Neutrino dark matter in seesaw models of various types

Dmitrii Kazarkin [1,2],
in collaboration with
Mikhail Dubinin[2] and Elena Fedotova[2]

[1] Lomonosov Moscow State University, Moscow
[2] Skobeltsyn Institute of Nuclear Physics research, Moscow

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Outline

Neutrino warm dark matter

Minimal seesaw. νMSM model.

Seesaw II + Seesaw I in the framework of Left Right Symmetric model

Inverse seesaw model

Conclusions

Appendix

Neutrino mass problem

- Neutrinos have extremely small non-zero masses. Neutrino oscillation data:

$$m_{\text{light}} = \boxed{?} \sqrt{|\Delta m^2_{21}|} \simeq 0.009 \text{ eV} \quad \sqrt{|\Delta m^2_{31}|} = 0.049 \text{ eV} \quad \sqrt{|\Delta m^2_{32|}} = 0.050 \text{ eV}$$

- Neutrino mixing matrix U_{PMNS}

$$\nu_i = \sum_{\alpha} (U_{\mathrm{PMNS}})_{\alpha i} \nu_\alpha$$

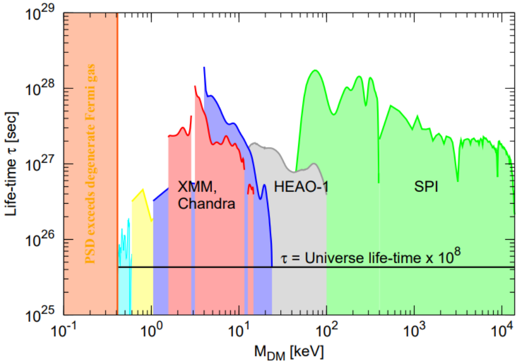
- In Standard model neutrino are massless particles
- Possible natural explanation - **Seesaw mechanism** (and its variations)
  1. Seesaw I or Minimal Seesaw
  2. Seesaw II or VEVs seesaw relation
  3. Inverse seesaw (ISS)
- They can be used as models with fermionic warm dark matter (WDM) candidate $m_{DM} \sim \mathcal{O}(\text{keV})$

keV 'sterile' neutrino as warm DM

Warm Dark Matter: The lightest sterile neutrino with mass $\sim 1 - 10$ keV.

- **Lifetime:** quasi-stable because of very small mixing with active neutrino
$$\tau_{\nu_s} = 10^{19} \left(\frac{m_s}{1 \text{ keV}} \right)^{-5} \frac{1}{\sin^2(2\theta)} \text{ sec} > H_0^{-1} \simeq 10^{17} \text{ sec}$$
- **Non-observation of radiative one-loop decay**

[Aliev, Vysotsky, Sov. Phys. Usp. 24 (1981)]



X-ray bound:
$N_1 \to \gamma, \nu$ with $E_\gamma \simeq M_1/2$
lead to strong lifetime limit
$\tau_{\nu_s} > 10^{25}$ sec

$$\sin^2(2\theta) < 10^{-6} \left(\frac{m_s}{1 \text{ keV}} \right)^{-5}$$

[Boyarsky et al, arXiv:0811.2385v1]

DM relic density: production via oscillations

Framework of 1 $\nu_\alpha + 1\ \nu_s$ states. Mixing parameter is $\sin(\theta)$

- Boltzmann equation for DM production via $\nu_a \leftrightarrow \nu_s$

$$\begin{aligned}
\frac{\partial}{\partial t} f_s(p,t) \quad &- \quad H\,p\,\frac{\partial}{\partial p} f_{N_1}(p,t) = C_{\mathrm{oscl}}(p,t,T) \\
\text{where } C_{oscl} \quad &\approx \quad \frac{\Gamma_\alpha(p)}{2} \sin^2(2\theta_{\mathrm{eff}}) \left[1 + \left(\frac{\Gamma_\alpha(p) l_m}{2}\right)^2\right]^{-1} [f_{\nu_\alpha}(p,t) - f_s(p,t)] \\
\sin^2(\theta_{\mathrm{eff}}) \quad &= \quad \frac{\Delta^2(p)\sin^2 2\theta}{\Delta^2(p)\sin^2 2\theta + \left[\Delta(p)\cos 2\theta - V^D - V^T(p)\right]^2} \qquad \Delta = (m_2^2 - m_1^2)/2p
\end{aligned}$$

