Emission of gravitational waves by constant tension domain walls

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Introduction

- Pulsar timing arrays: NANOGrav, EPTA with InPTA, PPTA, and Chinese PTA $\rightarrow$
- stochastic gravitational waves background
- supermassive black hole binaries are the most likely source
- primordial sources are also possible
- we focus on domain walls
- CosmoLattice computer code – Figueroa et al., 2021

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{4}\lambda_\chi \left(\chi^2 - \eta^2\right)^2 \tag{1}$$

Scaling regime

- One domain wall per horizon
- Distance between walls = Hubble radius

$$\rho_{wall} \sim \frac{H^{-2}\sigma_{wall}}{H^{-3}} \sim \sigma_{wall}/t \qquad (2)$$

$\xi_{dw} = \frac{\rho_{wall}}{\sigma_{wall}} t$ – scaling-parameter; $\sigma_{dw} = \frac{2\sqrt{2\lambda}\eta^3}{3}$ – tension of domain walls

$\Delta^2 = \frac{2}{\lambda\eta^2}$ – domain wall width

$\rightarrow \xi = \xi_{sc} = \text{const}$

$k\tau \ll 1 \quad \frac{d\rho_{gw}}{d\ln k} \propto k^3$ Caprini et al., 2009

Scaling regime

$$P \sim \dddot{Q}_{ij}^2/(40\pi M_{pl}^2) \tag{3}$$

$$Q_{ij} \sim M_{wall}/H^2 \tag{4}$$

$$M_{wall} \sim \sigma_{wall}/H^2 \tag{5}$$

$$\rho_{gw} \sim PtH^3 \sim \frac{\sigma_{wall}^2}{40\pi M_{pl}^2} \tag{6}$$

Initial conditions

$$\langle \phi(\mathbf{k})\phi(\mathbf{q})\rangle = A(k)\delta(\mathbf{k}+\mathbf{q}), \qquad \langle \dot{\phi}(\mathbf{k})\dot{\phi}(\mathbf{q})\rangle = B(k)\delta(\mathbf{k}+\mathbf{q})$$

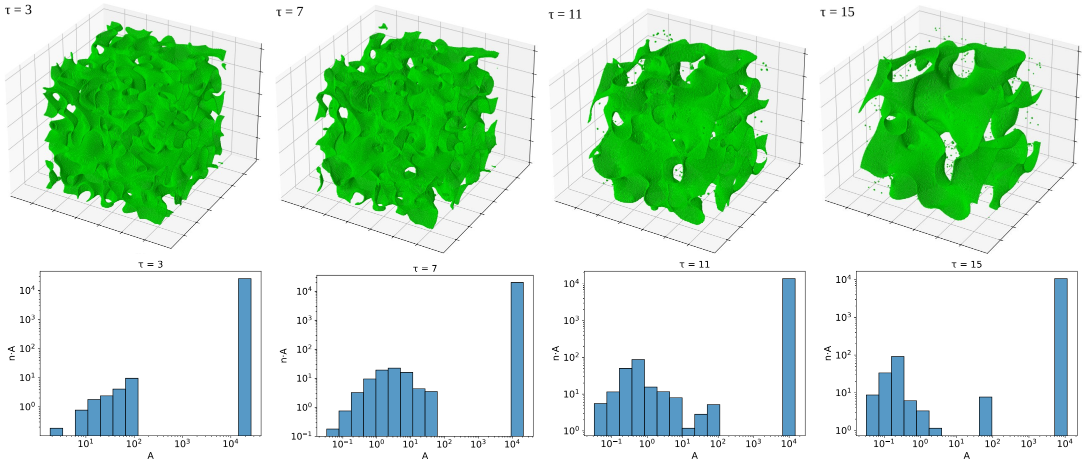
Vacuum initial conditions:

$$A(k) = \frac{\Theta\left(k - k_{cut}\right)}{16\pi^3 k}, \qquad B(k) = \frac{k}{16\pi^3} \cdot \Theta\left(k - k_{cut}\right) \tag{7}$$

Thermal initial conditions:

