Massive Domain Wall Fermions

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Historical Remark 1: Domain Wall Fermions in 1+1 Dim

Zero modes in Jackiw-Rebbi model (1976):

The Jackiw-Rebbi model describes a one-dimensional Dirac field coupled to a non-linear topological scalar field (ϕ^4 – model, Sine-Gordon – model …).

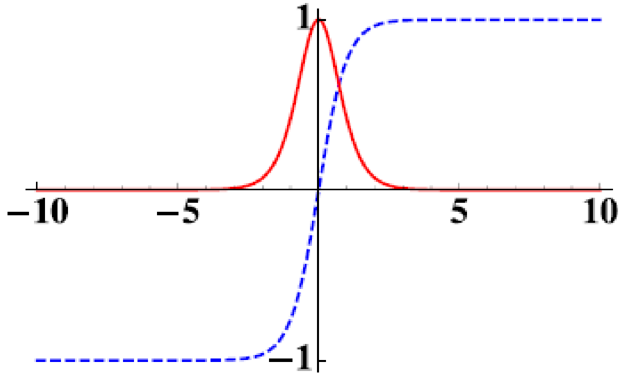
Neglecting the dynamics of the scalar field, a topological soliton in the background yields a topologically protected Zero-energy mode for the Dirac field.

Wave-function of this Zero-mode is localized on the topological soliton.

$$i\partial_t \Psi = \left(\alpha c p_z + \frac{\beta m c^2}{\kappa} \phi(z) \right) \Psi$$

$$\partial_\mu \partial^\mu \phi(z) + \frac{\lambda^2}{2\kappa^2} \left(\kappa^2 - \phi(z)^2\right) \phi(z) = 0 \longrightarrow \phi_s(z) = \pm\ \kappa \tanh(\lambda z)$$

$$\Psi_0(z) = \exp\left[\mp \frac{mc}{\kappa} \int_0^z dx \phi_s(x) \right] \chi$$

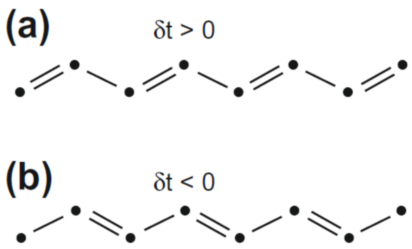


Jackiw-Rebbi Zero mode: application

Polymer physics – Su, Schrieffer, Heeger (SSH) Model (1980):

The SSH model was introduced as a model of the conducting polymer **polyacetylene**.

There are two topologically distinct ground-states of the SSH model:



Tight-binding Hamiltonian of the SSH model:

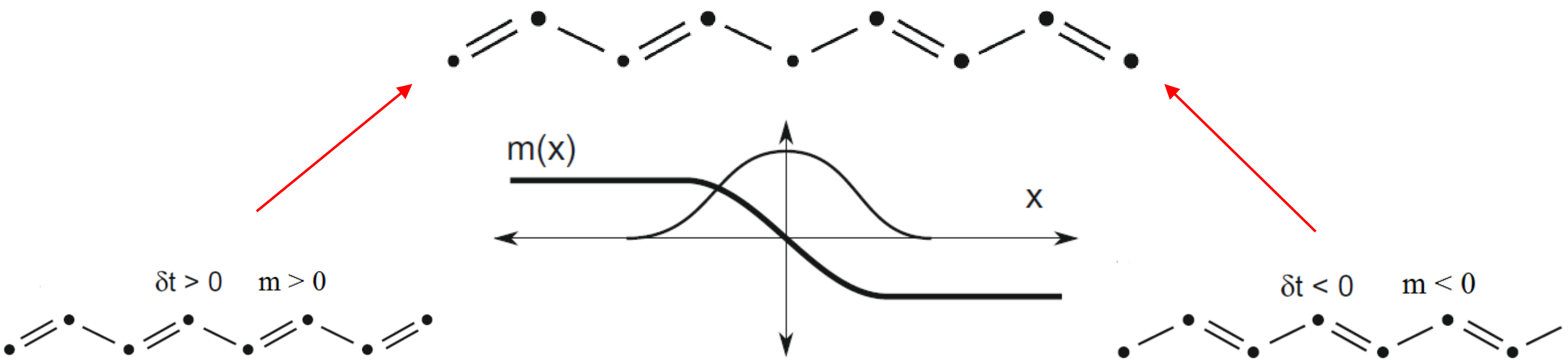
$$H = \sum_i (t + \delta t) c^{\dagger}_{Ai} c_{Bi} + (t - \delta t) c^{\dagger}_{Ai+1} c_{Bi} + h.c.$$

In the continuous limit, the Hamiltonian has the form of 1+1 D Dirac field:

$$H = -i v_F \sigma^x \partial_x + m \sigma^y \quad \text{where } v_F = ta \text{ and } \boxed{m = 2\delta t}$$

Jackiw-Rebbi mode in polyacetylene

Let us consider the SSH model polyacetylene **with structure defect**:

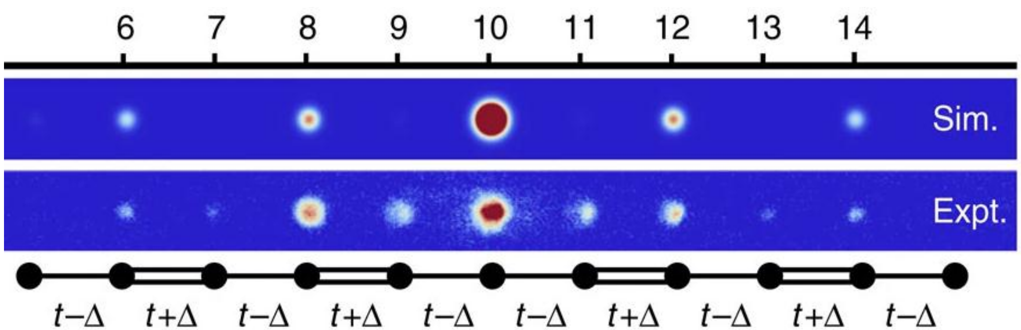


$$H = -iv_F\sigma^x\partial_x + m\sigma^y$$

Another application:

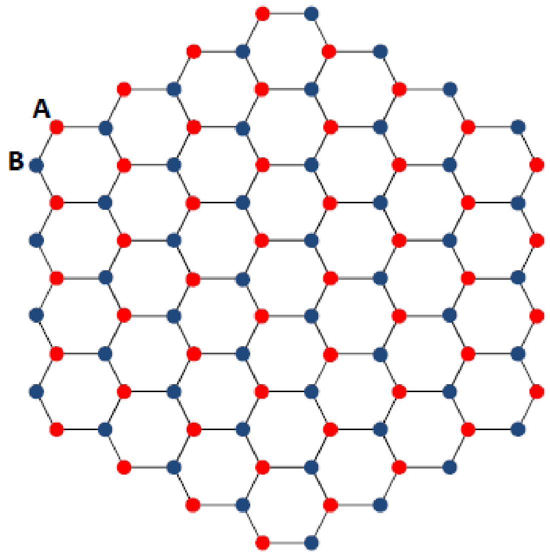
"Fermi-Dirac gas in an optical lattice" J Ruostekoski, … (PRL, 2002)

"Observation of the topological soliton state in the SSH model."
E. Meier ... *Nat Commun* (2016).

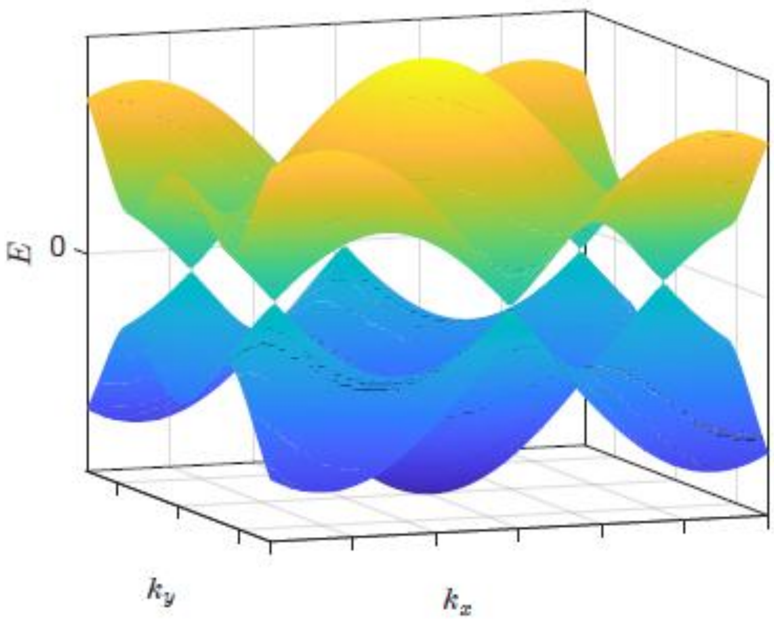


2+1 Domain Wall fermions

Domain Wall fermions in hexagonal structure 2D materials. G. Semenoff, … (2008)



P.R. Wallace 1947



But what if the sublattice symmetry is broken?