[Abazajian et al, Phys.Rev. D64 (2001) 023501]

- It gives estimation for relic density of sterile neutrino

$$\Omega_{\nu_s} h^2 \quad = \quad K_\alpha(m_s) \left(\frac{\sin^2(2\theta)}{10^{-8}}\right) \left(\frac{m_s}{1\ \mathrm{keV}}\right)^2$$

where $K_\alpha \sim 0.3$ with weak dependency on M_1 within the considered limits, $\alpha = e, \mu, \tau$

Seesaw type I mechanism

Additional fields: $SU(2)_L \times U(1)_Y$ - singlets $\nu_{R,k}$, $k = \overline{1,3}$ (flavour basis) with heavy Majorana mass term $\sim M_R$. Mass states N_J, $J = \overline{1,3}$ are heavy neutral leptons (HNL).

Lagrangian: $\mathcal{L} = \mathcal{L}_{SM} + i\overline{\nu}_R \partial_\mu \gamma^\mu \nu_R - \left(Y\, \bar{l}_L \widetilde{\phi} \nu_R + \frac{1}{2} \overline{\nu^c}_R M_R \nu_R + h.c \right),$

After SSB: $\mathcal{L} \supset (\overline{\nu_L},\ \overline{\nu_R^c}) \begin{pmatrix} \mathbb{O} & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}, \quad \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = P_L\, \mathrm{U} \begin{pmatrix} \nu \\ N \end{pmatrix}$

$$\mathrm{U} = \mathrm{WV} \qquad \mathrm{U} = \exp \begin{pmatrix} \mathbb{O} & -\theta \\ \theta^\dagger & \mathbb{O} \end{pmatrix} \simeq \begin{pmatrix} I - \frac{1}{2}\theta\theta^\dagger & -\theta \\ \theta^\dagger & I - \frac{1}{2}\theta^\dagger\theta \end{pmatrix} + \mathcal{O}(\theta^3)$$

$$\mathrm{V} = \begin{pmatrix} U_\nu & 0 \\ 0 & U_N \end{pmatrix} \qquad m_\nu \equiv U_\nu \hat{m} U_\nu^T \qquad M_N \equiv U_N \hat{M} U_N^T$$

here $\hat{\square} = \mathrm{diag}(\dots)$ - diagonal matrix.

$(\nu - N)$**-mixing:** $\quad \Theta \equiv \theta U_N$

PMNS: $\quad U_{\mathrm{PMNS}} = \left(I - \frac{1}{2}\theta^\dagger\theta \right) U_\nu$

Heavy neutrino mixing and νMSM

A system of matrix equations for diagonalizing transformation U:
(leading order θ-accuracy)

$$\begin{cases} \theta \simeq m_D M_R^{-1}, \\ m_\nu = -\theta M_R \theta^T, \\ M_N \simeq M_R \end{cases} \Rightarrow \qquad \overset{\text{seesaw I equation}}{\boxed{m_\nu = -m_D M_N^{-1} m_D^T}}$$

Seesaw I equation can be rewritten as a so-called **Casas-Ibarra parametrization:** `[Casas J., Ibarra A., Nucl.Phys.B 618 (2001) 171.]`

$$I = \Omega\Omega^T = \left[i\sqrt{\tilde{m}^{-1}}U_\nu^\dagger m_D U_N \sqrt{\hat{M}^{-1}}\right]^T \left[-i\sqrt{\tilde{m}^{-1}}U_\nu^\dagger m_D U_N \sqrt{\hat{M}^{-1}}\right],$$

$$m_D = iU_{\mathrm{PMNS}}^\dagger \sqrt{\hat{m}}\Omega\sqrt{\hat{M}^{-1}} \rightarrow \Theta = iU_{\mathrm{PMNS}}^\dagger \sqrt{\hat{m}}\Omega\sqrt{\hat{M}^{-1}}$$

νMSM - model `[T.Asaka, M.Shaposhnikov,Phys.Lett.B 620, 17(2005)]`

3 sterile neutrino:
N_1 - WDM with $M_1 \sim \mathcal{O}(\mathrm{keV})$
N_2 and N_3 heavy neutrinos with masses $M_2 \simeq M_3 \sim \Lambda_{EW}$,
$\Delta = |M_2 - M_3| \ll M_{2,3}$ need for Resonant leptogenesis (lepton asym. → baryon asym.)

