On the CBK relation in QCD : 15 years later

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Decoding of the title

- 15 years later after personal talk at QFTHEP-2010 (Golitsyno); see AK, Mikhailov,"New extended Crewther-type relation and the consequences of multiloop perturbative results," PoS **QFTHEP2010** (2010), 014
- Definite progress in the studies of the consequences of the Quark parton model Crewther (72) and similar QCD Broadhurst, AK (93) relation

Processes to be considered

- The process e^+e^- - annihilation to hadrons. It is tested at different colliders. It is possible to consider e^+e^- annihilation into γ^* or Z^0, creating then hadrons. Novosibirsk, Bejing, KEK existing colliders and CEPC, CERN FCC etc.
- DIS processes give possibility to understand better the content of nucleon; JLAB, EIC (BNL)
- Is it possible to relate characteristics of definite annihilation (s-channel) and DIS (t-channel) processes ? The answer is - yes, through applying OPE to the AVV triangle diagram. What are the outcomes ?

May be useful in view of not yet appeared QCD CEPC white paper (Prof. J. Gao talk) and disagreement with PMC/BLM scale-scheme fixing studies Brodsky (partly), Di Guistino (partly), Xing-Gan Wu and his colleagues . Continuation of contacts from St.Petersburg Gatchina June 2024 Workshop.

Definitions of basic quantities in the CBK relation

The e^+e^- to hadrons D function in QCD with $a_s = \alpha_s/\pi$

$$D(a_s(Q^2)) = Q^2 \int\limits_0^\infty ds \frac{R^{th}_{e^+e^-}(a_s(s))}{(s+Q^2)^2} \to Q^2 \int\limits_0^\infty ds \frac{R^{exp+th}_{e^+e^-}(s)}{(s+Q^2)^2},$$

$$R^{th}_{e^+e^-} = \sigma_{tot}^{e^+e^- \to hadrons}(a_s)/(\sigma_0(e^+e^- \to \mu^+\mu^-) = 4\pi\alpha^2/(3s)).$$

$$\left(\frac{\partial}{\partial \ln \mu^2} + \beta(a_s)\frac{\partial}{\partial a_s}\right) D(a_s) = 0, \; \frac{\partial a_s}{\partial \ln \mu^2} = \beta(a_s)$$

$$D\left(a_s\right) = (\mathbf{N_c = 3}) \left(\sum_i q_i^2\right) D^{NS}\left(a_s\right) + \left(\sum_i q_i\right)^2 D^{SI}\left(a_s\right)$$

While considering $\sigma_0(e^+e^- \to \gamma^* \to \mu^+\mu^-) = 4\pi\alpha^2/(3s)$ we fix RS procedure $\alpha = Z_3(\alpha)\alpha_B$ with $Z_3 = 1$. Then the expression for $R^{th}(s)$ is defined in the model $SU_c(3) + U(1)$ and not $SU_c(3) \times U(1)$ (in THEORY : γ^* (or decaying Z^0) is virtual

$\overline{\text{MS}}$-scheme results for $D^{NS}(a_s) = 1 + \sum_{n\geq 1} d_n a_s^n$

$$d_1 = \frac{3}{4}C_F; T_f = T_F n_F = n_f/2$$

$$d_2 = -\frac{3}{32}C_F^2 - \left(\frac{11}{8} - \zeta_3\right)C_F T_f + \left(\frac{123}{32} - \frac{11}{4}\zeta_3\right)C_F C_A$$

$$= 1.9857 - 0.1153 n_f$$

Chetyrkin, Kataev, Tkachov (79); numerical Dine, Sapirstein (1979); analytical Celmaster, Gonsalves (1980); unpublished Ross, Terrano, Wolfram (1978-1980-corrected by Ch,K,T)

$$d_3 = -\frac{69}{128}C_F^3 - \left(\frac{29}{64} - \frac{19}{4}\zeta_3 + 5\zeta_5\right)C_F^2 T_f + \left(\frac{151}{54} - \frac{19}{9}\zeta_3\right)C_F T_f$$

$$-\left(\frac{127}{64} + \frac{143}{16}\zeta_3 - \frac{55}{4}\zeta_5\right)C_F^2 C_A + \left(\frac{90445}{3456} - \frac{2737}{144}\zeta_3 - \frac{55}{24}\zeta_5\right)C_F C_A^2$$

$$-\left(\frac{485}{27} - \frac{112}{9}\zeta_3 - \frac{5}{6}\zeta_5\right)C_F C_A T_f = 18.243 - 4.216 n_f + 0.086 n_f^2$$