2+1 Domain Wall fermions

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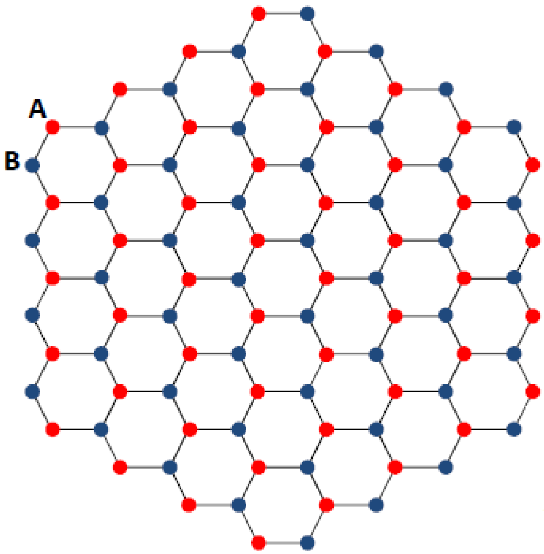
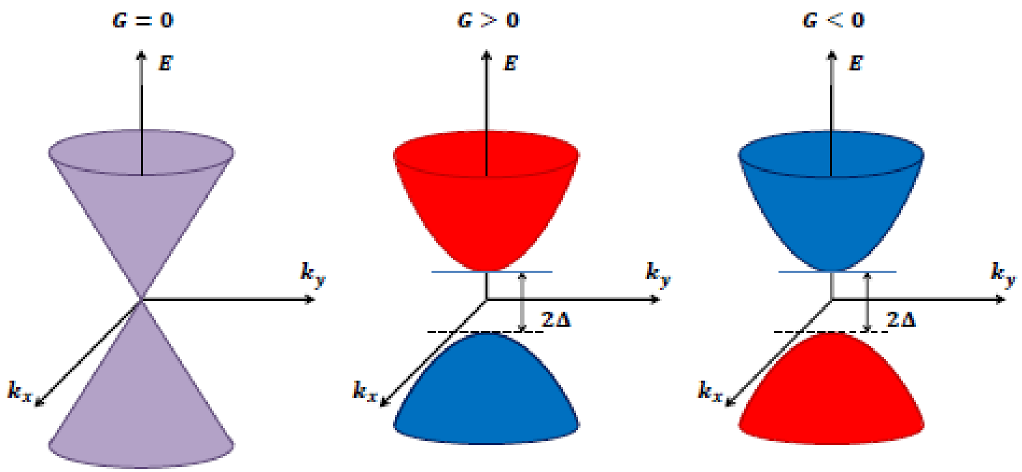


$$H = \sum_{A,i} t\, b^\dagger_{A+\vec{s}_i} a_A + \sum_{B,i} t\, a^\dagger_{B-\vec{s}_i} b_B + \sum_{A} \mu a^\dagger_A a_A - \sum_{B} \mu b^\dagger_B b_B$$

$$E = \pm\sqrt{\mu^2 + t^2\left(2\cos^2\frac{\sqrt{3}a}{2}k_x + \cos\frac{3a}{2}k_y\right)^2 + t^2\sin^2\frac{3a}{2}k_y}$$



2+1 Domain Wall fermions

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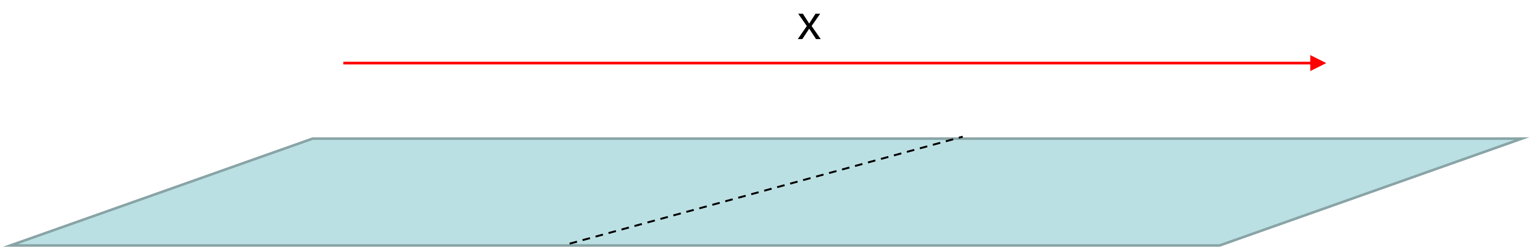
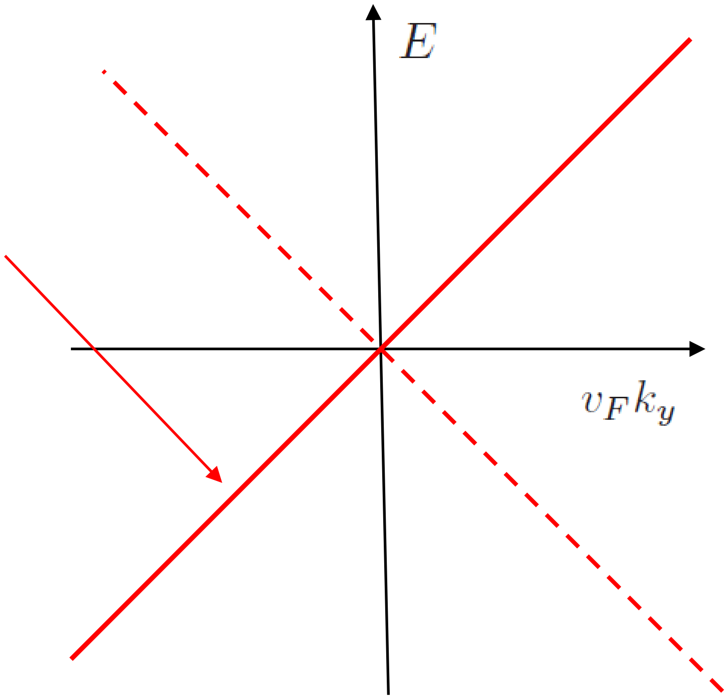


$$\lim_{x\to-\infty} m(x) = -m < 0 \quad , \quad \lim_{x\to\infty} m(x) = m > 0$$

$$\psi_L(x,y) = e^{ik_y y/\hbar - \frac{v_F}{\hbar^2}\int_0^x dx' m(x')} \begin{bmatrix} i \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad E = v_F k_y$$

$$\psi_R(x,y) = e^{ik_y y/\hbar - \frac{v_F}{\hbar^2}\int_0^x dx' m(x')} \begin{bmatrix} 0 \\ 0 \\ 1 \\ i \end{bmatrix} \quad E = -v_F k_y$$



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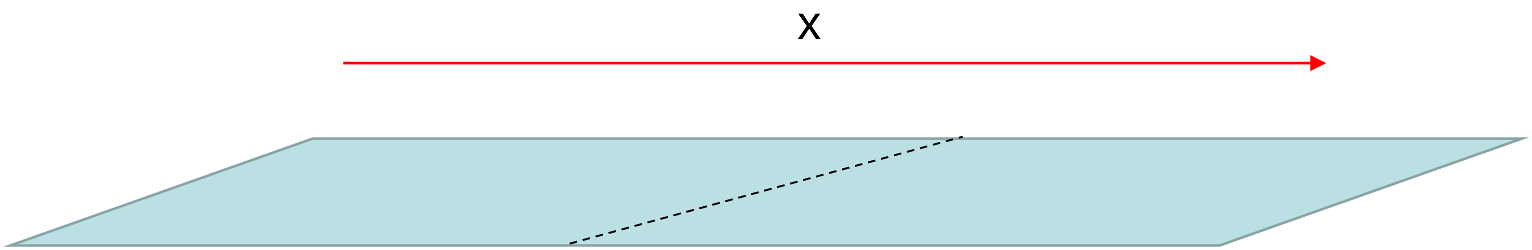

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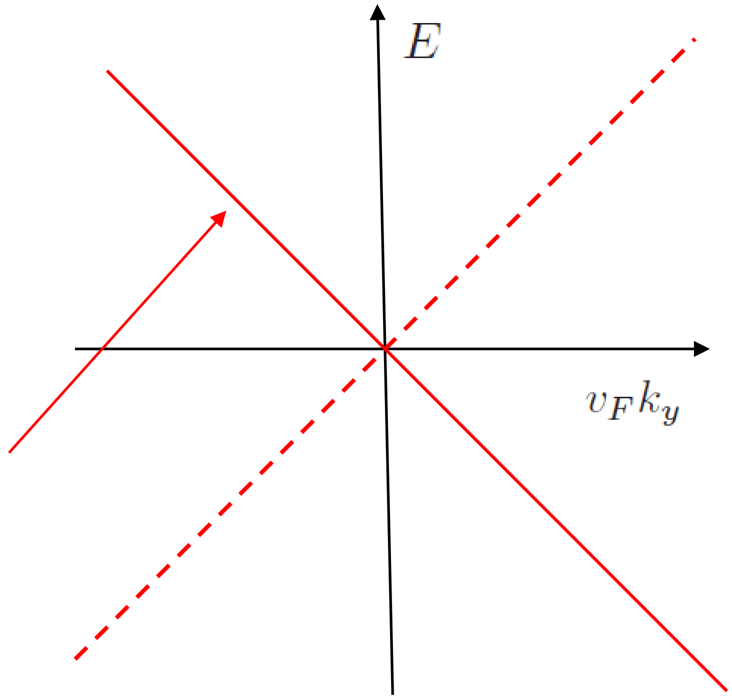
$$\psi_R(x,y) = e^{ik_y y/\hbar - \frac{v_F}{\hbar^2}\int_0^x dx' m(x')} \begin{bmatrix} 0 \\ 0 \\ 1 \\ i \end{bmatrix} \quad E = -v_F k_y$$