Short overview of LRSM: Higgs fields

Higgs fields	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$
$\Delta_L = \begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & -\frac{\delta_L^+}{\sqrt{2}} \end{pmatrix}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	2
$\Delta_R = \begin{pmatrix} \frac{\delta_R^+}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & -\frac{\delta_L^+}{\sqrt{2}} \end{pmatrix}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	2
$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	0

Table: Representations of the Higgs fileds in LRSM

$$\mathcal{L}_{Higgs} = tr|D_\mu \Phi|^2 + tr|D_\mu \Delta_R|^2 + tr|D_\mu \Delta_L|^2 - V(\Phi, \Delta_L, \Delta_R)$$

`[P.S. Bhupal Dev et al.  JHEP 02 (2019) 154]`

Short overview of LRSM

1. **LR-model gauge group:** $G_{LR} = SU(3)_c \times SU(2)_R \times SU(2)_L \times U(1)_{B-L}$
2. $SU(2)_R \times U(1)_{B-L} \xrightarrow[M_R \sim \mathcal{O}(10\ \text{TeV})]{\langle \Delta_R \rangle} U(1)_Y/\mathbb{Z}_2$
   with VEV of Higgs right triplet Δ_R
3. $SU(2)_L \times U(1)_Y \xrightarrow{\langle \Phi \rangle, \langle \Delta_L \rangle} U(1)_{em}, \quad Q_{\rm em} = T_{3R} + T_{3L} + \frac{B-L}{2}$

Vacuum structure of Higgs fields

$$\langle \Delta_{L,R} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix} \quad \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}, \quad \sqrt{k_1^2 + k_2^2} = 246\ \text{GeV}$$

Leptons	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$
$L_{\alpha_L} = \begin{pmatrix} \nu_{\alpha_L} \\ l_{\alpha_L} \end{pmatrix}$	**1**	**2**	**1**	-1
$L_{\alpha_R} = \begin{pmatrix} \nu_{\alpha_R} \\ l_{\alpha_R} \end{pmatrix}$	**1**	**1**	**2**	-1

Table: Representations of the lepton fields in LRSM

Seesaw relation for Higgs triplet VEVs

β-sector of the Higgs potential:

$$\begin{aligned} V_\beta(\Phi,\Delta_L,\Delta_R) \quad = \quad & \beta_1\Big(Tr[\phi\Delta_R\phi^\dagger\Delta_L^\dagger]+Tr[\phi^\dagger\Delta_L\phi\Delta_R^\dagger]\Big)+\beta_2\Big(Tr[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger]+Tr[\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger]\Big)+ \\ & \beta_3\Big(Tr[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger]+Tr[\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger]\Big), \end{aligned}$$

Additional GUT and/or SUSY assumptions → $\beta_i = 0$ or $\beta_i \simeq 0$

Seesaw relation between v_L and v_R

$$\boxed{v_L = \gamma\,\frac{(246\ \text{GeV})^2}{v_R},} \qquad \text{where} \quad \gamma \equiv \frac{\beta_2 k_1^2 + \beta_1 k_1 k_2 + \beta_3 k_2^2}{(2\rho_1 - \rho_3)(246\ \text{GeV})^2}\ ,$$

$\beta_i = 0:\quad (2\rho_1 - \rho_3)v_R v_L = 0$

$\boxed{v_L = 0}$

General LR-condition:
→ $v_R \neq 0$
Vacuum stability
→ $(2\rho_1 - \rho_3) \neq 0$

