34) has a pole of the order $n + 1$ at $x = i\kappa + i\pi$. Therefore, the integral in the solution (33) can be calculated using the basic theorem of residue theory, where the integration is performed in the complex z -plane along the contour C in Fig. 2. In particular, for the level $n = 1$ of the s -state of the composite system we have

$$\varphi_0^{(1)}(\rho, \kappa_1) = C_0^{(1)}(\kappa_1)(\rho - \rho'_{\kappa_1})e^{-(\rho - \rho'_{\kappa_1})\kappa_1}, \quad (35)$$

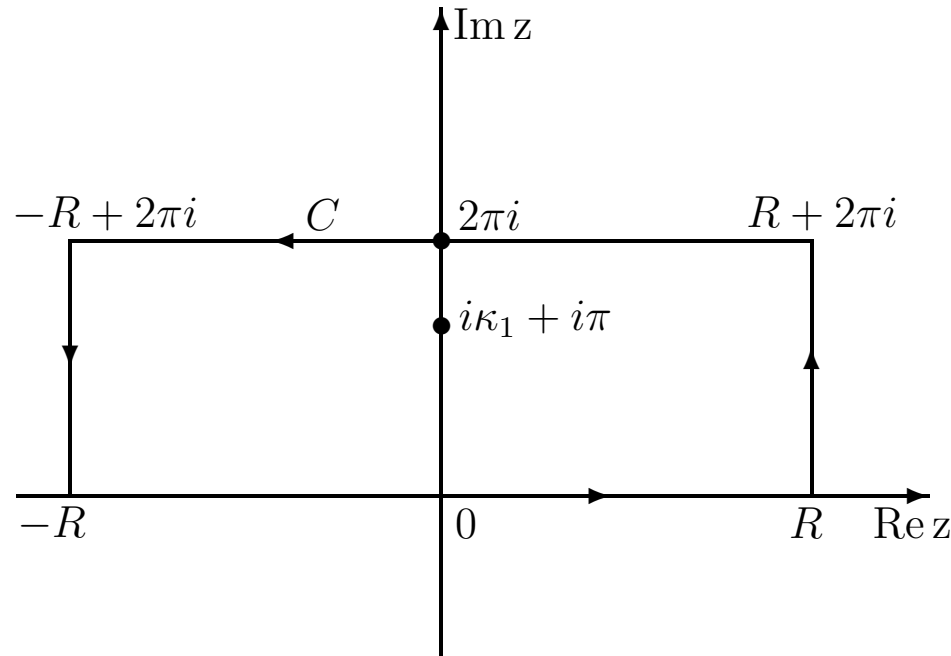


Fig. 2: The integration contour C .

where

$$C_0^{(1)}(\kappa_1) = 2\pi C_0(\kappa_1) \exp \left[\frac{\tilde{\alpha}'_s a'}{4} \sin \kappa_1 - \frac{\kappa_1 \rho'_{\kappa_1}}{2} \right], \quad \rho'_{\kappa_1} = \frac{\tilde{\alpha}'_s a'}{2} \cos \kappa_1, \quad (36)$$

the rapidity κ_1 is determined from (34) at $n = 1$, and the normalization factor $|C_0^{(1)}(\kappa_1)|^2$ gives by expression

$$|C_0^{(1)}(\kappa_1)|^2 = m' \kappa_1^3 e^{-2\kappa_1 \rho'_{\kappa_1}} / \pi (2\kappa_1^2 \rho'^2_{\kappa_1} - 2\kappa_1 \rho'_{\kappa_1} + 1), \quad (37)$$

and it can be found from the normalization condition

$$4\pi \int_0^{\infty} dr |\varphi_0^{(1)}(r, \kappa_1)|^2 = 1.$$

Then, in accordance with expressions (29)–(37), the form factor (28) and the r.m.s. (30) with interaction (32) for the ground level of bound s -state with the energy M_1 (in units of GeV) can be represented in the form