MC simulation of Gapped Graphene

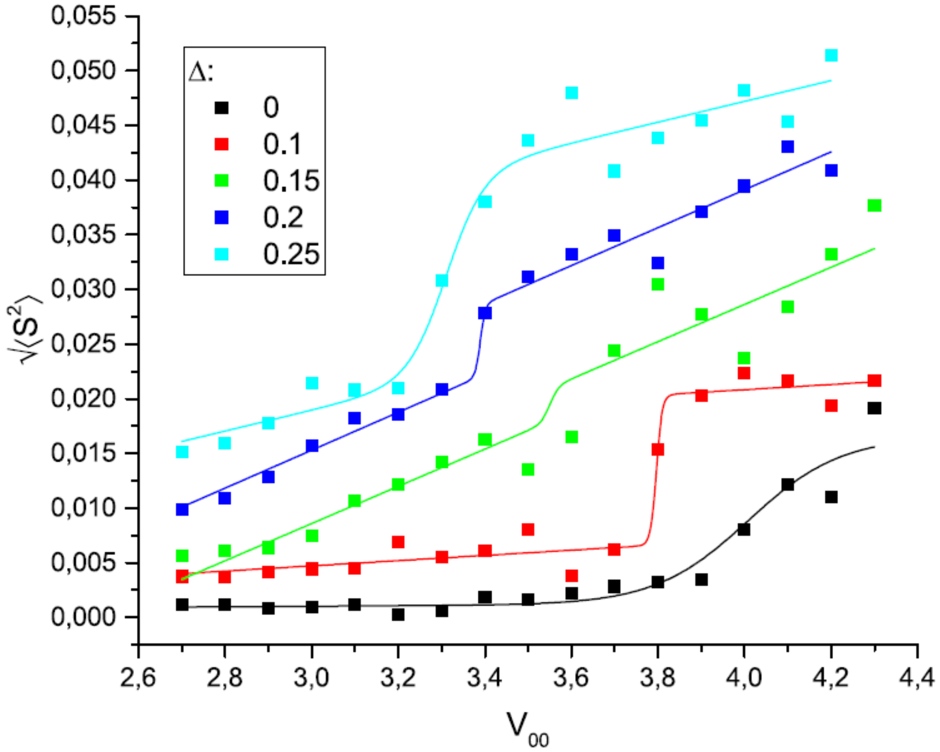
Simulation of sublattice symmetry breaking due to interaction with substrate.
S. Mostovoy, O. Pavlovsky (2023)

$$\hat{H} = -\kappa \sum_{x \neq y, \sigma} \hat{c}^{\dagger}_{x,\sigma} \hat{c}_{y,\sigma} + \frac{1}{2} \sum_{x,y} \hat{q}_x V_{xy} \hat{q}_y.$$

$$\hat{q} = \hat{c}^{\dagger}_{x,+1} \hat{c}_{x,+1} + \hat{c}^{\dagger}_{x,-1} \hat{c}_{x,-1} - 1$$

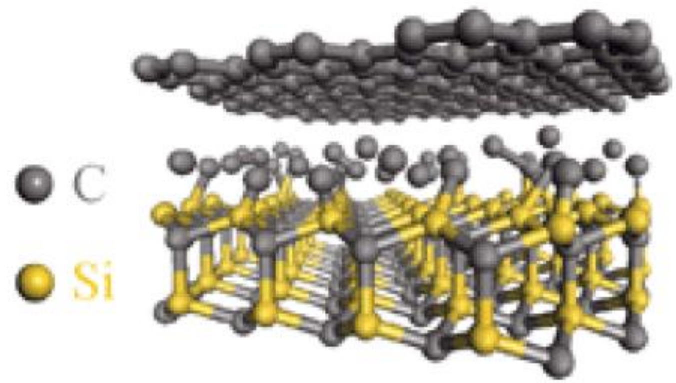
$$V_{00} = \frac{U - \Delta}{2} \sum_{x \in A} \hat{q}_x^2 + \frac{U + \Delta}{2} \sum_{x \in B} \hat{q}_x^2.$$

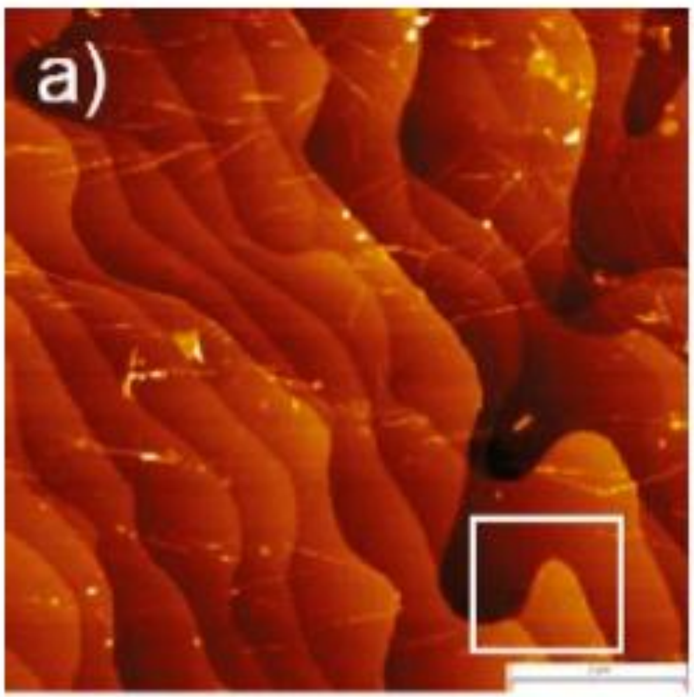
$$\langle S_z^2 \rangle = \frac{1}{2} \left\{ \frac{1}{L^4} \left\langle \left(\sum_{x \in A} \hat{S}_{zx} \right)^2 \right\rangle + \frac{1}{L^4} \left\langle \left(\sum_{x \in B} \hat{S}_{zx} \right)^2 \right\rangle \right\}$$



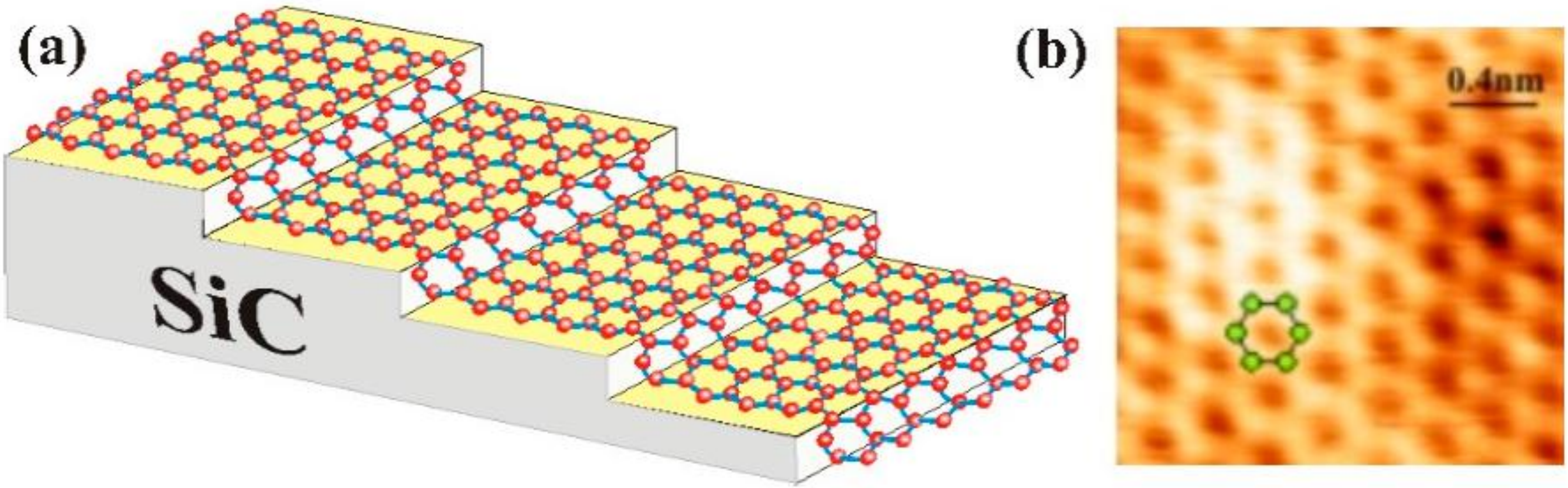
Graphene on Silicone Carbide

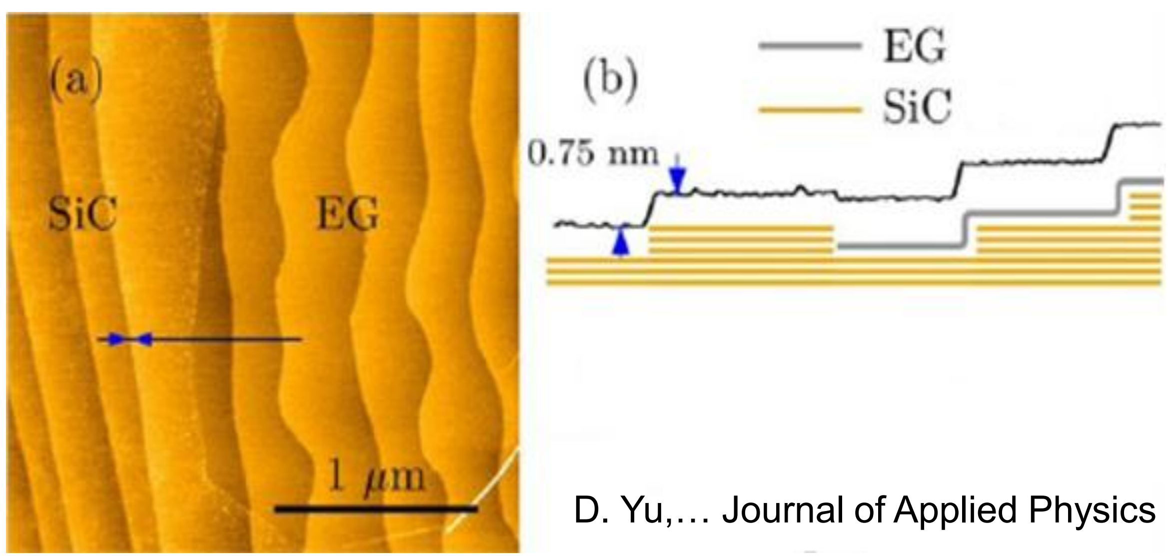
Graphene + Silicone Carbide





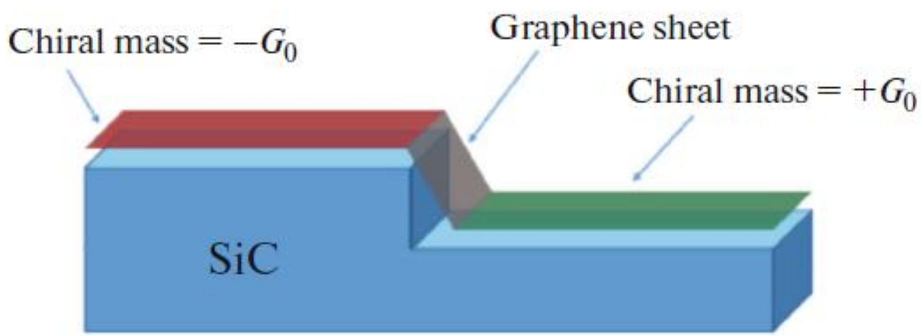
Graphene on Silicone Carbide: Terrace-Stepped Structure