$\beta_i \to 0$

$$\boxed{v_L \simeq \frac{(246\ \text{GeV})^2}{v_R}}$$

$(v_R \gg 246\ \text{GeV} \quad \text{or}\ \gamma \ll 1)$

Neutrino mixing in LRSM: Seesaw I + II

$$\begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} \qquad \begin{array}{c} M_D = \frac{1}{\sqrt{2}}(h_L k_1 + \tilde{h}_L k_2), \\ M_L = \sqrt{2} h_M v_L, \quad M_R = \sqrt{2} h_M v_R, \end{array}$$

here $h_L, \ \tilde{h}_L, \ h_M$ are **Yukawa couplings** with left triplet Δ_L and bi-doublet Φ

Mixing matrix in Casas-Ibarra parametrization

seesaw II equation: $\boxed{m_\nu = M_L - M_D M_N^{-1} M_D^T}$

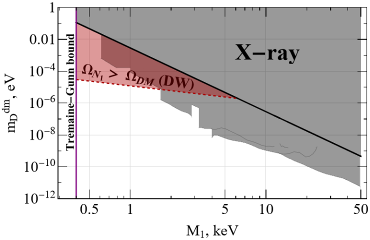
$$\Theta = i U_\nu \left(\sqrt{\tilde{m}}\right) \Omega \sqrt{\hat{M}^{-1}}, \qquad \sqrt{\tilde{m}} = \sqrt{\hat{m} - U^\dagger_{\mathrm{PMNS}} M_L U^*_{\mathrm{PMNS}}}$$

Assumption: $U_N = I, \ \theta^2 \ll 1$

$$h_M \simeq \frac{\hat{M}}{\sqrt{2} v_R} \Rightarrow \boxed{\tilde{m} = \hat{m} - \frac{v_L}{v_R} U^\dagger_{\mathrm{PMNS}} \hat{M} U^*_{\mathrm{PMNS}}}$$

DM mixing: seesaw I and LRSM

$$m_D^{dm} = |\sqrt{\tilde{m}}|^2_{kn}\Omega_{n1}\Omega^*_{k1} \qquad |\Theta_{DM}|^2 = \frac{m_D^{dm}}{M_1}$$

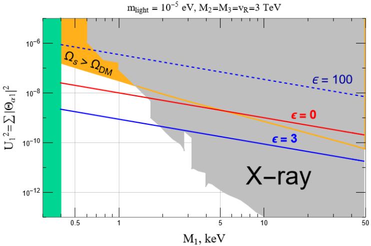


ν-MSM limit: pure seesaw I

$v_L = 0 \to \sqrt{\tilde{m}} = \sqrt{\hat{m}}$ Strong DM constraints and ν-oscillation data lead to a fine-tuned (FT) form of Ω

$$\Omega_{NH} : \quad \Omega_{j1} \;\to\; \delta_{j1}$$
$$\Omega_{IH} : \quad \Omega_{j3} \;\to\; \delta_{j3}$$

$$\boxed{m_D^{dm}(v_L = 0, \Omega \to \mathrm{FT}) = m_{\text{light}}}$$



LRSM with $v_L \neq 0$: seesaw I + II

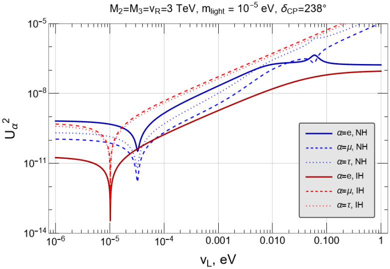
- $m_{\text{light}} \gg v_L \frac{\max M_J}{v_R}$ – νMSM-limit;
- $m_{\text{light}} \ll v_L \frac{\max M_J}{v_R}$ – **seesaw II dominance** → strong increase in mixing, inconsistent with DM constraints;
- $\hat{m} \simeq U^\dagger_{\text{PMNS}} M_L U^*_{\text{PMNS}}$ → DM mixing decreases by 1-2 order due to **seesaw I – seesaw II** cancellation for some entries of $|\sqrt{\tilde{m}}|^2_{kn}$

