Dark Matter, domain walls, gravitational waves

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Outline

- Dark matter via inverse phase transition.

  Spoiler: neither freeze-in, nor freeze-out,
  closer to vacuum misalignment.

- Domain walls and gravitational waves.

  Probing dark matter couplings
  with pulsar timing array (PTA) data.

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{M^2 \cdot \chi^2}{2} - \frac{\lambda \cdot \chi^4}{4} + \frac{g^2 \chi^2 \phi^\dagger \phi}{2} \; .$$

χ is a cold dark matter field ϕ is in thermal equilbrium with plasma

$$\boxed{0 < g^2 \ll 1}$$

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$T \propto \frac{1}{a(t)} \Longrightarrow Z_2$-symmetry breaking at early times

$$\langle \chi \rangle = \sqrt{\frac{Ng^2 T^2(t)}{12\lambda} - \frac{M^2}{\lambda}}$$

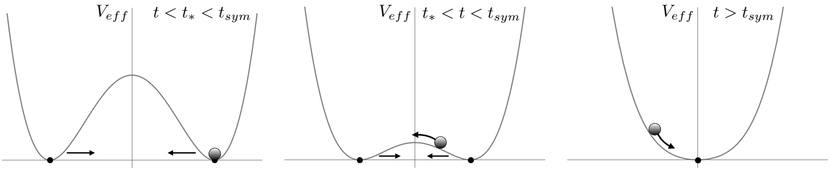
$T \rightarrow T_{sym} = \sqrt{\frac{12M^2}{Ng^2}} \Longrightarrow \langle \chi \rangle \rightarrow 0$ Inverse phase transition

$$\langle\chi\rangle = \sqrt{\frac{Ng^2T^2}{12\lambda} - \frac{M^2}{\lambda}}$$

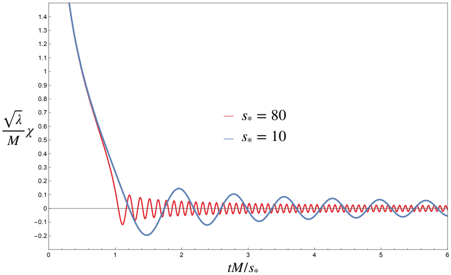
At early times $\chi = \langle\chi\rangle$

Later times: $\frac{d\langle\chi\rangle}{dt} \propto \frac{1}{\langle\chi\rangle} \to \infty$ as $\langle\chi\rangle \to 0$

Tracking stops and χ starts oscillating when $\frac{|\dot{M}_{eff}|}{M_{eff}^2} \sim 1$



Z_2-symmetry + feeble couplings involved protect stability
$\Longrightarrow$ these oscillations naturally feed into dark matter



Dark matter abundance is fulfilled provided that

$$M \simeq 25\ \text{eV} \cdot \frac{\beta^{3/5}}{\sqrt{N}} \cdot \left(\frac{g}{10^{-8}}\right)^{7/5} \qquad \beta \equiv \frac{\lambda}{g^4} \gtrsim 1$$

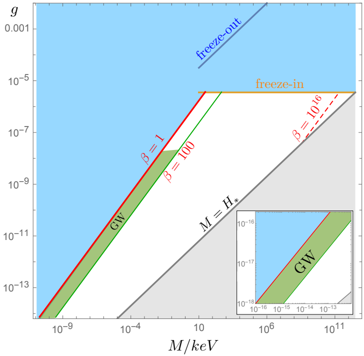
Particles ϕ are relativistic all the way through the phase transition.

$$M \simeq 30\ \text{eV} \cdot \frac{\beta^{3/10}}{\sqrt{N}} \cdot \left(\frac{g}{10^{-8}}\right)^{6/5} \cdot \sqrt{\frac{m_\phi}{10\ \text{GeV}}} \cdot \left(\frac{g_*(T_{sym})}{100}\right)^{1/5}$$

If particles ϕ become non-relativistic at the phase transition.

Inverse phase transition makes possible Dark Matter production in the regions "inaccessible" by freeze-out and freeze-in.

This is in essence "beyond freeze-in mechanism".