Gorishny,K,Larin (87-bug in SCHOONSCHIP programs); corrected and recalculated further in Gorishny,K,Larin (91); Surguladze, Samuel (91); Chetyrkin (97)

Perturbative QCD for LEP

31 March 1995 (30 Years Ago)
Reports of the Working Group on Precision Calculations for the Z Resonace
CERN 95-03 Yellow Report ; 410 pages
Eds. Dmitry Bardin
Wolfgang Hollik
Gian Piero Passarino ;

- K.G. Chetyrkin, J. H. Kuhn, A. Kwiatkowski, QCD Corrections to the e^+e^- Cross-Section and the Z Boson Decay Rate , pp.175-263 ;
- S.A.Larin, T. van Ritbergen, J. A. M. Vermaseren, The Large Quark Mass Expansion of $\Gamma(Z^0 \to hadrons)$ in Order α_s^3, pp. 265-274;
- J. Chyla, A. L. Kataev, Theoretical Ambiguities of QCD Predictions at the Z^0 Peak, pp. 313-340

d_4 in the $\overline{\text{MS}}$-scheme

$$d_4 = \frac{d_F^{abcd} d_A^{abcd}}{d_R}\left(\frac{3}{16} - \frac{1}{4}\zeta_3 - \frac{5}{4}\zeta_5\right) + n_f \frac{d_F^{abcd} d_F^{abcd}}{d_R}\left(- \frac{13}{16} - \zeta_3 - \frac{5}{2}\zeta_5\right)$$

$$+C_F^4\left(\frac{4157}{2048} + \frac{3}{8}\zeta_3\right) + C_F^3 T_f\left(\frac{1001}{384} + \frac{99}{32}\zeta_3 - \frac{125}{4}\zeta_5 + \frac{105}{4}\zeta_7\right)$$

$$+C_F^2 T_f^2\left(\frac{5713}{1728} - \frac{581}{24}\zeta_3 + \frac{125}{6}\zeta_5 + 3\zeta_3^2\right)$$

$$+C_F T_f^3\left(- \frac{6131}{972} + \frac{203}{54}\zeta_3 + \frac{5}{3}\zeta_5\right)$$

$$+C_F^3 C_A\left(- \frac{253}{32} - \frac{139}{128}\zeta_3 + \frac{2255}{32}\zeta_5 - \frac{1155}{16}\zeta_7\right)$$

$$+C_F^2 T_f C_A\left(\frac{32357}{13824} + \frac{10661}{96}\zeta_3 - \frac{5155}{48}\zeta_3 - \frac{33}{4}\zeta_3^2 - \frac{105}{8}\zeta_7\right) +$$

d_4-continuation

$$
\begin{aligned}
&+C_F C_A T_f^2 \left(\frac{340843}{5184} - \frac{10453}{288}\zeta_3 \right) \\
&+C_F^2 C_A^2 \left(-\frac{592141}{18432} - \frac{49325}{384}\zeta_3 + \frac{6505}{48}\zeta_5 + \frac{1155}{32}\zeta_7 \right) \\
&+C_F C_A^2 T_f \left(-\frac{4379861}{20736} + \frac{8609}{77}\zeta_3 + \frac{18805}{288}\zeta_5 - \frac{11}{2}\zeta_3^2 + \frac{35}{16}\zeta_7 \right) \\
&+C_F C_A^3 \left(\frac{52207039}{248832} - \frac{426223}{3456}\zeta_3 - \frac{77995}{1152}\zeta_5 + \frac{605}{32}\zeta_3^2 - \frac{385}{64}\zeta_7 \right) \\
&= 135.792 - 34.440 n_f + 1.875 n_f^2 - 0.010 n_f^3
\end{aligned}
$$