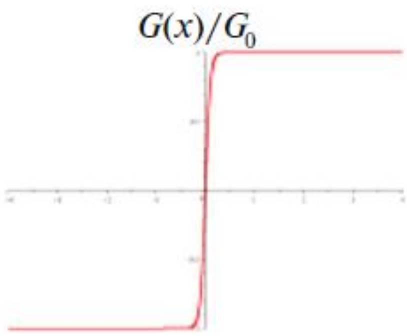




D. Yu,… Journal of Applied Physics (2012)

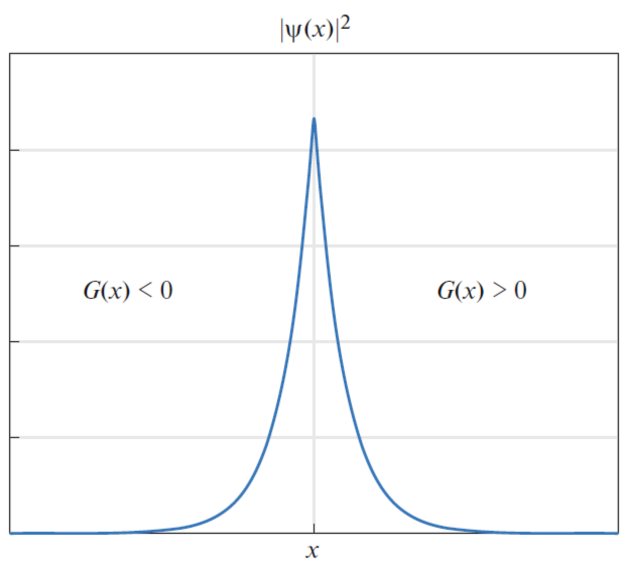
Graphene on Silicone Carbide: Two-Terrace Structure





$$\left(-i\hbar v_f(\sigma_x\partial_x + \sigma_y\partial_y) + G(x,y)\sigma_z\right)\psi(x,y) = E\psi(x,y),$$

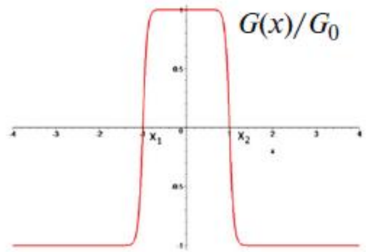
$$\psi(x,y) = \psi(x)e^{ik_y y} = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} e^{ik_y y}$$



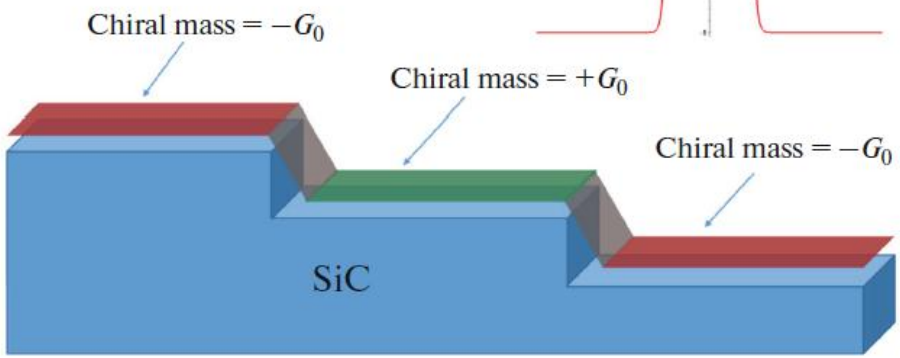
$$\psi = C \begin{pmatrix} 1 \\ i \end{pmatrix} e^{ik_y y} e^{\int_0^x -\frac{G(x')}{G_0} dx'}$$

$$G(x) = G_0 \tanh(ax) \Longrightarrow E = k_y$$

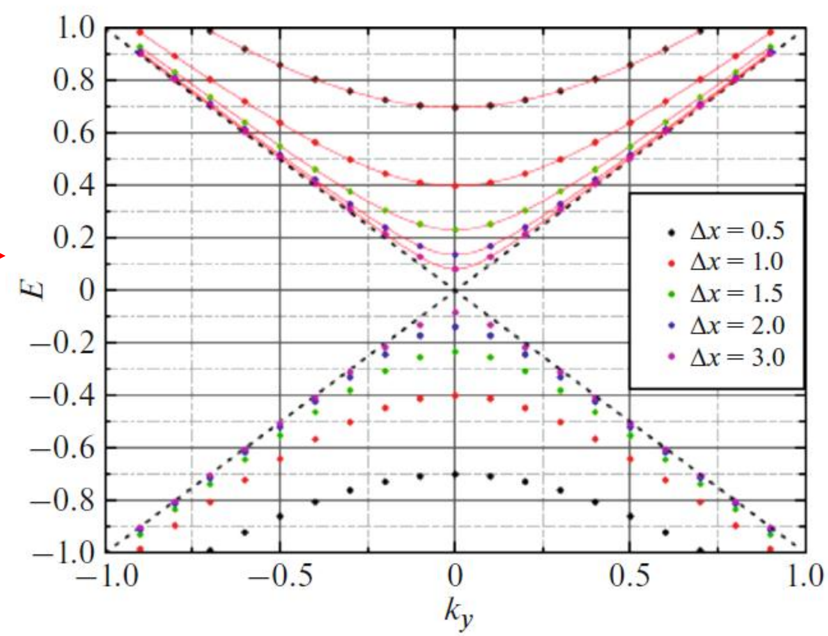
Graphene on Silicone Carbide: Three-Terrace Structure



$$G(x) = G_0 \left(\tanh\left(a(x - x_1)\right) - \tanh\left(a(x - x_2)\right) - 1\right).$$