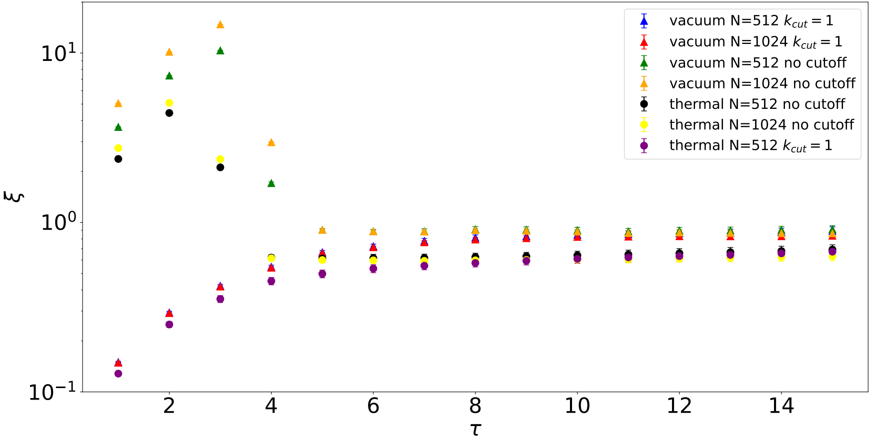
$$A(k) = \frac{1}{8\pi^3 \cdot k\left(e^{\frac{k}{T}} - 1\right)}, \qquad B(k) = \frac{k}{8\pi^3 \cdot \left(e^{\frac{k}{T}} - 1\right)} \tag{8}$$

Snapshots



Vacuum initial conditions, $N = 512$. $S = \Delta x^2 \sum_{links} \delta \frac{|\nabla \chi|}{|\chi_{,x}|+|\chi_{,y}|+|\chi_{,z}|}$

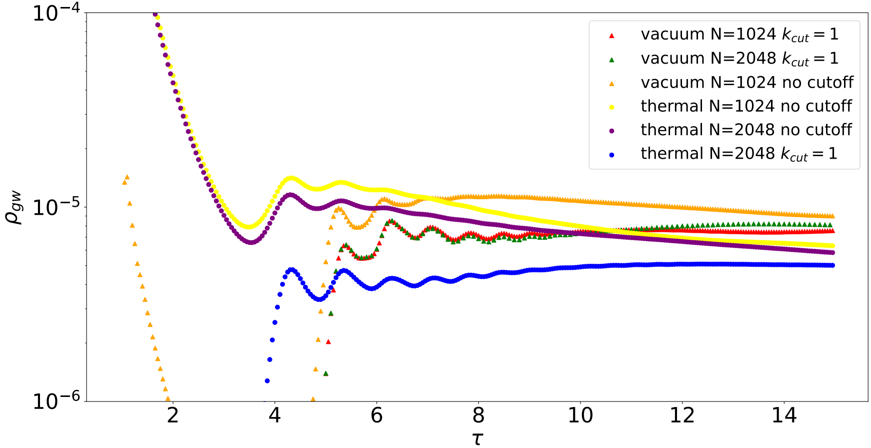
Scaling-parameter



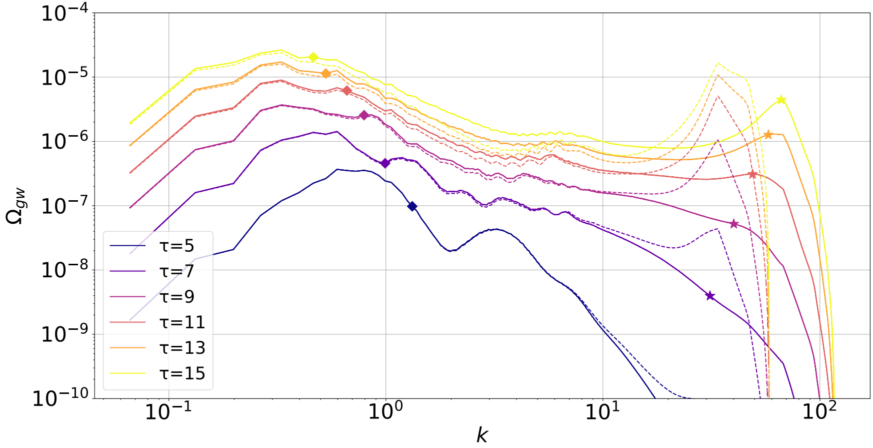
$$\xi = 0.85 \pm 0.04 \qquad \text{(vacuum)} \tag{9}$$