Cancellation effect in the mixing: seesaw I + II

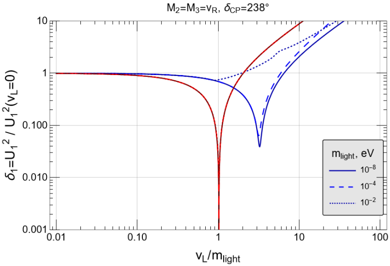
ν**MSM-benchmark for** Ω : $\quad m_D^{dm} = \left| \sqrt{\hat{m} - \frac{v_L}{v_R} U^{\dagger}_{\mathrm{PMNS}} \hat{M} U^{*}_{\mathrm{PMNS}}} \right|^2_{11\text{ (NH) or }33\text{ (}}$

$$\Omega^{(FT)}_{\mathrm{NH}} = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & \multicolumn{2}{c}{\multirow{2}{*}{$\Omega_{2\times 2}$}} \\ 0 & & \end{array} \right), \qquad \Omega^{(FT)}_{\mathrm{IH}} = \left(\begin{array}{ccc} 0 & \multicolumn{2}{c}{\multirow{2}{*}{$\Omega_{2\times 2}$}} \\ 0 & & \\ 1 & 0 & 0 \end{array} \right).$$

$$U^2_{\alpha} = \sum_{I=1}^{3} |\Theta_{\alpha I}|^2$$



$$U^2_I = \sum_{\alpha=e,\mu,\tau} |\Theta_{\alpha I}|^2, \quad |\Theta_{\mathrm{DM}}|^2 = U^2_1$$



Inverse seesaw: General approach $ISS(\mathrm{p},\mathrm{q})$

Field content of the $ISS(\mathrm{p},\mathrm{q})$

Consider **three types** of neutral lepton fields:

– left-handed flavour neutrino $\nu_{L\ \alpha}$, $\alpha = e, \mu, \tau$,
– right-handed neutrino $N_{R\ a}$ $a = \overline{1,\mathrm{p}}$
– right-handed sterile fermions $S_{R\ b}$ $b = \overline{1,\mathrm{q}}$

Lagrangian after SSB: **(Naturalness condition:** $\mu \ll m_D < M_R$)

$$\mathcal{L}_{ISS} = \frac{1}{2}(\overline{\nu}_L, \ \overline{\nu^c_R}, \ \overline{\Sigma^c_R}) \begin{pmatrix} \mathbb{O}_{3\times 3} & m_{D\ 3\times p} & \mathbb{O}_{3\times q} \\ m^T_{D\ p\times 3} & \mathbb{O}_{p\times p} & M_{R\ p\times q} \\ \mathbb{O}_{q\times 3} & M^T_{R\ p\times q} & \mu_{\ q\times q} \end{pmatrix} \begin{pmatrix} \nu^c_L \\ \nu_R \\ \Sigma_R \end{pmatrix}$$

Diagonalization: step 1

Rewrite mass matrix to the **seesaw I-like** form $\tilde{m}_D \equiv (m_D, \ 0)$

$$\mathrm{M}_{9\times 9} = \begin{pmatrix} \mathbb{O}_{\ 3\times 3} & \widetilde{m}_{D\ 3\times (p+q)} \\ \widetilde{m}^T_{D\ (p+q)\times 3} & \mathcal{X}_{\ q\times q} \end{pmatrix} \quad \text{where} \quad \mathcal{X} = \begin{pmatrix} \mathbb{O}_{\ p\times p} & M_{R\ p\times q} \\ M^T_{R\ q\times p} & \mu_{\ q\times q} \end{pmatrix}$$

$$\mathrm{U}^T \mathrm{M} \mathrm{U}, \quad \mathrm{U} = \mathrm{W} \begin{pmatrix} U_{\nu\ 3\times 3} & \mathbb{O}_{3\times (p+q)} \\ \mathbb{O}_{(p+q)\times 3} & \mathcal{U}_{\ (p+q)\times(p+q)} \end{pmatrix} \quad \mathrm{W} = \exp(\omega) \simeq \mathrm{I} + \omega + \ldots$$

Inverse seesaw: ISS(p,q) diagonalization

Shur complement

Let $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ then Shur complement is $(M|D) \equiv A - BD^{-1}C$