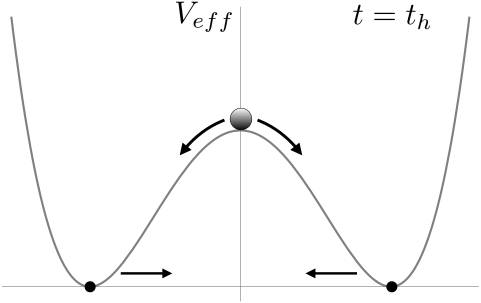
$$\beta \equiv \frac{\lambda}{g^4} \qquad 1 \lesssim \beta \lesssim 10^{18}$$

Green stripes $\Longrightarrow$ potentially testable regions of parameter space.

Melting domain walls

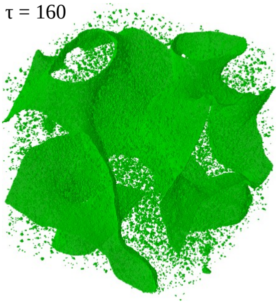
Vilenkin'81 Babichev, Gorbunov, SR, Vikman'21

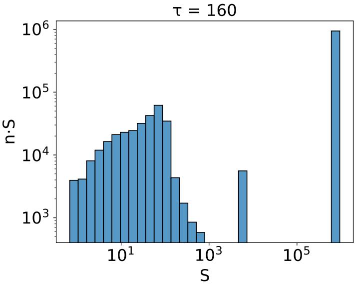
Domain walls are common in models with spontaneous breaking of Z_2-symmetry Zel'dovich et al'74



NB Setting to zero through $\xi R \chi^2/2$ for $\xi \gtrsim 1$

Domain wall evolution





Domain wall: embedding of kink into $1+3$ spacetime

Kink: $\chi(z) = v \cdot \tanh\left(\frac{\sqrt{\lambda}\langle\chi\rangle z}{2}\right)$

CosmoLattice Figueroa, Florio, Torrenti, Valkenburg'20 21

No domain wall problem in our case

$$\boxed{\rho_{wall} \propto \frac{1}{a^5} \quad vs \quad \rho_{rad} \propto \frac{1}{a^4}}$$

Melting domain walls never dominate the Universe and completely vanish at inverse phase transition Vilenkin'81

NB. Conventional domain walls overclose the Universe

$$\sigma_{wall} = \text{const} \Longrightarrow \frac{\rho_{wall}}{\rho_{rad}} \propto a^2(t)$$

Do melting domain walls leave any trace?

Domain walls emit gravitational waves

By construction, domain walls are spatially inhomogeneous scalar field configurations.

$$\left(\frac{\partial^2}{\partial\tau^2} + \frac{2a'}{a}\frac{\partial}{\partial\tau} - \frac{\partial^2}{\partial \mathrm{x}^2}\right) h_{ij}^{TT} = 16\pi G_N T_{ij}^{TT}$$

$$\rho_{gw} = \frac{1}{32\pi G_N a^2} \cdot \langle \frac{\partial h_{ij}^{TT}}{\partial\tau}\frac{\partial h_{ij}^{TT}}{\partial\tau}\rangle$$

Most energetic gravitational waves are emitted, when the domain wall network is being created at the time t_i

Properties of GW emission can be estimated analytically or numerically with CosmoLattice.

GW spectroscopy

$$\Omega_{gw}(f) = \frac{1}{\rho_{tot}} \cdot \frac{d\rho_{gw}}{d\ln f}$$

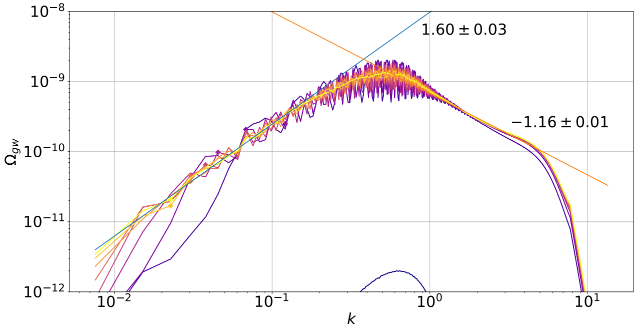
$$\boxed{f_{gw,peak} \approx \frac{15\ \text{nHz}\sqrt{N}}{g_*^{1/3}(T_i)} \cdot \left(\frac{g}{10^{-18}}\right)}$$

$$\boxed{\Omega_{gw,peak} \cdot h_0^2 \approx \frac{5 \cdot 10^{-11} \cdot N^4}{g_*^{7/3}(T_i) \cdot \beta^2}}$$