$$\begin{aligned}
F_{\ell=0,n=1}^{\text{Coul}}(t) &= \frac{32(z_1 + z_2)m'^3 \kappa_1^3 G(\chi_\Delta)}{M_1 g' (2\kappa_1^2 \rho'_{\kappa_1} - 2\kappa_1 \rho'_{\kappa_1} + 1) \sinh \chi_\Delta} \times \\
&\times \left\{ \frac{4\kappa_1 \tilde{A}_1 \chi_\Delta}{(\chi_\Delta^2 + 4\kappa_1^2)^2} + \frac{\tilde{B}_1 \chi_\Delta}{\chi_\Delta^2 + 4\kappa_1^2} + \tilde{C}_1 \arctan \frac{\chi_\Delta}{2\kappa_1} - \right. \\
&- \frac{t}{2M_1^2} \left(\frac{4\kappa_1 \tilde{A}_0 \chi_\Delta}{(\chi_\Delta^2 + 4\kappa_1^2)^2} + \frac{\tilde{B}_0 \chi_\Delta}{\chi_\Delta^2 + 4\kappa_1^2} + \tilde{C}_0 \arctan \frac{\chi_\Delta}{2\kappa_1} \right) + \\
&+ \frac{8M_1^4 (g'^2 - 1)}{g'^2 (2M_1^2 - t)^2} \left[\frac{4\kappa_1 \tilde{A}_2 \chi_\Delta}{(\chi_\Delta^2 + 4\kappa_1^2)^2} + \frac{\tilde{B}_2 \chi_\Delta}{\chi_\Delta^2 + 4\kappa_1^2} + \tilde{C}_2 \arctan \frac{\chi_\Delta}{2\kappa_1} - \right. \\
&\left. \left. - \frac{t}{2M_1^2} \left(\frac{4\kappa_1 \tilde{A}_3 \chi_\Delta}{(\chi_\Delta^2 + 4\kappa_1^2)^2} + \frac{\tilde{B}_3 \chi_\Delta}{\chi_\Delta^2 + 4\kappa_1^2} + \tilde{C}_3 \arctan \frac{\chi_\Delta}{2\kappa_1} \right) \right] \right\}, \tag{38}
\end{aligned}$$

$$\begin{aligned}
& \langle r_0^2 \rangle_{\ell=0, n=1}^{\text{Coul}} \\
& = \frac{0.03894}{M_1^2} \left\{ 2 + \frac{1}{2\kappa_1^2} \left[1 + \frac{(1 + \tan^2 \kappa_1) \hat{E}_1 + \frac{2(g'^2 - 1)}{g'^2} \hat{E}_2}{(1 + \tan^2 \kappa_1) \hat{D}_1 + \frac{2(g'^2 - 1)}{g'^2} \hat{D}_2} \right] - \right. \\
& \quad \left. - \frac{3 \left[(1 + \tan^2 \kappa_1) \hat{D}_0 + \frac{2(g'^2 - 1)}{g'^2} (\hat{D}_3 - 2\hat{D}_2) \right]}{(1 + \tan^2 \kappa_1) \hat{D}_1 + \frac{2(g'^2 - 1)}{g'^2} \hat{D}_2} \right\} \text{fm}^2,
\end{aligned} \tag{39}$$

where were introduced the notations

$$\begin{aligned}
\tilde{A}_0 &= 2\tilde{a}g'^2 \cos^2 \kappa_1, \tilde{B}_0 = 4\tilde{a}g'^2 \cos \kappa_1 \sin \kappa_1 \frac{b' - a' \cos^2 \kappa_1}{b' + a' \cos^2 \kappa_1}, \\
\tilde{C}_0 &= 2\tilde{a}g'^2 \sin^2 \kappa_1 \left(\frac{b' - a' \cos^2 \kappa_1}{b' + a' \cos^2 \kappa_1} \right)^2, \tilde{A}_1 = \tilde{a} + \tilde{b} - 2\tilde{b}g'^2 \sin^2 \kappa_1, \\
\tilde{B}_1 &= -4\tilde{b}g'^2 \cos \kappa_1 \sin \kappa_1, \tilde{C}_1 = 2\tilde{a}g'^2 \sin^2 \kappa_1, \\
\tilde{A}_2 &= - \left(\tilde{a} + \tilde{b} - 2\tilde{b}g'^2 \sin^2 \kappa_1 \right) \cos^2 \kappa_1, \tilde{B}_2 = -\frac{4b'\tilde{b}g'^2}{a'} \cos \kappa_1 \sin \kappa_1, \\
\tilde{C}_2 &= - \left[\tilde{a} + \tilde{b} - 2\tilde{b}g'^2 \sin^2 \kappa_1 - 2g'^2 \left(3\tilde{a} + 2\tilde{b} \right) \cos^2 \kappa_1 \right] \sin^2 \kappa_1, \\
\tilde{A}_3 &= 6\tilde{a}g'^2 \cos^4 \kappa_1, \tilde{B}_3 = \frac{24b'\tilde{a}g'^2 \cos^3 \kappa_1 \sin \kappa_1}{b' + a' \cos^2 \kappa_1}, \\
\tilde{C}_3 &= \frac{24b'^2 \tilde{a}g'^2 \cos^2 \kappa_1 \sin^2 \kappa_1}{(b' + a' \cos^2 \kappa_1)^2},
\end{aligned} \tag{40}$$