Neutrino effective mass operator

$$m_\nu = (\mathrm{M}|\mathcal{X}) \equiv -\tilde{m}_D \mathcal{X}^{-\mathbf{1}} \tilde{m}_D^T = -(m_D,\ \mathbb{O}) \begin{pmatrix} (\mathcal{X}|\mu)^{-\mathbf{1}} & \star \\ \star & \star \end{pmatrix} \begin{pmatrix} m_D^T \\ \mathbb{O} \end{pmatrix} = m_D \left((M_R)\mu^{-\mathbf{1}}(M_R)^T \right)^{-\mathbf{1}} m_D^T$$

Only if $p = q$:

$$\boxed{m_\nu = m_D (M_R^T)^{-1} \mu (M_R)^{-1} m_D^T \sim \frac{\mu m_D^2}{M^2}}$$

Diagonalization: step 2 (Sterile block)

Case 1: $\boxed{p = q}$ All sterile fields form pseudo-Dirac pairs

$$\mathcal{X}' = \mathcal{U} \begin{pmatrix} \mathbb{O}_{p\times p} & M_{R\ p\times q} \\ M_{R\ q\times p}^T & \mu \end{pmatrix} \mathcal{U}^T = \begin{pmatrix} \sim -M_R + \mu & \mathcal{O}(\mu) \\ \mathcal{O}(\mu) & \sim M_R + \mu \end{pmatrix}$$

Inverse seesaw: Toy-model ISS(1,1)

Toy-model ISS(1,1): $m_\pm \equiv \frac{\mu \pm \sqrt{\mu^2 + 4M^2}}{2}$

$$\mathcal{X} = \begin{pmatrix} 0 & M \\ M & \mu \end{pmatrix} \to \begin{pmatrix} m_- & 0 \\ 0 & m_+ \end{pmatrix} \simeq \begin{pmatrix} \mu - M & 0 \\ 0 & \mu + M \end{pmatrix}$$

$$U = \begin{pmatrix} \frac{1}{\sqrt{1+\frac{m_-^2}{M^2}}} & \frac{1}{\sqrt{1+\frac{m_+^2}{M^2}}} \\ \frac{m_-}{M\sqrt{1+\frac{m_-^2}{M^2}}} & \frac{m_+}{M\sqrt{1+\frac{m_+^2}{M^2}}} \end{pmatrix} \simeq \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

S'', N'' are mass-states, N', S' - "middle" basis states (after the 1st step of diagonalization) **Pseudo-dirac** HEAVY ($\sim M$) states with small mass splitting $\sim \mu$.

$$N'' \equiv N_- \simeq \frac{1}{\sqrt{2}}(N' - S')$$
$$S'' \equiv N_+ \simeq \frac{1}{\sqrt{2}}(N' + S')$$

There is no WDM candidate!

Inverse seesaw: Toy-model ISS(1,2)

Toy-model ISS(1,2): $\boxed{m^0_\pm \simeq \pm\sqrt{M_1^2+M_2^2}+\mathcal{O}(\mu) \qquad m_3 \simeq \mathcal{O}(\mu)}$

$$\mathcal{X} = \begin{pmatrix} 0 & M_1 & M_2 \\ M_1 & \mu_1 & 0 \\ M_2 & 0 & \mu_2 \end{pmatrix} = \mathcal{X}_0 + \delta\mathcal{X}(\mu) \qquad \mathcal{X}_I \to \begin{pmatrix} \boxed{\sim 0} & 0 & 0 \\ 0 & m_- & 0 \\ 0 & 0 & m_+ \end{pmatrix}$$

Mass of light sterile state: $\boxed{m_3 \approx \frac{M_2^2\mu_1 + M_1^2\mu_2}{M_1^2+M_2^2} \sim \mu \sim \mathcal{O}(\text{keV})}$

$$\begin{pmatrix} \frac{M_1}{\sqrt{2(M_1^2+M_2^2)}} & \frac{M_1}{\sqrt{2(M_1^2+M_2^2)}} & \frac{M_2}{\sqrt{M_1^2+M_2^2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{M_2}{\sqrt{2(M_1^2+M_2^2)}} & \frac{M_2}{\sqrt{2(M_1^2+M_2^2)}} & -\frac{M_1}{\sqrt{M_1^2+M_2^2}} \end{pmatrix} \qquad \begin{pmatrix} N' \\ S_1' \\ S_2' \end{pmatrix} = U \underbrace{\begin{pmatrix} N_+ \\ N_- \\ \boxed{\mathcal{S}_3} \end{pmatrix}}_{phys.states}$$