Vanilla region: $\boxed{\beta \equiv \frac{\lambda}{g^4} \simeq 1}$ $\boxed{N \gg 1}$

For $10^{-18} \lesssim g \lesssim 10^{-8}$ one covers the frequency range of all current and planned GW experiments: PTAs, LISA, Einstein Telescope.

GW spectrum

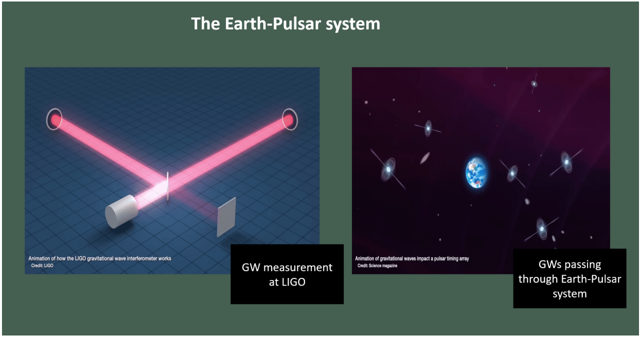


- While the peak is saturated at $t \simeq t_i$, the slope $\Omega_{gw} \propto f^{1.6}$ is due to GW emission at the times $t > t_i$.
- The slope of GWs can be obtained theoretically in the case of infinite duration source: $n = 2$.

$$\rho_{source} \propto \frac{1}{a^{\beta}} \Longrightarrow n = 2\beta - 8$$

Strong evidence of stochastic GW background has been reported: NANOGrav, EPTA+InPTA, CnPTA, PPTA

$$\Omega_{gw}(f) \simeq 5.8 \cdot 10^{-8} \cdot \left(\frac{f}{30\ \text{nHz}}\right)^{n} \qquad n = 1.8 \pm 0.6$$



GW detection with PTAs: Sazhin'78, Detweiler'79, Hellings and Downs'83

Melting walls can explain the observed PTA signal.

Other candidates:

-Supermassive black hole binary (SMBHB) mergers are often quoted as the most common source of the background found, but...

- GW driven SMBHBs predict $n = 2/3$
  versus the PTA $n = 1.8 \pm 0.6$
  excluded at more than 2σ CL
- final pc problem
- SMBHs are difficult to produce, $M \sim 10^{10}\ M_{\odot}$

-Conventional domain walls/first order phase transitions typically lead to the low-frequency behavior $\Omega_{gw} \propto f^3$ (causality tail), which is too steep relative to PTA measurements.

Metastable cosmic strings $\Omega_{gw} \propto f^2$ Buchmuller et al'23

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$g^2 \simeq 10^{-36}$ $\lambda \simeq 10^{-72}$ $N = 24$ $1\ \text{MeV} \lesssim m_\phi \lesssim 10\ \text{MeV}$

$T_i \sim 100\ \text{MeV}$ $1\ \text{MeV} \lesssim T_{sym} \lesssim 10\ \text{MeV}$ BBN!!!

The field χ is extremely weakly coupled!

Not an unfamiliar situation in physics, cf. axions, but we deal with a different group of underlying symmetries.

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DM abundance constraint $\Longrightarrow M \simeq 10^{-13}$ eV $\Longrightarrow$ superradiance of stellar mass Kerr black holes

Zel'dovich'71, Starobinskii'73, Arvanitaki et al'09

Summary

- Inverse phase transition in the early Universe allows for dark matter production in a very feebly coupled regime.
- Melting domain walls naturally arising in this setup produce potentially visible gravitational waves.
- The spectral shape of emitted gravitational waves is in a very good agreement with the recent pulsar timing array (PTA) measurements.

Thanks for your attention!!!