$$\begin{aligned}
\hat{E}_1 &= 10\beta' (1 + \tan^2 \kappa_1) - 2(4\kappa_1 \tan \kappa_1 - 5), \\
\hat{E}_2 &= -2\beta' (1 + \tan^2 \kappa_1) (4\kappa_1 \tan \kappa_1 + 5) - 10, \\
\hat{D}_0 &= \tilde{\beta} \left[\left(\frac{2\kappa_1 \tan \kappa_1 [\beta' (1 + \tan^2 \kappa_1) - 1]}{\beta' (1 + \tan^2 \kappa_1) + 1} + 1 \right)^2 + 1 \right], \\
\hat{D}_1 &= 2\beta' (1 + \tan^2 \kappa_1) + \tilde{\beta} \left[(2\kappa_1 \tan \kappa_1 - 1)^2 + 1 \right] + \\
&\quad + 2(\tilde{\beta} - 1) (2\kappa_1 \tan \kappa_1 - 1), \\
\hat{D}_2 &= -\beta' (1 + \tan^2 \kappa_1) \left[(2\kappa_1 \tan \kappa_1 + 1)^2 + 1 \right] + \\
&\quad + 4(3\tilde{\beta} + 1)\kappa_1^2 \tan^2 \kappa_1 - 2, \\
\hat{D}_3 &= 3\tilde{\beta} \left[\left(\frac{4\beta' \kappa_1 \tan \kappa_1 (1 + \tan^2 \kappa_1)}{\beta' (1 + \tan^2 \kappa_1) + 1} + 1 \right)^2 + 1 \right], \quad \beta' = \frac{b'}{a'}, \quad \tilde{\beta} = \frac{\tilde{a}}{\tilde{b}}.
\end{aligned} \tag{41}$$

We emphasize that, in expressions (40) and (41), we excluded the coupling constant $\tilde{\alpha}'_s$ by means of not only the quantization condition of the energy levels in (34) at $n = 1$, but also the identity that we installed for the spin parameters a', b', \tilde{a} and \tilde{b} in the process of the calculations and which has the form^{a)}

$$a'(\tilde{a} + \tilde{b}) - 2\tilde{b}g'^2(a' + b') \equiv 0. \quad (42)$$

For the spin parameters β' and $\tilde{\beta}$ the identity (42) takes the form

$$\tilde{\beta} + 1 - 2g'^2(\beta' + 1) \equiv 0. \quad (43)$$

^{a)} Analogous identity was installed and for the component system of two spinor particles with equal masses ($g' = 1$) in [Yu. D. Chernichenko, Phys. At. Nucl. **82**, 334 (2019)].