Baikov, Chetyrkin, Kuhn (2008-2010); Confirmed by Herzog, Ruijl, Ueda, Vermaseren, Vogt (2017)

Sum rules of lN and νN deep-inelastic scattering

$$S_{Bjp}(Q^2) = \int_0^1 dx[g_1^{(lp)}(x,Q^2) - g_1^{(ln)}(x,Q^2)] = \frac{1}{6}|\frac{g_A}{g_V}|C_{Bjp}(a_s(Q^2))$$

$$S_{GLS}(Q^2) = \frac{1}{2}\int_0^1 dx[F_3^{(\nu p)}(x,Q^2) + F_3^{(\nu n)}(x,Q^2)] = 3C_{GLS}(a_s)$$

$$C_{Bjp}(a_s) = C^{NS}(a_s) + C_{Bjp}^{SI}(a_s)$$

$$C_{GLS}(a_s) = C^{NS}(a_s) + C_{GLS}^{SI}(a_s)$$

Sum rules are used for extraction of α_s values and study of the contributions of high-twist non-perturbativec effects Non-singlet and singlet coefficient functions should be analysed separately

$\overline{\text{MS}}$-scheme analytical results for $C^{NS}(a_s) = 1 + \sum_{n\geq 1} c_n a_s^n$

$$c_1 = -\frac{3}{4} C_F$$

$$c_2 = \frac{21}{32} C_F^2 + \frac{1}{2} C_F T_f - \frac{23}{16} C_F C_A = -4.583 + 0.333 n_f$$

Gorishny, Larin (1986)

$$c_3 = -\frac{3}{128} C_F^3 - \left(\frac{133}{576} + \frac{5}{12}\zeta_3\right) C_F^2 T_f - \frac{115}{216} C_F T_f^2$$
$$+ \left(\frac{1241}{576} - \frac{11}{12}\right) C_F^2 C_A + \left(-\frac{5437}{864} + \frac{55}{24}\zeta_5\right) C_F C_A^2$$
$$+ \left(\frac{3535}{864} + \frac{3}{4}\zeta_3 - \frac{5}{6}\zeta_5\right) C_F C_A T_f = -41.4399 + 7.6077 n_f - 0.1775 n_f^2$$

Larin, Vermaseren (1991)

Neutrino DIS phenomenologically related QCD studies, etraction of α_s

- A. L. Kataev and A. V. Sidorov, "The Jacobi polynomials QCD analysis of the CCFR data for xF3 and the Q**2 dependence of the Gross-Llewellyn-Smith sum rule," Phys. Lett. B **331** (1994), 179-186
- L. S. Barabash (JINR), ... A. V. Sidorov (JINR), ..., M. M. Kirsanov (IHEP) [IHEP-JINR Neutrino Detector], "Measurement of xf3, f2 structure functions and Gross-Llewellyn-Smith sum rule with IHEP-JINR neutrino detector," [arXiv:hep-ex/9611012 [hep-ex]]., LO QCD fits
- J. H. Kim, D. A. Harris, C. G. Arroyo, L. de Barbaro, P. de Barbaro, A. O. Bazarko, R. H. Bernstein, A. Bodek, T. Bolton and H. S. Budd, *et al.* "A Measurement of alpha(s)(Q**2) from the Gross-Llewellyn Smith sum rule," Phys. Rev. Lett. **81** (1998), 3595-3598 ; NNLO extractions from CCFR data