There are light sterile fermions ($q-p$ states) with μ-scale mass when $p<q$ in $ISS(p,q)$ model.
This fact has already been highlighted in `[Asmaa Abada et al JCAP10(2014)001]`

Conclusions

- We explored a possible scenario for warm dark matter (WDM) realization within the **Minimal Left-Right Model (MLRM)** and **Inverse Seesaw Model (ISS)**.
- ISS(2,3) or ISS($p, p+s$), $p, s \in \mathbb{N}$: this Inverse Seesaw models can naturally provide a light warm dark matter candidate - either . In this scenario, the lepton number violation scale μ should be of the order of keV.
- A modified mixing matrix formula was derived, incorporating the effects of non-zero vacuum expectation value (VEV) of the left Higgs triplet:

$$\Theta = iU_{\text{PMNS}}\sqrt{\tilde{m}}\Omega\sqrt{\hat{M}^{-1}}, \qquad \sqrt{\tilde{m}} = \sqrt{\hat{m} - U^{\dagger}_{\text{PMNS}} M_L U^{*}_{\text{PMNS}}}$$

- A suppression regime was identified, where the mixing of the dark matter candidate N_1 is significantly reduced (**by 1–3 orders of magnitude**) due to competing contributions from **Type-I and Type-II seesaw mechanisms** (i.e., for $v_L \neq 0$ and $m_{\text{light}}/v_L \sim \max M_J/v_R$).
- The ν**MSM** case was embedded into the left-right symmetric model as a **limiting scenario**.

Acknowledgments

Thank you for your attention

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Backup slides

Масштабы масс калибровочного и хиггсовского сектора

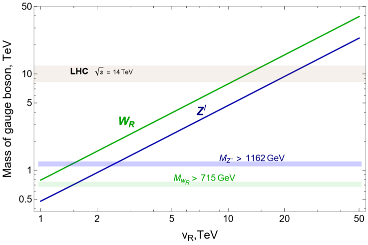


Figure: Masses of new vector gauge bosons Z_2 and W_2

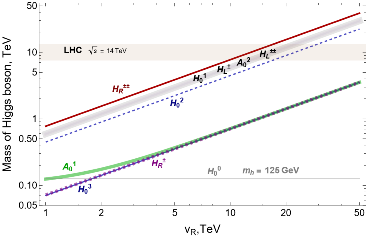
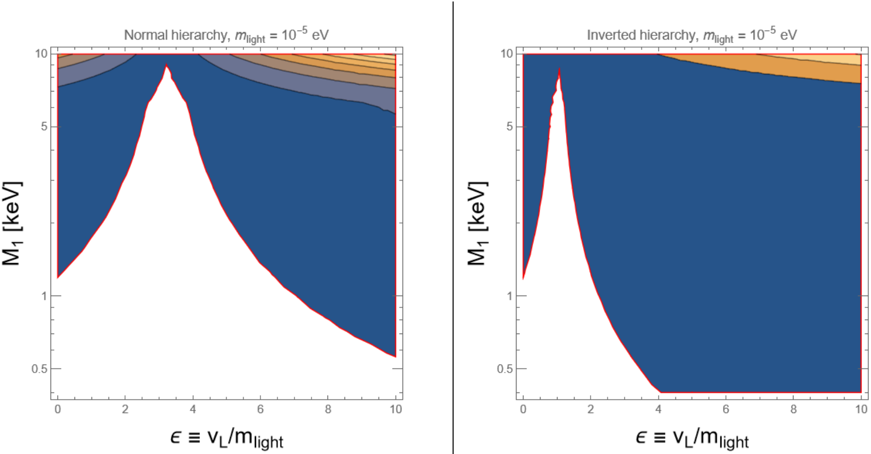


Figure: Masses of all 14 Higgs bosons in LRSM with tuning of self-interaction constants: $\alpha_3 = 0.01$ $\rho_1 = 0.1$, $\rho_2 = 0.3$, $\rho_3 = 0.9$,
$\lambda_1 = \lambda_{SM} = 0.118$,
$\lambda_2 = 0.01$, $\lambda_3 = 0.1$