For large $-t = Q^2 \gg M_1^2$ the rapidity behaves as $\chi_\Delta \approx \ln(Q/M_1)^2$ and, consequently, the leading behavior of form factor (38) gives as

$$F_{\ell=0,n=1}^{\text{Coul}}(Q^2) \approx \tag{44}$$

$$\approx \frac{32\pi(z_1 + z_2)m'^3 \kappa_1^3 \tilde{C}_0}{M_1 g' (2\kappa_1^2 \rho'_{\kappa_1} - 2\kappa_1 \rho'_{\kappa_1} + 1)} \frac{\ln(Q/M_1)^2}{(Q/M_1)^2} \left\{ 1 + O\left[\ln^{-1}(Q/M_1)^2\right] \right\}.$$

Such behavior of the form factor under large $|t|$ complies with the prediction of the dimensional quark counting rules [V. A. Matveev, R. M. Muradyan, A. N. Tavkhelidze, *Lett. Nuovo Cimento* **5**, 907 (1972)], which gives $F_\pi \sim |t|^{-1}$. In the case of relativistic spinless particles the decrease of the form factor occurs under the law $F_\pi \sim (|t| \ln^3 |t|)^{-1}$ [Yu. D. Chernichenko, *Phys. At. Nucl.* **77**, 229 (2014)].

By using obtained here result for the Coulomb wave function (35), we calculate the r.m.s. for the ground level of bound s -state of pseudoscalar π^\pm -, K^\pm - and K_0 -mesons

($n = 1, \ell = 0, a' = g'^2, b' = 1 - a', \tilde{a} = \tilde{b} = 1$) with masses: $M_{\pi^\pm} = 0.13957 \text{ GeV}$, $M_{K^\pm} = 0.49368 \text{ GeV}$ and $M_{K_0} = 0.49760 \text{ GeV}$ [K.A. Olive et al. (Particle Data Group), Chin. Phys. C **38**(9) 34 (2014) 090001].

For this, using the expressions (39), (41) and the values masses M_1 for π^\pm -, K^\pm - and K_0 -mesons, we find those values of the factor g' and the rapidity κ_1 that lead to the values of the r.m.s. for the ground level of bound s -state of pseudoscalar π^\pm -, K^\pm - and K_0 -mesons, belonging to the confidence intervals of their experimental values [S. R. Amendolia et al., Phys. Lett. B **146**, 116 (1984); Nucl. Phys. B **277**, 168 (1986); S. R. Amendolia et al., Phys. Lett. B **178**, 435 (1986); A. Lai et al., Eur. Phys. J. C **30**, 33 (2003)].

The values for the masses m'_{π^\pm} , m'_{K^\pm} and m'_{K_0} are calculated from the values of the masses M_1 , the factor g' and the rapidity κ_1 for π^\pm -, K^\pm - and K_0 -mesons from the Tabl. 1 using expressions

$$M_{\pi^\pm} = 2m'_{\pi^\pm} g'_{\pi^\pm} \cos \kappa_{\pi^\pm}, \quad M_{K^\pm} = 2m'_{K^\pm} g'_{K^\pm} \cos \kappa_{K^\pm}, \quad M_{K_0} = 2m'_{K_0} g'_{K_0} \cos \kappa_{K_0}.$$

The values for the quark masses m_u, m_d and m_s are calculated uniquely from the values of the factor g' and the masses m' for π^\pm -, K^\pm - and K_0 -mesons from the [the Tabl. 1](#) using a system of equations

$$\left\{ \begin{array}{l} m_u = m'_{\pi^\pm} (g'_{\pi^\pm} - \sqrt{g'^2_{\pi^\pm} - 1}), m_d = m'_{\pi^\pm} (g'_{\pi^\pm} + \sqrt{g'^2_{\pi^\pm} - 1}); \\ m_u = m'_{K^\pm} (g'_{K^\pm} - \sqrt{g'^2_{K^\pm} - 1}), m_s = m'_{K^\pm} (g'_{K^\pm} + \sqrt{g'^2_{K^\pm} - 1}); \\ m_d = m'_{K_0} (g'_{K_0} - \sqrt{g'^2_{K_0} - 1}), m_s = m'_{K_0} (g'_{K_0} + \sqrt{g'^2_{K_0} - 1}). \end{array} \right.$$

The values of the Coulomb interaction constant $\tilde{\alpha}'_s$ for the ground level of the s -state of π^\pm -, K^\pm - and K_0 -mesons were calculated from the condition for quantization of energy levels (34) at $n = 1$ using the values of the factor g' and the rapidity κ_1 from the [the Tabl. 1](#). The results of calculations for [the r.m.s. \(39\)](#) and the values of all parameters for [the ground level of bound \$s\$ -state of pseudoscalar \$\pi^\pm\$ -, \$K^\pm\$ - and \$K_0\$ -mesons](#) are provided in [Tabl. 1](#).