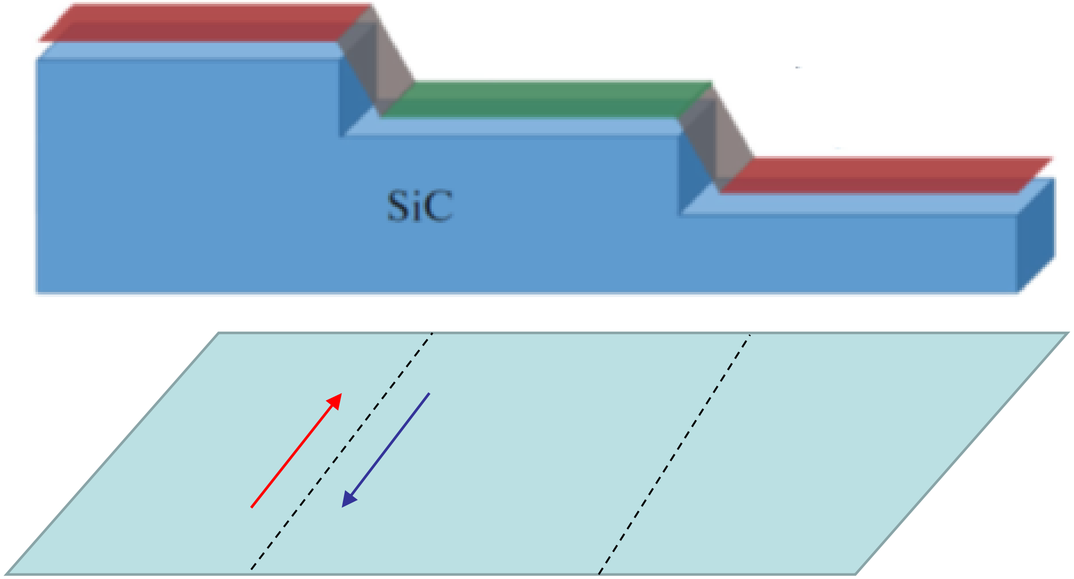


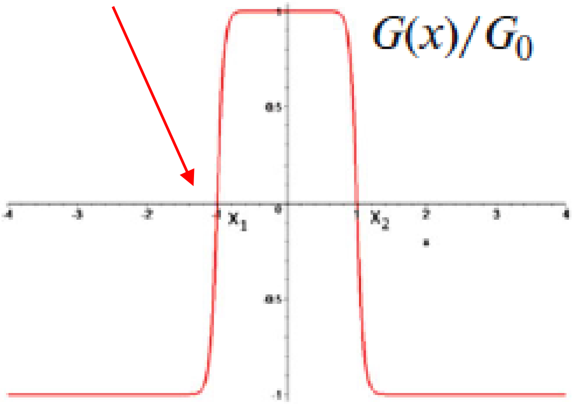
$$E = \pm\sqrt{k_y^2 + M_e^2}.$$

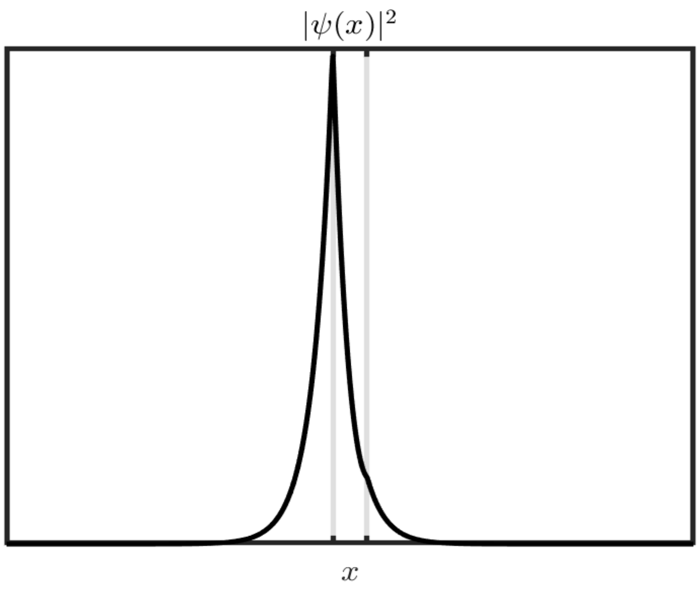


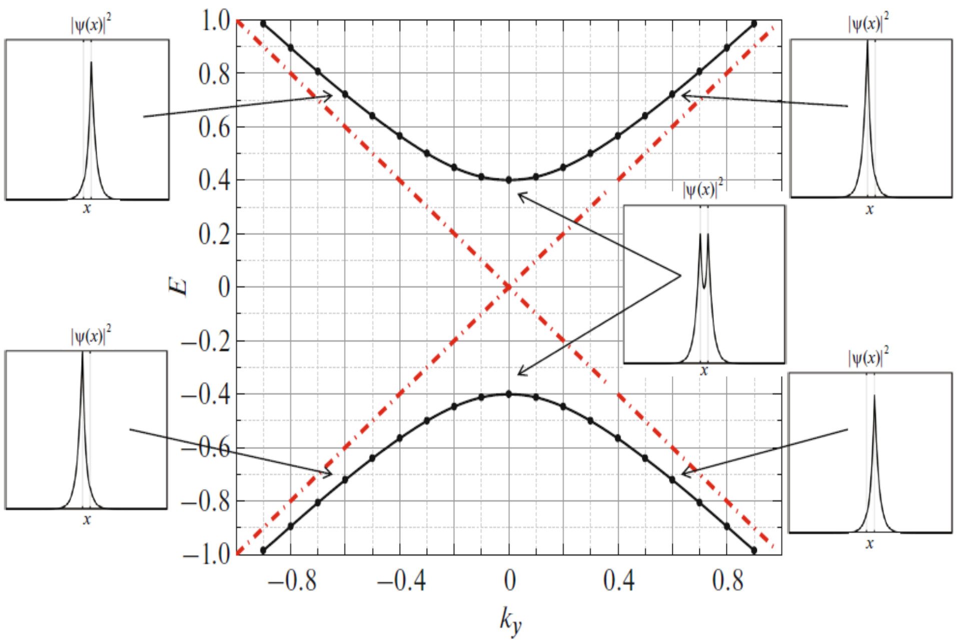
$$M_e = \frac{G_0}{\kappa(a)^2 \Delta x^2 + 1}$$