$$\xi = 0.63 \pm 0.04 \qquad \text{(thermal)} \tag{10}$$

GW energy density

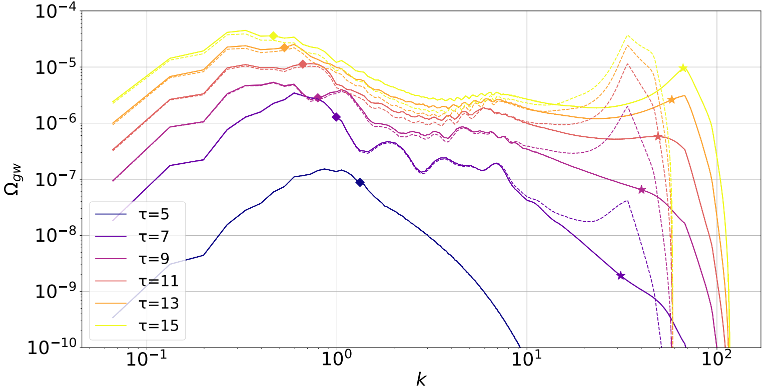


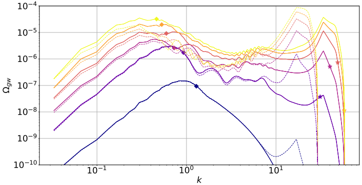
Numerical artefact

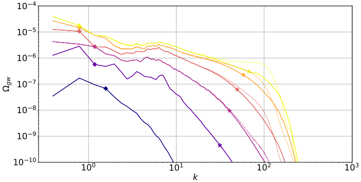


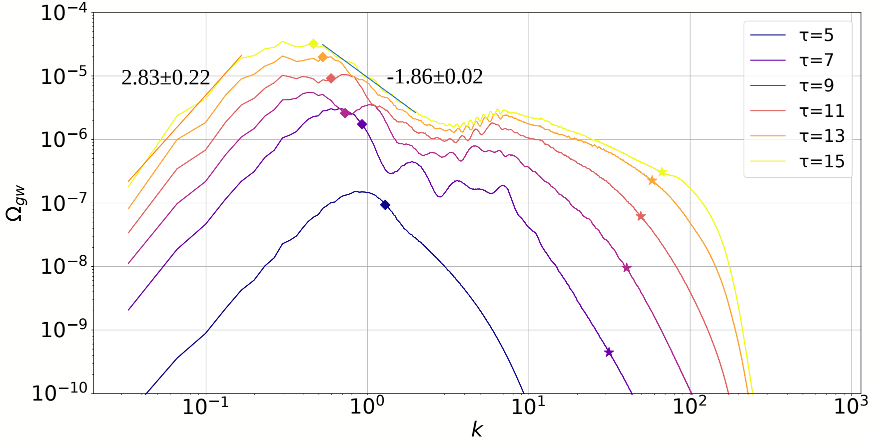
Thermal initial conditions, $k_{cut} = 1,\ N = 2048$

Spectra of GW: vacuum initial conditions





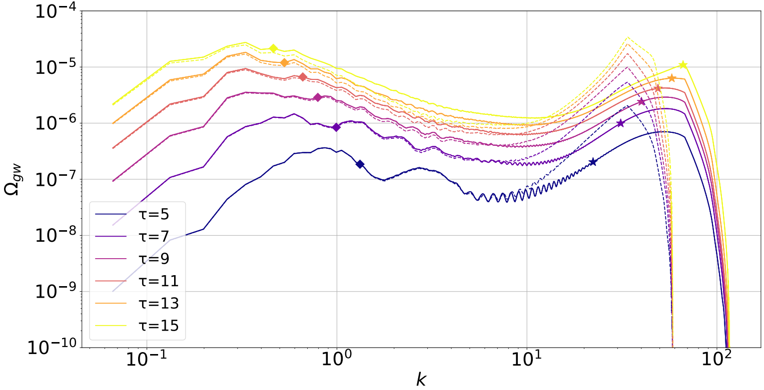


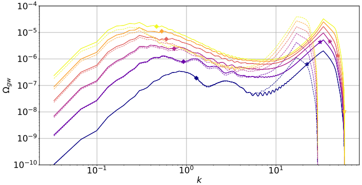


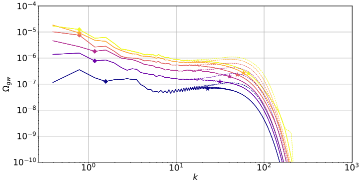
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
Ω_{gw}
2.83±0.22
-1.86±0.02
τ=5
τ=7
τ=9
τ=11
τ=13
τ=15
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
k