Tabl. 1: Values of the r.m.s. and parameters for π^\pm -, K^\pm - and K_0 -mesons

Mesons	M_1 , GeV	m_u , GeV	m_d , GeV	m_s , GeV	m' , GeV
π^\pm	0.13957	0.10062	0.11527		0.10770
K^\pm	0.49368	0.10062		0.40478	0.20181
K_0	0.49760		0.11527	0.40478	0.21601

Mesons	g'	κ_1	$\tilde{\alpha}'_s$	$\langle r_0^2 \rangle_{\text{th}}, \text{fm}^2$	$\langle r_0^2 \rangle_{\text{exp}}, \text{fm}^2$
π^\pm	1.00231	0.86783	7.34912	0.44441	0.439 ± 0.007
K^\pm	1.25214	0.21573	0.92252	0.29118	0.34 ± 0.05
K_0	1.20378	0.29487	1.32457	-0.06925	-0.090 ± 0.021

In Fig. 3 we built the graphs of function $\langle r_0^2 \rangle = \langle r_0^2 \rangle (\kappa)$, represented by expression (39), as the function of variable $\kappa_1 = \kappa$ for the ground level of bound s -state of pseudoscalar π^\pm -, K^\pm - and K_0 -mesons with the values of parameters from Tabl. 1.

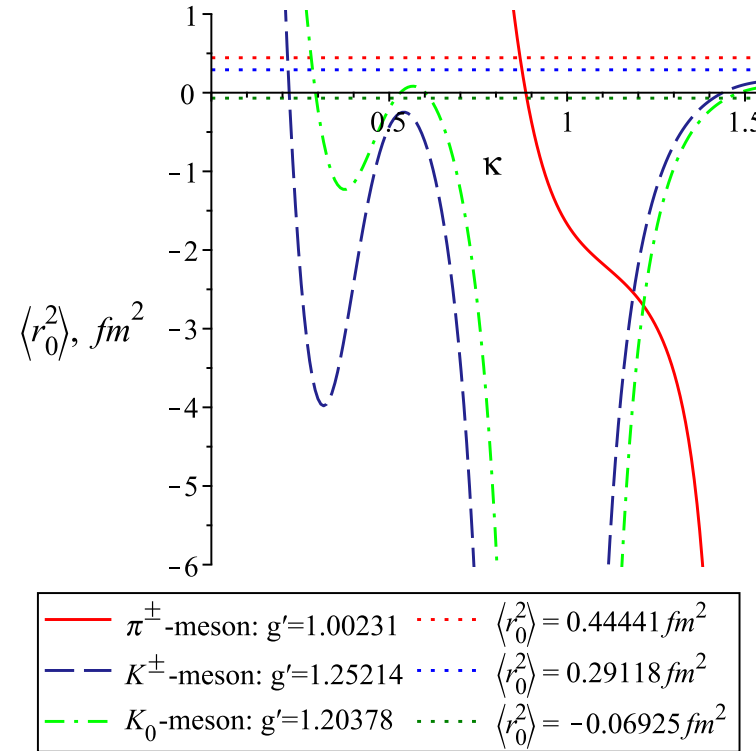


Fig. 3: Behavior of function $\langle r_0^2 \rangle$ as a function of the variable κ for the ground level of bound s -state of pseudoscalar π^\pm -, K^\pm - and K_0 -mesons with the values of parameters from Tabl. 1.