Three-Terrace Structure: Localization at large k_y

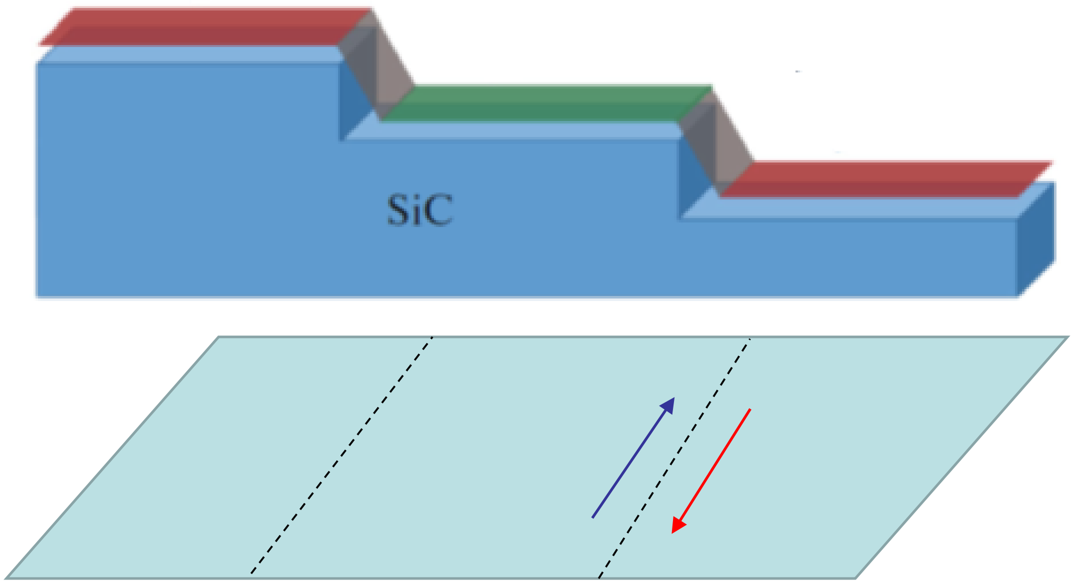


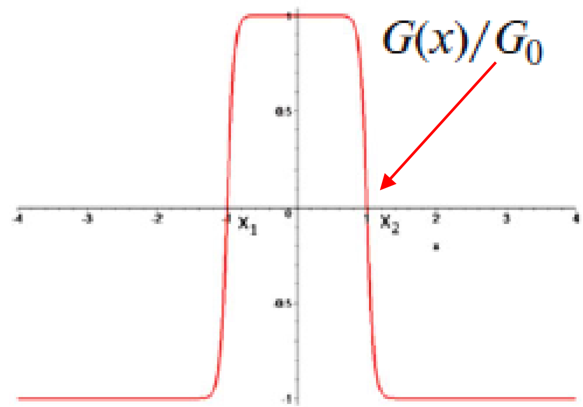


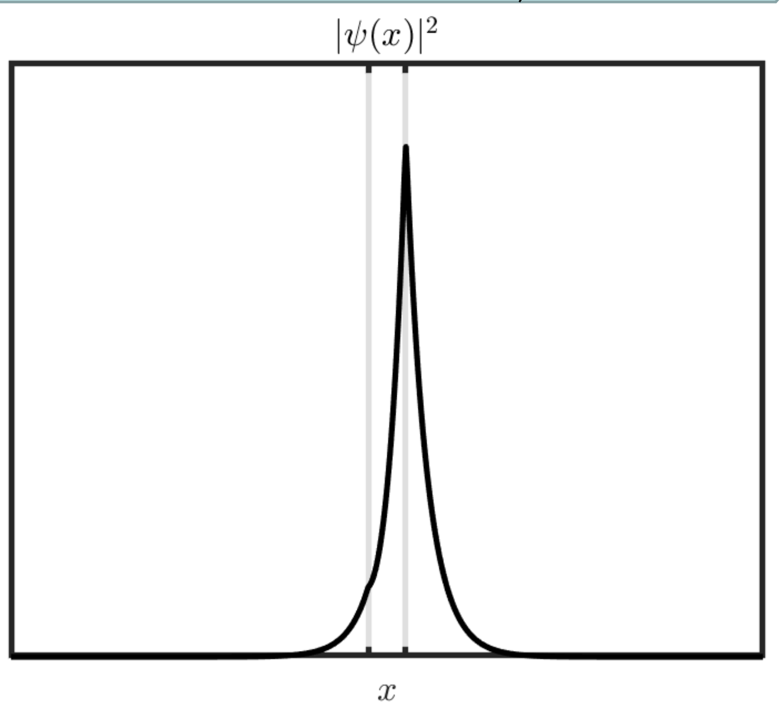


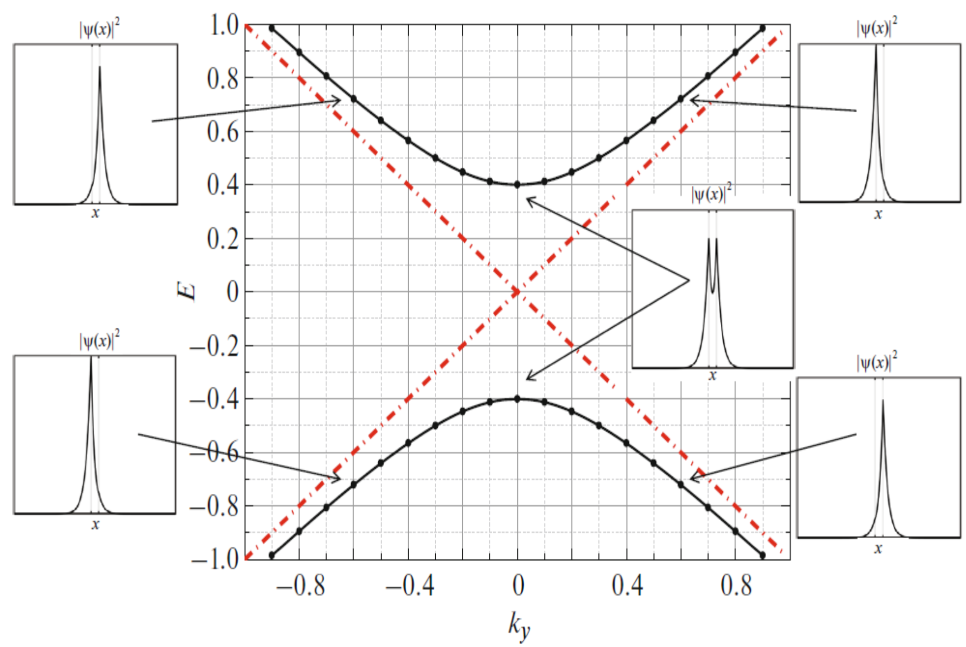


Three-Terrace Structure: Localization at large k_y

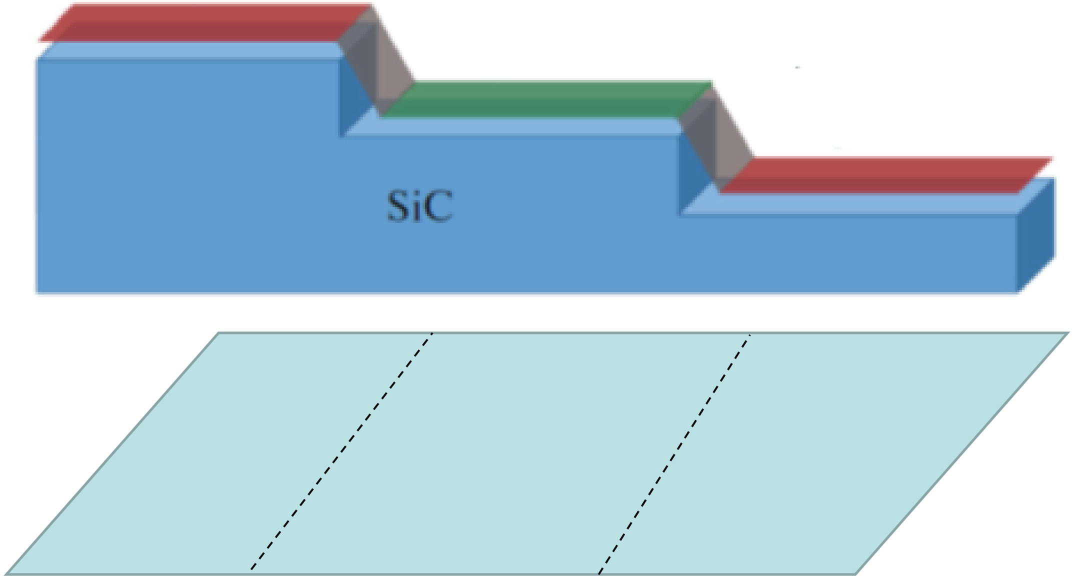


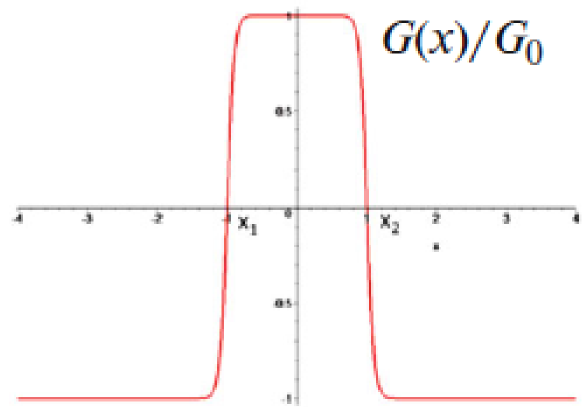


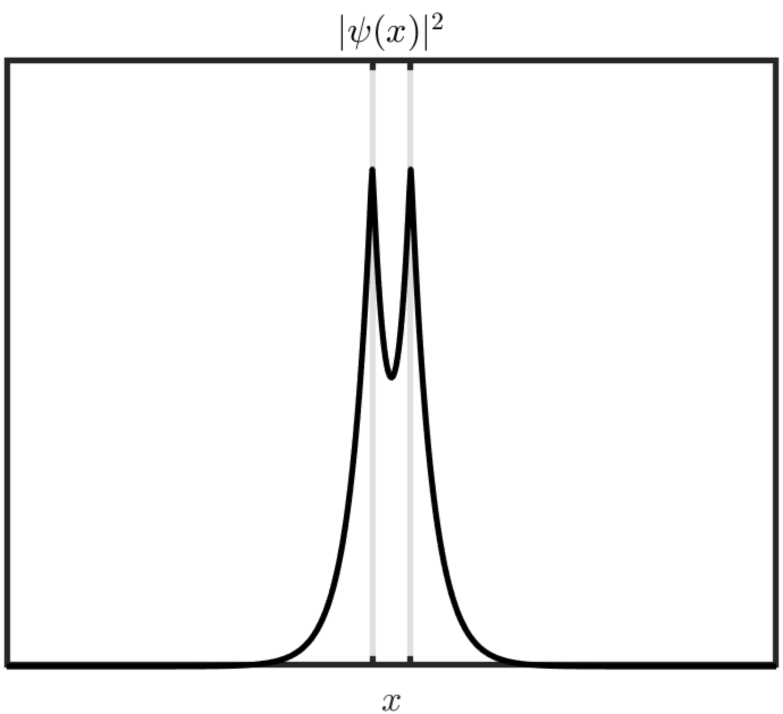