c_4 coefficient in the $\overline{\text{MS}}$-scheme

$$
\begin{aligned}
c_4 = {} & \frac{d_F^{abcd} d_A^{abcd}}{d_R}\left(-\frac{3}{16}+\frac{1}{4}\zeta_3+\frac{5}{4}\zeta_5\right)+n_f\frac{d_F^{abcd} d_F^{abcd}}{d_R}\left(\frac{13}{16}+\zeta_3-\frac{5}{2}\zeta_5\right)\\
& +C_F^4\left(-\frac{4157}{2048}-\frac{3}{8}\zeta_3\right)+C_F^3 T_f\left(\frac{839}{2304}+\frac{451}{96}\zeta_3-\frac{145}{24}\zeta_5\right)\\
& +C_F^2 T_f^2\left(-\frac{265}{576}+\frac{29}{24}\zeta_3\right)+C_F T_f^3\left(\frac{605}{972}\right)\\
& +C_F^3 C_A\left(-\frac{3707}{4608}-\frac{971}{96}\zeta_3+\frac{1045}{48}\zeta_5\right)\\
& +C_F^2 T_f C_A\left(-\frac{37403}{13824}-\frac{1289}{144}\zeta_3+\frac{275}{144}\zeta_5+\frac{105}{8}\zeta_7\right)
\end{aligned}
$$

c_4-continuation

$$+C_F C_A T_f^2\left(-\frac{165283}{20736}-\frac{43}{144}\zeta_3+\frac{5}{12}\zeta_5-\frac{1}{6}\zeta_3^2\right)$$
$$+C_F^2 C_A^2\left(\frac{1071641}{55296}+\frac{1591}{144}\zeta_3-\frac{1375}{144}\zeta_5+\frac{385}{16}\zeta_7\right)$$
$$+C_F C_A^2 T_f\left(-\frac{1238827}{41472}+\frac{59}{64}\zeta_3-\frac{18855}{288}\zeta_5+\frac{11}{12}\zeta_3^2-\frac{35}{16}\zeta_7\right)$$
$$+C_F C_A^3\left(-\frac{8004277}{248832}+\frac{1069}{576}\zeta_3+\frac{12545}{1152}\zeta_5-\frac{121}{96}\zeta_3^2+\frac{385}{64}\zeta_7\right)$$

Baikov, Chetyrkin, Kuhn (2010)

Special analytical structure of the CBK relation

Conformal symmetry based study Crewther (1972) and respecting QCD asymptotic freedom effects at $O(a_s^3)$ level Broadhurst, Kataev (1993) CBK relation . Property of the factorization of the QCD β-function is outlined. Confirmed at the $O(a_s^4)$-level by Chetyrkin, Baikov, Kuhn (2010)

$$C_{NS}(a_s)D^{NS}(a_s) = 1 + ZERO(C_F^n a_s^n) + ZERO(C_F^k C_A^m a_s^{k+m}) +$$

$$+\frac{\beta^{(3)}(a_s)}{a_s}\Bigg[S_1 C_F a_s + \Bigg(S_2 T_f N_f + S_3 C_A + S_4 C_F\Bigg)C_F a_s^2$$

$$+a_s^3\Bigg(S_5 C_F^3 + S_6 C_F^2 T_f + S_7 C_F T_f^2 + S_8 C_F C_A^2 + S_9 C_F C_A T_f + S_{10} C_F C_A^2$$

$$S_1 = -\frac{21}{8} + 3\zeta_3, S_2 = \frac{163}{24} - \frac{19}{3}\zeta_3$$

$$S_3 = -\frac{629}{32} + \frac{221}{12}\zeta_3\ , S_4 = \frac{397}{96} + \frac{17}{2}\zeta_3 - 15\zeta_5, S_5 - S_{10} - analytical$$

In view of conformal symmetry applied to AVV scale-independent coefficients in $C_{NS}(a_s)$ and $D^{NS}(a_s)$ are cancelling out.