$N = 2048, \; k_{cut} = 1$

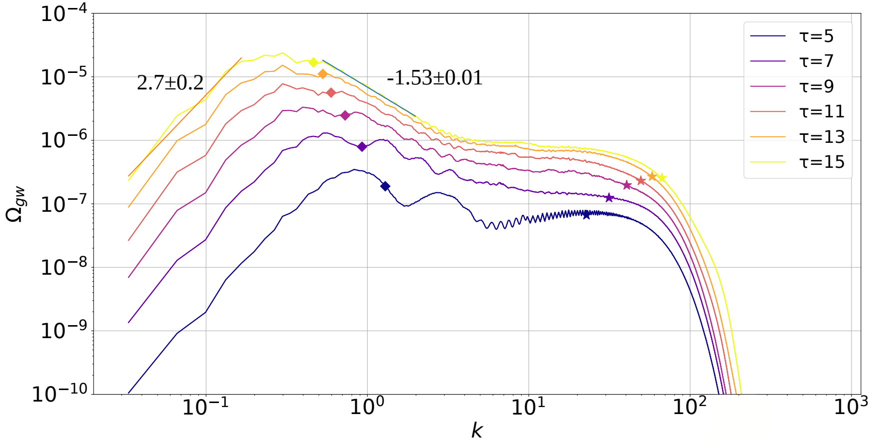
Spectra of GW: thermal initial conditions







Spectra of GW: thermal initial conditions



$N = 2048, \ k_{cut} = k_{max} = \sqrt{3}\pi N/L$

Results

$$\Omega_{gw,peak}(\tau) \approx 7.7(4.6)\times 10^{-10}\cdot\left(\frac{H_i}{H(\tau)}\right)^2\cdot\left(\frac{\eta}{6\cdot 10^{16}\ \text{GeV}}\right)^4 \qquad \text{vacuum (thermal)} \quad (11)$$

$$\Omega_{gw,peak}h_0^2 \approx 1.0(0.6)\times 10^{-10}\cdot\left(\frac{100\ \text{MeV}}{T_{dec}}\right)^4\cdot\frac{\sigma_{dw}^2}{(100\ \text{TeV})^6}\cdot\left(\frac{10}{g_*(T_{dec})}\right)^{4/3} \qquad \text{vacuum (thermal)} \quad (12)$$

$$f_{peak} = F_{peak}(\tau_{dec})\cdot\frac{a_{dec}}{a_0} \simeq 0.7H_{dec}\cdot\frac{a_{dec}}{a_0} \simeq 7.5\ \text{nHz}\left(\frac{T_{dec}}{100\ \text{MeV}}\right)\cdot\left(\frac{g_*(T_{dec})}{10}\right)^{1/6} \quad (13)$$

Summary

- Domain wall network settles to scaling regime: ξ and energy density of GW are constant.
- Different value of ξ in the cases of thermal and vacuum conditions may have physical origin.
- It is possible to get rid of numerical artifact in the UV part of the spectrum.

Potential bias

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{4}\cdot\lambda(\chi^2 - v^2)^2 - V_{breaking}\right] \tag{14}$$

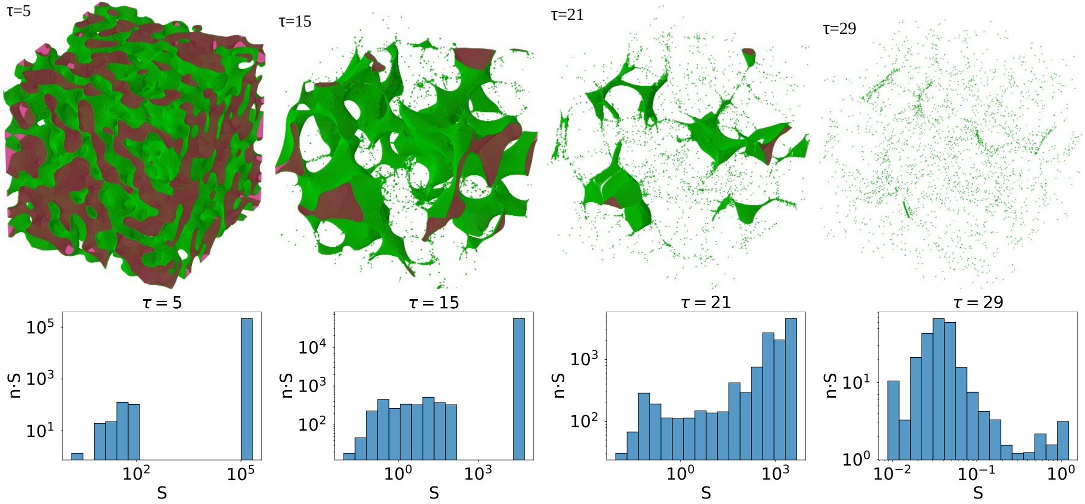
$$V_{breaking} = \epsilon\chi^3 \tag{15}$$