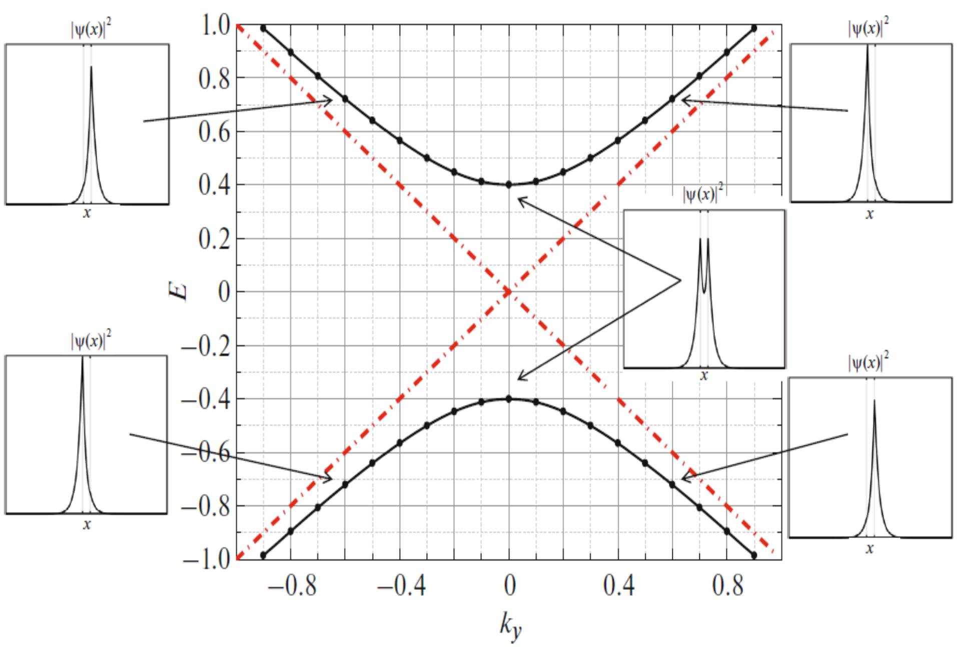


Three-Terrace Structure: Localization at k_y=0

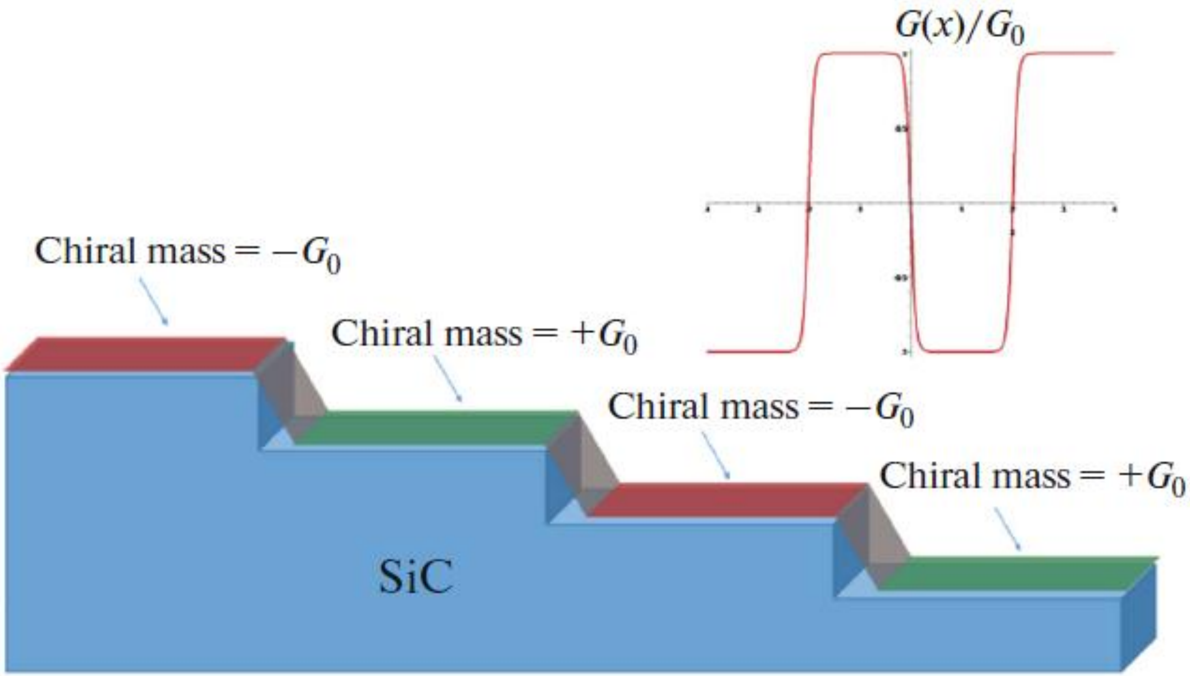




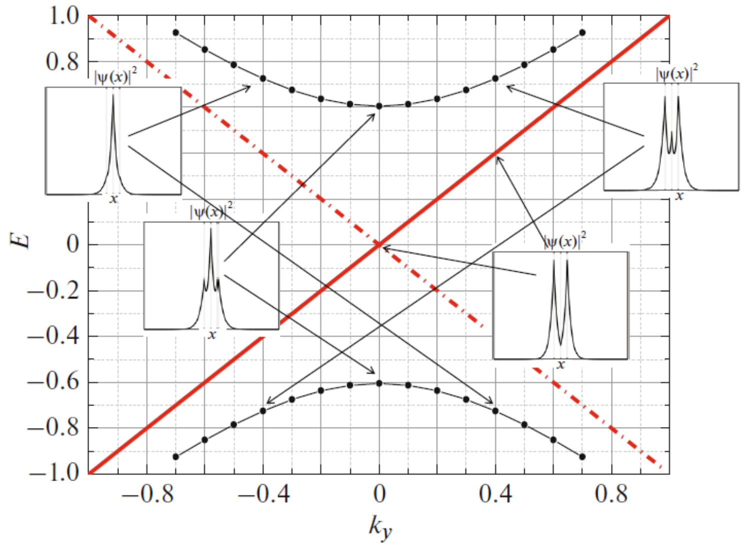




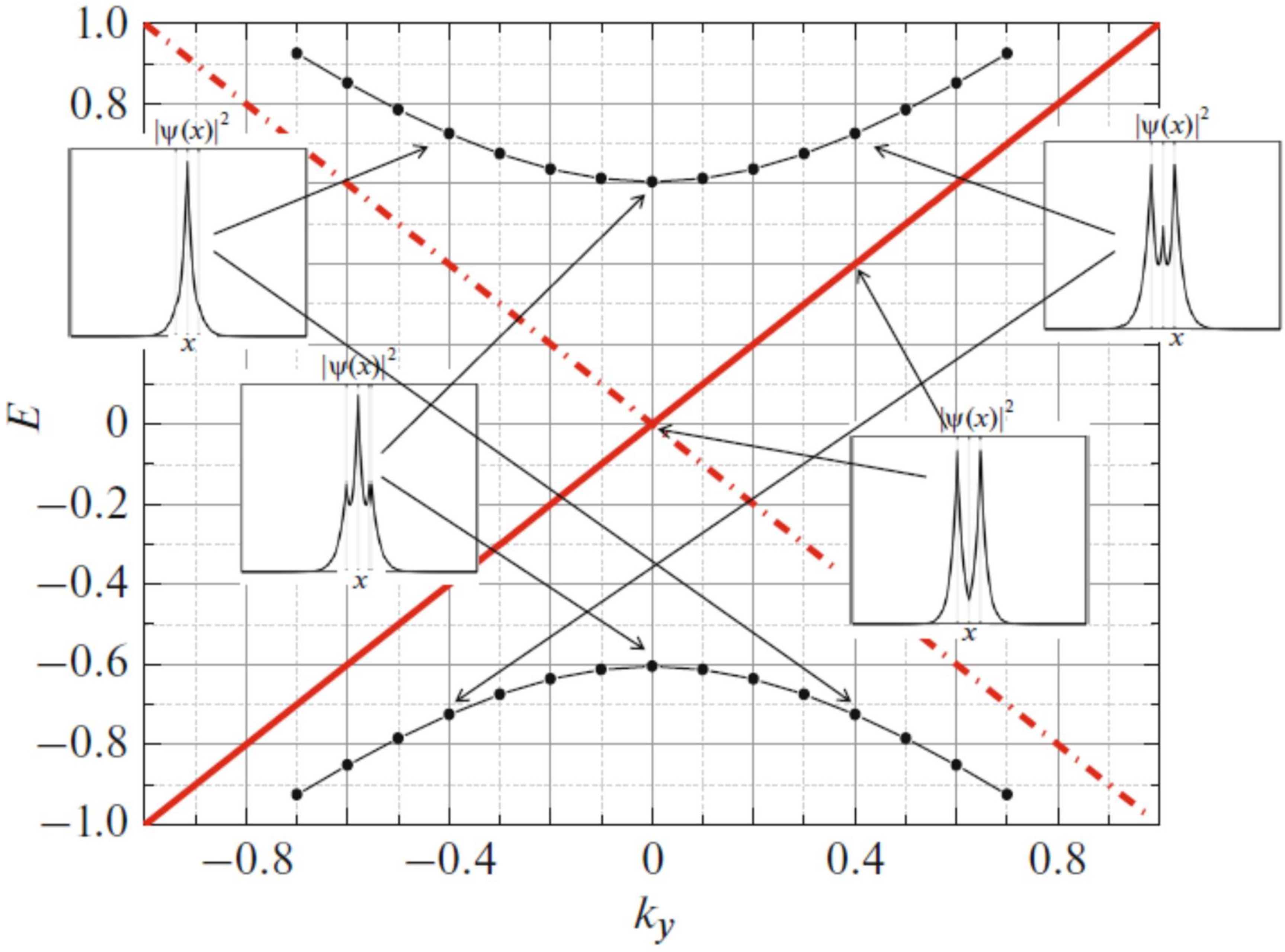
Graphene on Silicone Carbide: Four-Terrace Structure



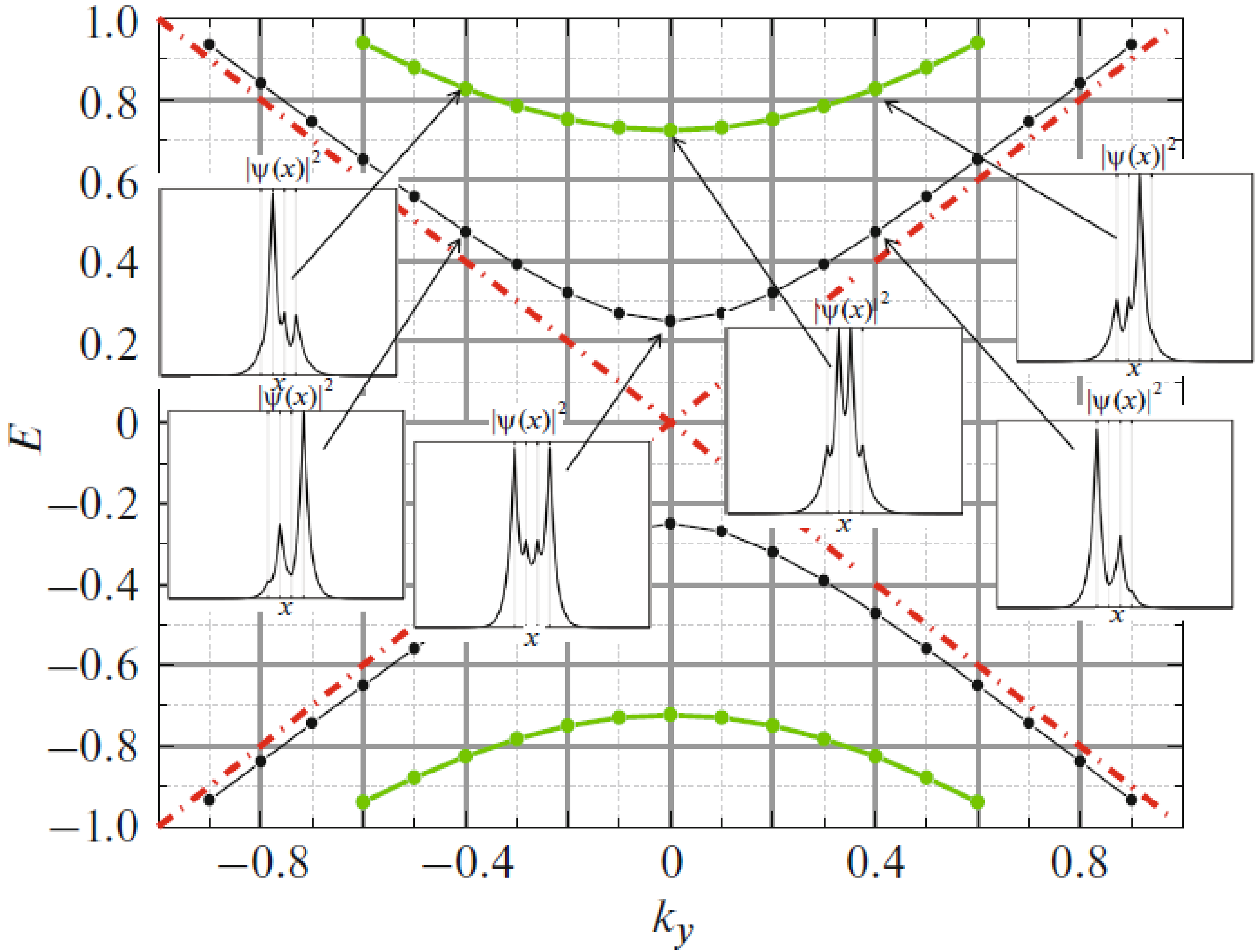
$$G(x) = \tanh\left(a(x - x_1)\right) - \tanh\left(a(x - x_2)\right) + \tanh\left(a(x - x_3)\right).$$