From Fig. 3 we see that graphs to functions $\langle r_0^2 \rangle$ for the r.m.s. (39) of the ground level of bound s -state of pseudoscalar π^\pm -, K^\pm - and K_0 -mesons with the values of parameters from Tabl. 1 has singularities at “critical” the values of the rapidity κ . The “critical” the values of the rapidity $\kappa_{\pi^\pm}^{\text{conf}} \approx 1.50289$, $\kappa_{K^\pm}^{\text{conf}} \approx 0.92501$, $\kappa_{K_0}^{\text{conf}} \approx 0.98039$ are defined the boundary of the quarks confinement region, forming mesons: at $\kappa \geq \kappa_i^{\text{conf}}$ ($i = \pi^\pm, K^\pm, K_0$) the Coulomb constant $\tilde{\alpha}'_s < 0$. Exactly the same singularity for the r.m.s. π^\pm -meson occurred in the case of equal masses quarks ($m_u = m_d$) for both scalar and vector currents [Yu. D. Chernichenko, Phys. At. Nucl. **81**, 360 (2018); Phys. At. Nucl. **82**, 334 (2019)].

At $\kappa < \kappa_i^{\text{conf}}$ ($i = \pi^\pm, K^\pm, K_0$) their the Coulomb constant $\tilde{\alpha}'_s > 0$, and for small values of the rapidity κ their the Coulomb constant are small.

Note that the dimensionless factor

$$\sigma_{\text{conf}} = \frac{\tilde{\alpha}'_s a'}{4} \sin \kappa_1 - \frac{\kappa_1 \rho'_{\kappa_1}}{2} = \frac{\tilde{\alpha}'_s a'}{4} (\sin \kappa_1 - \kappa_1 \cos \kappa_1) \quad (45)$$

in (36) (will include in the normalization constant of the solution (35)) coincides in the form with the left side of the quasiclassical condition for quantization of energy levels s -state of a bound system of two spinless particles of arbitrary masses with a linear potential $V(r) = \sigma r$

$$\chi_n \cosh \chi_n - \sinh \chi_n = \frac{\pi \sigma}{2g' m'^2} \left(n - \frac{1}{4} \right), \quad n = 1, 2, \dots, \quad (46)$$

taken when $\chi_n = -i\kappa_1$ and $n = 1$ (without $\tilde{\alpha}'_s a'/4, i$).

Obviously, $\tilde{\alpha}'_s a'/4$ is determined by $\tilde{\alpha}'_s$ and a' . At $a' = 0$ ($b' = 2/m'g'$), the factor (45) disappears, that corresponds to the case of the two spinless particles of arbitrary masses.

Thus, the nature of the dimensionless factor (45) is determined by the quark spins and the Coulomb potential, that leads to the confinement of meson quarks with a dimensionless linear interaction constant σ_{conf} in (45) depending on the factor a' and the values of the rapidity κ_1 , 'critical' the values of which determine the boundary of the quark confinement region of mesons ($\alpha_s < 0$ at $\kappa > \kappa^{\text{conf}}$). For κ_1 and g' π^\pm -, K^\pm - and K_0 -mesons from the Tabl. 1: $\sigma_{\text{conf}}^{\pi^\pm} = 0.37264$, $\sigma_{\text{conf}}^{K^\pm} = 0.00121$, $\sigma_{\text{conf}}^{K^0} = 0.00407$.

5. Conclusions

In the present study, we have obtained a new covariant expressions for the elastic form factor and mean-square radius for the scalar current of the composite system of two relativistic particles with arbitrary masses and spin 1/2. The pseudoscalar, vector and pseudovector composite systems were considered. For them is received identity, which installs the dependency between masses and quarks spin, forming composite systems.

As example, the expressions for the form factor and mean-square radius of a bound system that formed by two relativistic spin 1/2 particles with arbitrary masses in the case of Coulomb potential were obtained. It is installed that the covariant wave RQP-function of Coulomb potential at $Q^2 \gg 1$ give the decrease for form factor under the law $F_\pi \sim |t|^{-1}$, which predicts the dimensional quark counting rules.

Values of the mean-square radius for the ground level of bound s -state of pseudoscalar π^\pm -, K^\pm - and K_0 -mesons with the Coulomb interaction are calculated.

The influence of quarks masses on the behavior of form factor and mean-square radius for the ground level of bound s -state of pseudoscalar π^\pm -, K^\pm - and K_0 -mesons are learned.

The consideration is conducted within the framework of relativistic quasipotential approach on the basis of covariant Hamiltonian formulation of quantum field theory, by transition to the three-dimensional relativistic configurational representation in the case of bound system of two relativistic spin particles with arbitrary masses.

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