$$V_{bias} = V_{breaking}(\chi \approx v) - V_{breaking}(\chi \approx -v) \approx 2\epsilon v^3 \tag{16}$$

$$V_{bias} \ll \lambda v^4 \qquad \epsilon \ll \lambda v \tag{17}$$

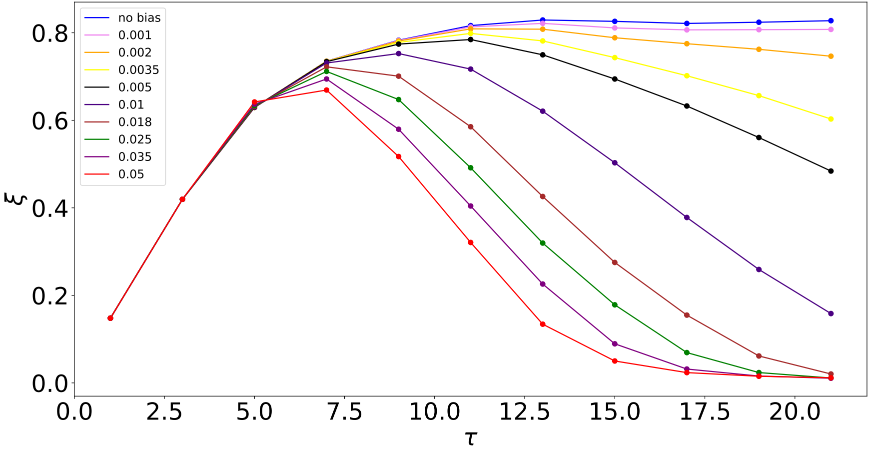
$$\langle\chi^2(\mathbf{x})\rangle = \int_{k_{min}}^{k_{cut}} \frac{dk\cdot k}{4\pi^2}, \qquad \langle\dot{\chi}^2(\mathbf{x})\rangle = \int_{k_{min}}^{k_{cut}} \frac{dk\cdot k^3}{4\pi^2} \tag{18}$$

Snapshots



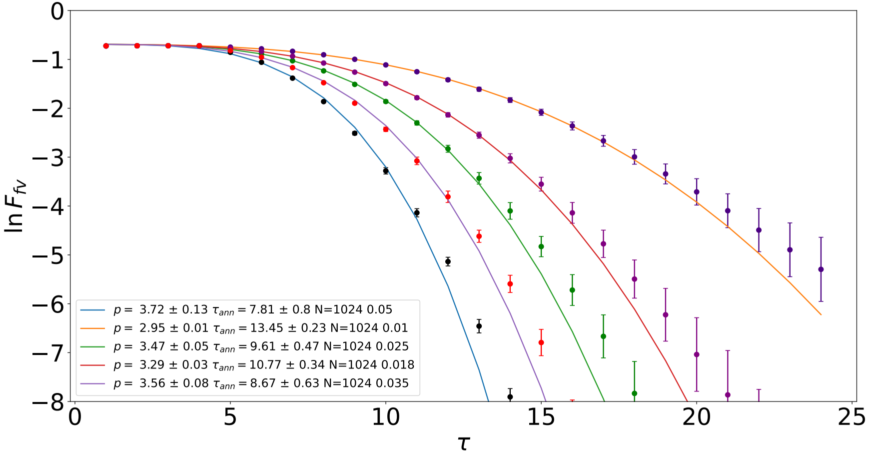
Vacuum initial conditions, $N = 2048$, $\epsilon = 0.01\lambda v$. $S = \Delta x^2 \sum_{links} \delta \frac{|\nabla\chi|}{|\chi_{,x}|+|\chi_{,y}|+|\chi_{,z}|}$