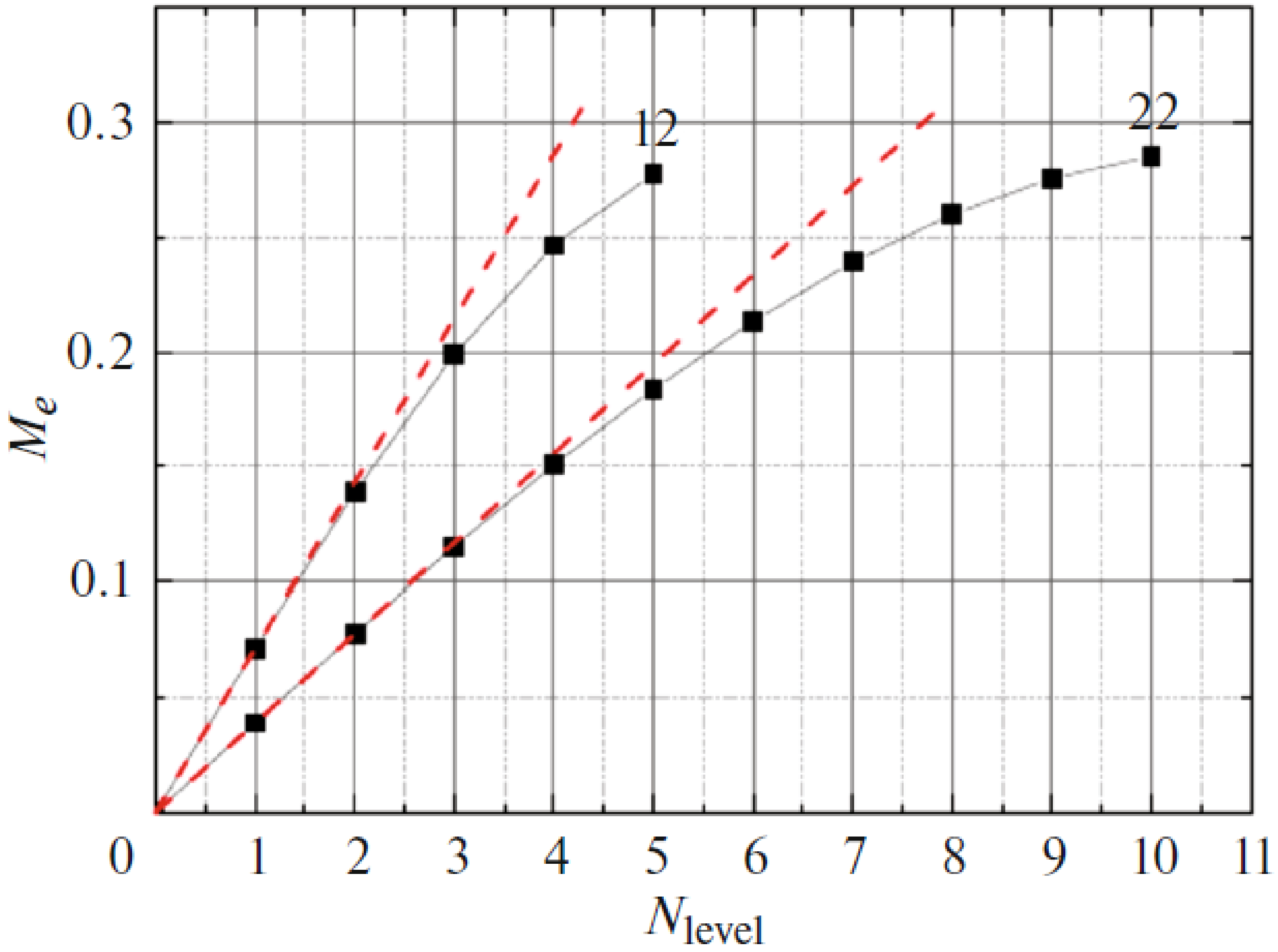
Four-Terrace Structure: Localization



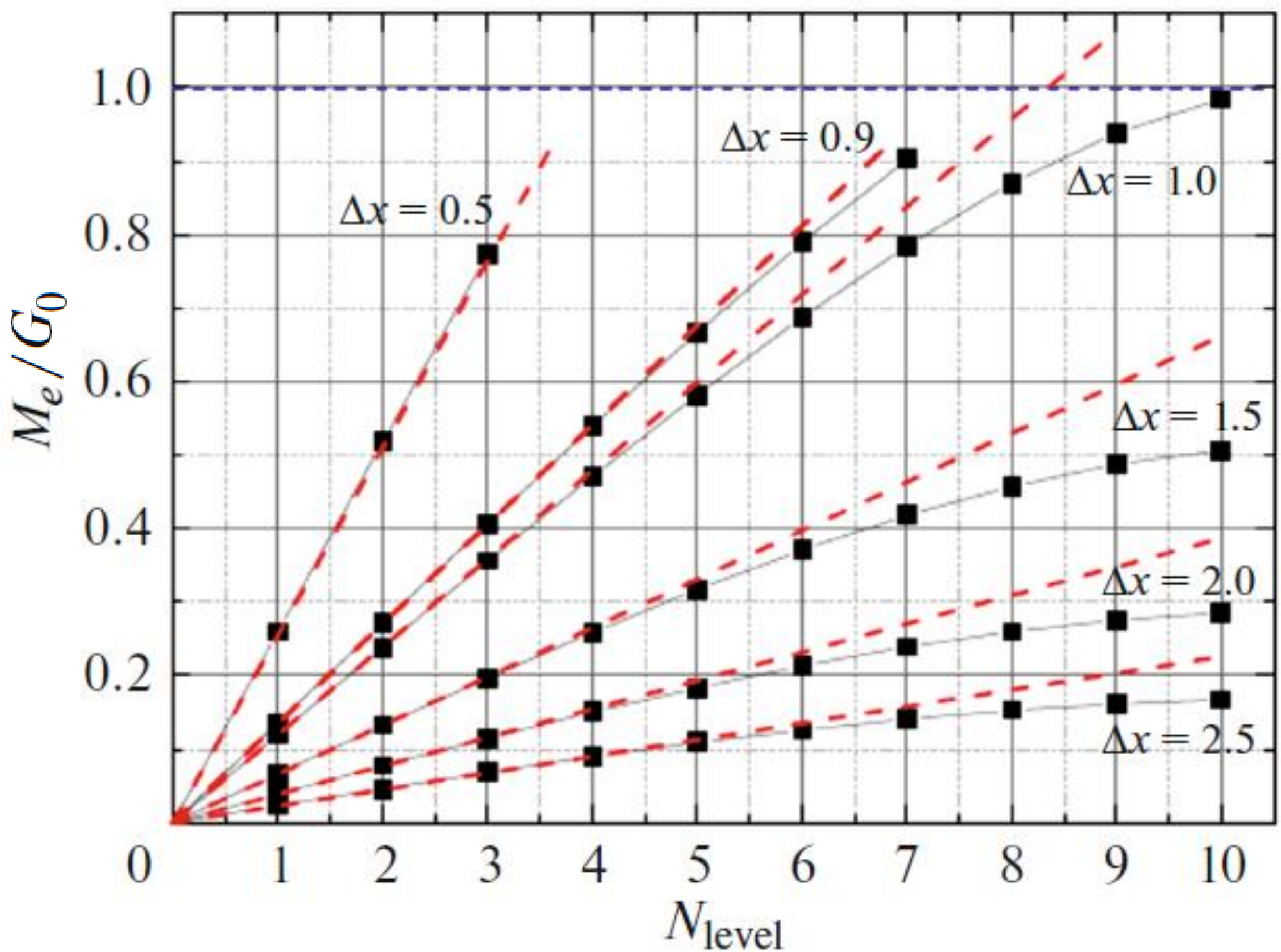
Five-Terrace Structure: Localization



Mass spectrum of Localized States



Mass spectrum of Localized States



Summary and Conclusions:

Graphene with a mass gap is of interest not only from the technological point of view, but also as a convenient model for studying complex quantum effects.

We investigated the possibility of observing one of such quantum effects— localized fermion states at the boundary of a chirality change in the material.

Localized electronic states in an effective model of graphene on a hexagonal silicon carbide substrate with a terrace-stepped surface structure were studied.

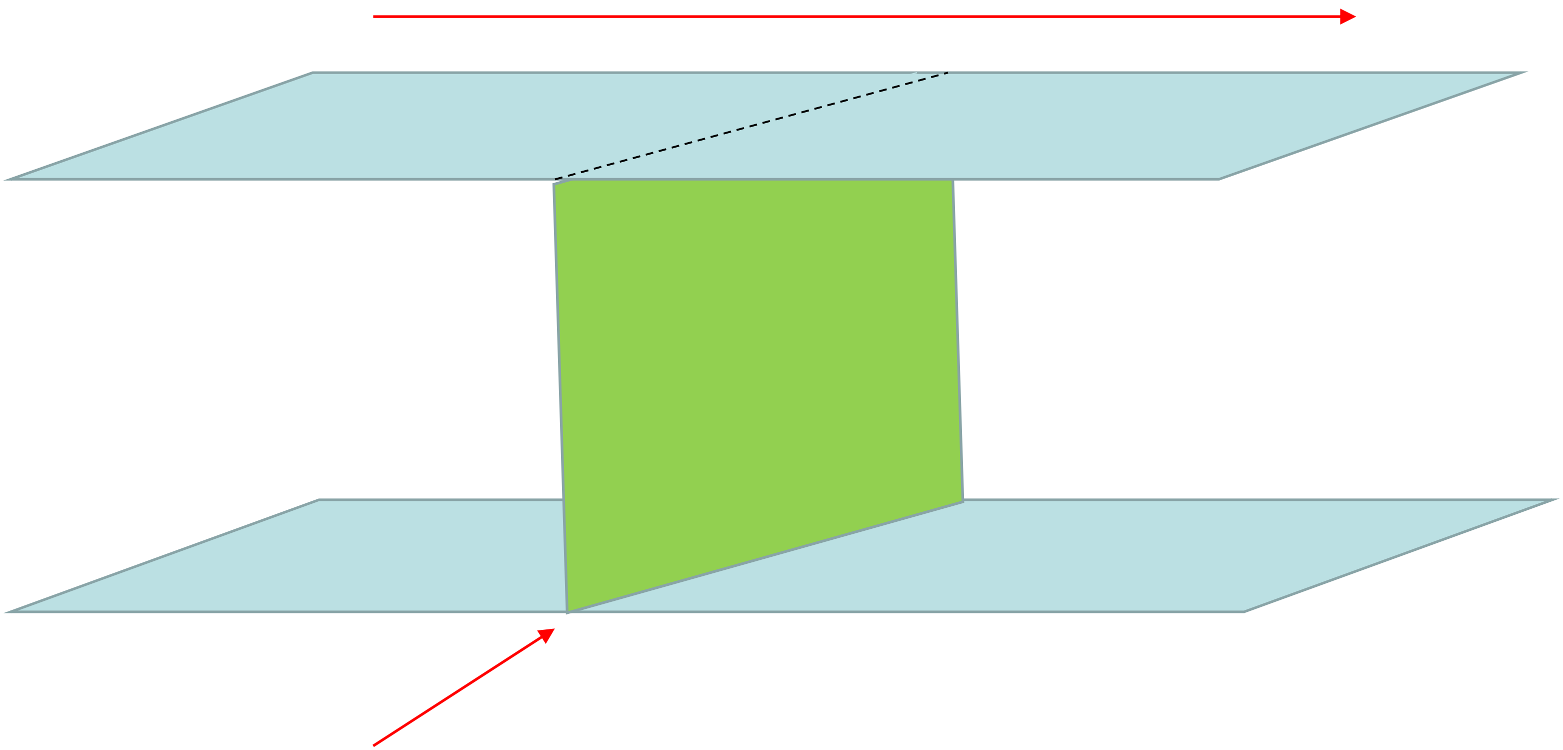
We found out that such a terrace-stepped structure generates not only the known massless topological electronic states, but also massive localized states, and the number of such states, their mass, and localization region are determined by the spatial arrangement of the terraces.

Domain Wall Chiral fermions on the lattice

Domain Wall Chiral fermions on the lattice. Kaplan (1992)

Extra Dimensional idea to solve the problem of chiral fermions on the lattice:

Extra dimension $s = x_5$



3+1 dimensional Minkowski (or 4 dimensional Euclidian) space

Domain Wall Chiral fermions on the lattice

Domain Wall Chiral fermions on the lattice. Kaplan (1992)

$$D_5 = D_4 + \gamma_5 \partial_5 - M(s)$$

$$\chi(x, s) = \exp(ipx) u(s)$$

$$[-\gamma_5 \partial_5 + M(s)] u(s) = 0$$

$$u = \exp\left(\pm \int_0^s ds' \, M(s')\right) v$$

This is Jackiw-Rebbi Zero mode
but in 3+1+1 dim.

$$D_4 = \gamma_\mu \partial_\mu$$

$$\partial_5 = \partial / \partial s$$