Заряженные и нейтральные токи

$$\begin{aligned}
\mathcal{L}_{CC}^{\nu} &= \frac{g}{\sqrt{2}}(U_{\mathrm{PMNS}})_{\alpha i}\ \bar{l}_{\alpha}\ \hat{W}_1\,(\cos\zeta P_L - \sin\zeta P_R)\,\nu_i \\
&\quad + \frac{g}{\sqrt{2}}(U_{\mathrm{PMNS}})_{\alpha i}\ \bar{l}_{\alpha}\ \hat{W}_2\,(\sin\zeta P_L + \cos\zeta P_R)\,\nu_i + h.c. && \text{(1a)} \\
\mathcal{L}_{NC}^{\nu} &= \frac{g}{2c_W}\left(U_{\mathrm{PMNS}}^{\dagger} U_{\mathrm{PMNS}}\right)_{ij} \bar{\nu}_i \sum_{X=Z_1,Z_2} \hat{X}\left(a_X^{(L)} P_L - a_X^{(R)} P_R\right)\nu_j && \text{(1b)} \\
\mathcal{L}_{CC}^{N} &= -\frac{g}{\sqrt{2}}\Theta_{\alpha J}\ \bar{l}_{\alpha}\ \hat{W}_1\,(\cos\zeta P_L - \sin\zeta P_R)\,N_J \\
&\quad - \frac{g}{\sqrt{2}}\Theta_{\alpha J}\ \bar{l}_{\alpha}\ \hat{W}_2\,(\sin\zeta P_L + \cos\zeta P_R)\,N_J + h.c. && \text{(1c)} \\
\mathcal{L}_{NC}^{N} &= \frac{g}{2c_W}(\Theta^{\dagger}\Theta)_{IJ}\ \bar{N}_I \sum_{X=Z_1,Z_2} \hat{X}\left(a_X^{(L)} P_L - a_X^{(R)} P_R\right) N_J + && \text{(1d)} \\
&+ \left(\frac{g}{2c_W}\left(U_{\mathrm{PMNS}}^{\dagger}\Theta\right)_{iJ}\ \bar{\nu}_i \sum_{X=Z_1,Z_2} \hat{X}\left(a_X^{(L)} P_L - a_X^{(R)} P_R\right) N_J + h.c.\right)
\end{aligned}$$

Эффект сокращения seesaw I и II (доп)



Inverse Seesaw and Casas-Ibarra Parametrization

ISS with (p=q)

Inverse Seesaw Formula

$$m_\nu = m_D (M_R^T)^{-1} \mu (M_R)^{-1} m_D^T \qquad \text{(ISS with } p = q\text{)}$$

$$\tilde{\theta} = \tilde{m}_D \mathcal{X}^{-1} = (m_D, \ \ \mathbb{O}) \begin{pmatrix} (\mathcal{X}|\mu)^{-1} & (\mathcal{X}|\mu)^{-1} M_R \mu^{-1} \\ \star & \star \end{pmatrix}$$

Casas-Ibarra Parametrization

Through matrix decomposition:

$$\Omega = \sqrt{\hat{m}}^{\ -1} U_\nu m_D (M_R^T)^{-1} \sqrt{\mu} \qquad\qquad m_D = U_\nu^\dagger \sqrt{\hat{m}}\, \Omega \sqrt{\mu}^{\ -1} M_R^T$$

where Ω is orthogonal ($\Omega\Omega^T = I$).

$\nu - N$ Mixing

$$\theta_1 = U_\nu^\dagger \sqrt{\hat{m}}\, \Omega \sqrt{\mu} M_R^{-1} \sim \mathcal{O}\left(\frac{\sqrt{m_\nu \mu}}{M}\right)$$

$\nu - S$ Mixing

$$\theta_2 = U_\nu^\dagger \sqrt{\hat{m}}\, \Omega \sqrt{\mu}^{\ -1} \sim \mathcal{O}\left(\sqrt{\frac{m_\nu}{\mu}}\right)$$