Factorization property of the QCD β-function

This property is non-trivial even in QED when $C_A = 0$. In general β-function is responsible for effects of conformal symmetry violation. Expressions in MS-like schemes

$$\beta(a_s) = -\sum_{n\geq 0} \beta_n a_s^n \ , \ \beta_0 = (\frac{11}{3}C_A - \frac{4}{3}Tf)\frac{1}{4}$$

$$\beta_1 = (\frac{34}{3}C_A^2 - \frac{20}{3}C_A T_f - 4C_F T_f)\frac{1}{16} \ ,$$

$$\beta_2 = (\frac{2857}{54}C_A^3 + 2C_F^2 T_f - \frac{205}{9}C_F C_A T_f - \frac{1415}{27}C_A^2 T_f + \frac{44}{9}C_F T_f^2 + \frac{158}{27}C_A T_f^2\Big)\frac{1}{64}$$

O.V. Tarasov,A.A. Vladimirov, A.A. Zharkov (1980); S.A.Larin, J.A.M. Vermaseren (1993); Available 4 and 5 loop QCD β-function terms will be not considered.

Few words on factorization of the β-function

- Is important feature. Measure of violation of the CS in renormalized QFT models. Step to proof in all orders Crewther (96).
- Step to independent confirmation V.Braun, Korchemsky, Muller (03)
- Valid in gauge-independent schemes Garkusha, AK,Molokoedov (18)

In non-diagrammatic t 'Hooft scheme no property of factorization, though effects of β_0 and β_1 are seen Garkusha, AK (11). **This feature will be used further on .** CBK relation can be re-written as

$$C_{NS}(a_s)D^{NS}(a_s) = 1 + \sum_{n\geq 1}\left(\frac{\beta(a_s)}{a_s}\right)^n P_n(a_s)$$

$P_0(a_s) = 0$ Effect of conformal symmetry. AK,Mikhailov (10-12)

The $\{\beta\}$-expansion for the RG-invariant quantities

Consider the PT expansion

$$D^{ns}(a_s) = 1 + d_1 a_s + d_2(n_f) a_s^2 + d_3(n_f) a_s^3 + d_4(n_f) a_s^4 + O(a_s^5)$$

In the MS-like schemes β-expansion prescription is:

$$d_1 = d_1[0]$$

$$d_2(n_f) = \beta_0 d_2[1] + \mathbf{d_2[0]} \text{– } \textbf{the Basis of BLM procedure}$$

$$d_3(n_f) = \beta_0^2 d_3[2] + \beta_1 d_3[0,1] + \beta_0 d_3[1] + \mathbf{d_3[0]},$$

$$d_4(n_f) = \beta_0^3 d_4[3] + \beta_2 d_4[0,0,1] + \beta_1\beta_0 d_4[1,1] + \beta_0^2 d_4[2] + \beta_1 d_4[0,1]$$

$$+\beta_0 d_4[1] + \mathbf{d_4[0]}; \ldots$$

Suggested by **Mikhailov (Quarks2004, hep-ph.0411397 ; JHEP(07))** Further on Bakulev,Mikhailov, Stefanis(10) ; Kataev, Mikhalov (12,15,16) ; Brodsky,Wu, Mojaza et al(12-25) ; Cvetic,Kataev(16) ; Kataev,Molokoedov (22,23) ; Baikov, Mikhailov (22-23) ; Mikhailov (24)

The $\{\beta\}$ expanded QCD terms for D^{ns} in $SU(N_c)$

Using the **Cvetic,Kataev (16)** $\overline{MS}$-scheme factorized model , motivated by **Kataev,Mikhailov (12)** CBK-related consideration

$$D^{ns}(a_s) = 1 + \sum_{n\geq 0} \left(\frac{\beta(a_s)}{a_s}\right)^n P_n(a_s)$$

we obtain the results, which differ in part from obtained in QCD+gluino theory Mikhailov (07), Kataev,Mikhailov (15) more diagrammatic analysis

$$d_1[0] = \frac{3}{4}C_F \; d_2[0] = \left(-\frac{3}{32}C_F^2 + \frac{1}{16}C_F C_A \right) \; d_2[1] = \left(\frac{33}{8} - 3\zeta_3\right) C_F$$

$$d_3[0] = -\frac{69}{128}C_F^3 - \left(\frac{101}{256} - \frac{33}{16}\zeta_3\right) \neq +\frac{\mathbf{71}}{\mathbf{64}} \;\; C_F^2 C_A$$

$$-\left(\frac{53}{192} + \frac{33}{16}\zeta_3\right) \neq +\left(\frac{\mathbf{523}}{\mathbf{768}} - \frac{\mathbf{27}}{\mathbf{8}}\zeta_{\mathbf{3}}\right) \;\; C_F C_A^2$$