Scaling parameter



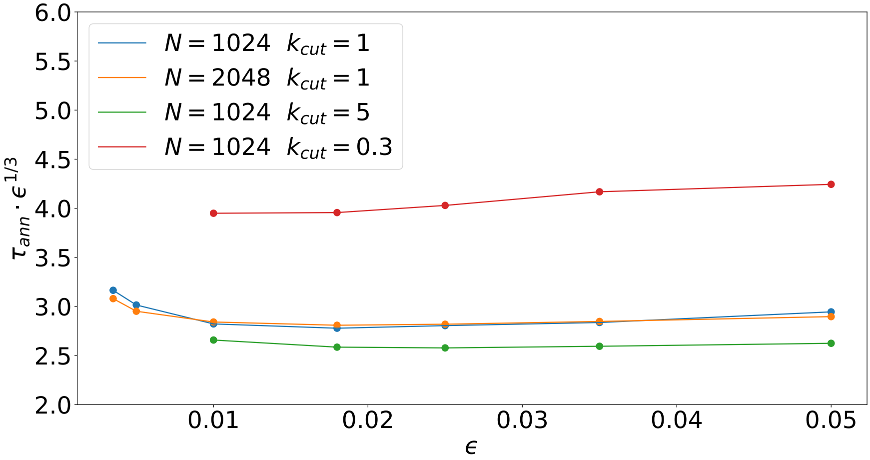
$k_{cut} = 1,\ N = 1024.$

False vacuum fraction



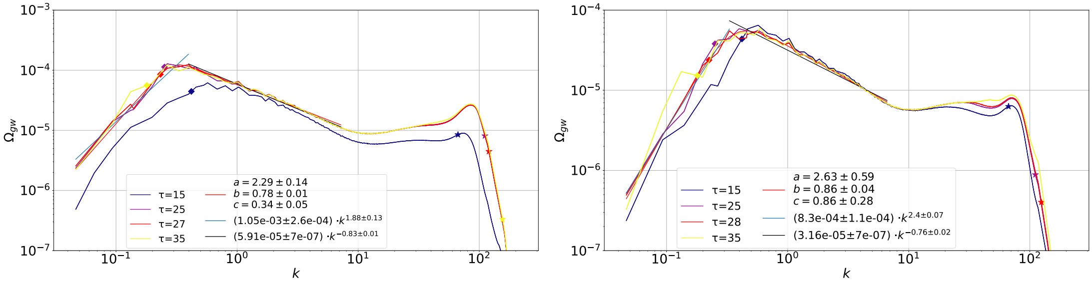
$\mathcal{F}_{fv} = \frac{V_{false}}{V}$. $\mathcal{F}_{fv} = \frac{1}{2} \cdot \exp\left[-\left(\frac{\tau}{\tau_{ann}}\right)^p\right]$. $k_{cut} = 1,\ N = 1024.$

False vacuum fraction



$\rho_{wall} \sim V_{bias} \Rightarrow \tau_{ann} \propto 1/\sqrt{\epsilon}$

Gravitational waves



$k_{cut} = 1,\ N = 2048,\ \epsilon = 0.025\lambda v$ (left) and $\epsilon = 0.05\lambda v$ (right).

$$\Omega_{gw} = \Omega_{gw,peak} \cdot \frac{(a+b)^c}{(bx^{-a/c}+ax^{b/c})^c}$$

Gravitational waves

$$f_{peak} \simeq 8 \text{ nHz}\, \lambda^{1/4} \cdot \left(\frac{\epsilon}{10^{-36} \cdot \lambda v}\right)^{1/3} \cdot \sqrt{\frac{v}{100 \text{ TeV}}} \cdot \left(\frac{100}{g_*(T_{ann})}\right)^{1/12} \quad (19)$$

$$\Omega_{gw,peak} h_0^2 \simeq 1 \cdot 10^{-10} \cdot \left(\frac{v}{100 \text{ TeV}}\right)^4 \cdot \left(\frac{10^{-36} \cdot \lambda v}{\epsilon}\right)^{4/3} \cdot \left(\frac{100}{g_*(T_{ann})}\right)^{1/3} \quad (20)$$

Summary

- Introduction of potential bias solves the DW-domination problem
- Domain walls decay faster than expected from the estimate $\sigma_{wall} H_{ann} \sim V_{bias} \sim \epsilon v^3$
- As a result, GW signal from domain walls is weaker than expected

Thank you for your attention!