As the result one has $d_3[0] = -23.227 \neq -\mathbf{35.87}$,

The $\{\beta\}$ expanded QCD expression for d_4

$$d_4[0] = \left(\frac{3}{16} - \frac{1}{4}\zeta_3 - \frac{5}{4}\zeta_5\right)\frac{d_F^{abcd}d_A^{abcd}}{d_R} - \left(\frac{13}{16} + \zeta_3 - \frac{5}{2}\zeta_5\right)\frac{d_F^{abcd}d_F^{abcd}}{d_R}n_f$$

$$+\left(\frac{4157}{2048} + \frac{3}{8}\zeta_3\right)C_F^4$$

$$-\left(\frac{3509}{1536} + \frac{73}{128}\zeta_3 + \frac{165}{32}\zeta_5\right) \neq -\left(\mathbf{\frac{2409}{512}} + \mathbf{\frac{27}{16}}\boldsymbol{\zeta_3}\right) \quad C_F^3 C_A$$

$$+\left(\frac{9181}{4608} + \frac{299}{128}\zeta_3 + \frac{165}{64}\zeta_5\right) \neq -\left(\mathbf{\frac{3105}{1024}} + \mathbf{\frac{81}{32}}\boldsymbol{\zeta_3}\right) \quad C_F^2 C_A^2$$

$$\left(-\frac{30863}{36864} - \frac{147}{128}\zeta_3 + \frac{165}{64}\zeta_5\right) \neq \left(\mathbf{\frac{68047}{12288}} + \mathbf{\frac{8113}{512}}\boldsymbol{\zeta_3} - \mathbf{\frac{3555}{128}}\boldsymbol{\zeta_5}\right)C_F C_A^3$$

$d_4[0] = +81.157 \neq \mathbf{-98}$ Differ from diagrammatic related expression of Mikhailov (22-24), obtained using Chetyrkin (22) and Zoller (16) extended QCD calculations . Supported by Ball, Beneke, V. Braun (95) .

D-function at the $O(a_s^4)$-level and PMC/BLM - representations

Compare MS-scheme D-function with the **scale independent** coefficients, but running a_* and a_{**} Cvetic,K (16) -model . Independent from RG-quantity non-diagrammatic with Mikhailov (09-25) diagrammatic quatity dependent Extended QCD

$$D_{\overline{\rm MS}}(a_s) = 3\sum_f Q_f^2\big(1 + a_s + (1.9857 - 0.1153n_f)a_s^2$$

$$+(18.243 - 4.216n_f + 0.086n_f^2)a_s^3$$

$$+(135.792 - 34.440n_f + 1.875n_f^2 - 0.010n_f^3)a_s^4\big)$$

$$D^{CK}_{PMC/BLM}(a_s) = 3\sum_f Q_f^2\big(1 + a_* + \frac{1}{12}a_*^2 - 23.227a_*^3$$

$$+(81.157 - 0.0080n_f)a_*^4\big)$$

$$D^{M}_{PMC/BLM}(a_s) = 3\sum_f Q_f^2\big(1 + a_{**} + \frac{1}{12}a_{**}^2 - 35.87a_{**}^3$$

$$+(-98 - 0.0080n_f)a_{**}^4\big)$$

Questions and ambiguites

Where is the IR Landau pole in the PMC/BLM ? Or how its removed in $a_* = a_s(Q^2/\Lambda^{BLM}(n_f))$?
Answer : by the proportional to powers of $\beta_0 a_s$ shifts of scales $\Lambda \to \Lambda^{PMC/BLM}$

$$a_* = a_s(Q^2/\Lambda^{BLM}(n_f)) \;\; where \;\; \Lambda^{BLM}(n_f) = \Lambda_{NLO} exp[-\frac{1}{12}\Delta_0]$$

$$\Lambda^{PMC/BLM}(n_f) = \Lambda_{NLO} exp[-\frac{1}{12}\Delta_0 + \beta_0 \Delta_1 a_s^{BLM}(Q^2/\Lambda^2_{BLM})]$$

$$\Lambda^{PMC/BLM}(n_f) = \Lambda_{NNLO} exp[-\frac{1}{12}\Delta_0 +$$

$$+\beta_0 \Delta_1 a_s^{PMC/BLM}(Q^2/\Lambda^2_{PMC/BLM}) + \beta_0^2 \Delta_2 (a_s^{BLM}(Q^2/\Lambda^2_{PMC/BLM}))^2]$$

Finally, there is **not yet fixed ambiguity due to the fact that** in the related to a_s and a_* defined in PMC/BLM for $k \geq 2$ $\beta_k^{PMC/BLM} \neq \beta_k$ WAS NOT YET taken into account.

PMC/BLM vs massless $\overline{\text{MS}}$: AK,Molokoedov PRD 108 (23)

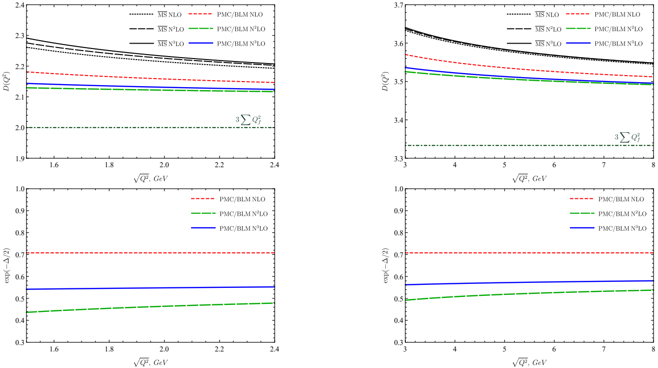

Figure: (1a) Adler function $D(Q^2)$ on $\sqrt{Q^2}$ at $n_f = 3, 4$ in the massless limit. Experimental related data higher $\overline{MS}$ Eidelman, Jegerlehner, AK, Veretin (98); Davier et al (23). $\overline{\text{MS}}$ more preferable from the point of view of the experimentally related EJKV data **which is in low energy region are HIGHER** .

Conclusion from K,Molokoedov PRD (23)

As seen from Fig. 1a, the application of the PMC-BLM procedure to the massless MS Adler function PT approximations is leading to moving the considered curves lower away fromthe experimentally based results for the Adler function in the considered kinematical region. Therefore, the PMC-BLM approach should be aplplied **WITH CARE AND MODIFIED** *in the process of comparison with the existing experimental data and, in particular, the ones provided by the e^+e^- colliders. PMC/BLM are qualitatively closer to "Finite QED" . Leads to UNDERESTIMATE of theory QCD ambiguity.*

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Different representations for the $D^{NS}(a_s)$ in QCD

It is possible to rewrite RG expressions the $e^+e^- \to hadrons$ D-function as formulated by Politzer (74) and in details by Chetyrkin et al (12)

$$D^{NS}(a_s) = 1 + \sum_{k\geq 1} D_k a_s^k = \gamma_{ph}(a_s) - \beta(a_s)\frac{\partial}{\partial a_s}\Pi(a_s)$$

where
$\Pi(a_s) = \sum_{k\geq 1} \Pi_k a_s^k, \gamma_{ph}(a_s) = \partial\Pi(a_s)/\partial ln(\mu^2) = \sum_{k\geq 0} \gamma_k a_s^k.$
Starting from S. J. Brodsky, M. Mojaza, X. G. Wu, Phys. Rev. D **89** 014027 (2014) Wu et al are using this represntation in the BLM-inspired analysis and have

$$D^{BMW}(\tilde{a}) = 3\sum_f Q_f^2 \gamma_{ph}(\tilde{a}) = 3\sum_f Q_f^2\big(1 + \tilde{a} + (2.604 - 0.153n_f)\tilde{a}^2$$

$$+(9.742 - 2.043n_f - 0.0198n_f^2)\tilde{a}^3 + (41.0141 - 12.911n_f$$

$$+0.489n_f^2 + 0.045n_f^3)\tilde{a}^4\big)$$

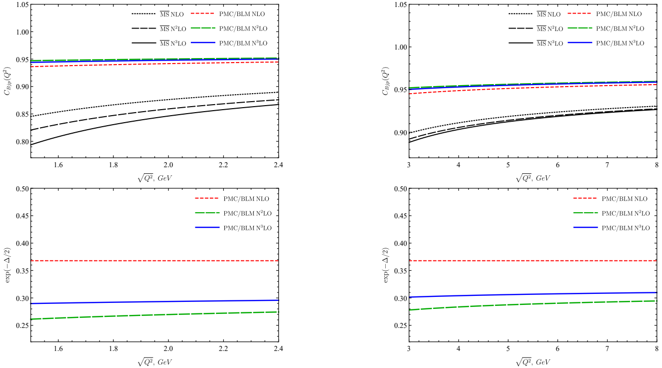
No SCALE-INDEPENDENT COEFFICIENTS . Special scheme

CBK relation and perturbative QCD for Bjorken polarized sum rule

$$C_{Bjp}^{\overline{\rm MS}}(a_s) = 1 + \beta(a_s)K(a_s)/D(a_s) =$$
$$1 - a_s + (-4.5833 + 0.3333n_f)a_s^2$$
$$+(-41.4399 + 7.6073n_f - 0.1775n_f^2)a_s^3$$
$$+(-479.448 + 123.390n_f - 7.697n_f^2 + 0.1037n_f^3)a_s^4,$$

$$C_{Bjp}^{CK}(a_*) = 1/D(a_*)$$
$$= 1 - a_* + 0.9167(a_*)^2 + 22.3894(a_*)^3 + (-126.8456 - 0.0802n_f)(a_*)^4$$

PMC/BLM vs massless $\overline{MS}$: Bjorken polarized SR at n_f=3,4 $S_{Bjp}(Q^2) = \frac{1}{6}(g_A/g_V)C_{Bjp}(Q^2)$ by AK and Molokoedov drawn @ 23



Experriment lower than $\overline{MS}$ Kotikov talk (!) Effects of conformal symmetry violation by both PT and non-PT effects ARE NOT SEEN in PMC but ARE SEEN in NATURE (!) . For the considerations see e.g. D.Kortlorz,Mikhailiov,A.Kotlorz (20); D.Kotlorz,Mikhailov(19); Ayala, Pineda (22) and AK (05).

HOWEVER

$$C_{Bjp}(NON\ expanded) = 1/D(\tilde{a}) + 0(\tilde{a}^3, \beta_0, \beta_1)$$
$$= 1 - \tilde{a} + 0.9167\tilde{a}_s^2 + 22.3894\tilde{a}^3$$
$$+(-126.8456 - 0.0802n_f)\tilde{a}^4 + MISSED\ O(\tilde{a}^3)$$

Results obtained in works given BY X.G. Wu and S. Brodsky et al FROM 2014 AND (e^+e^--annihilation, τ-decay width, hadronic Higgs, Z^0, W-decay widths, DIS sum rules,) SHOULD BE RECONSIDERED
Compare e.g. $\alpha_s(M_Z) = 0.1177 \pm 0.0011 \pm^{-0.0095}_{+0.0077}$ with $\alpha_s(M_Z) = 0.1191 \pm 0.0012 \pm 0.0006$ (!?) (Q. Yu, X. G. Wu et al, PRD111 (25)) Underestimate theory incertainty Aim of High Energy Colliders- TESTS OF QCD with MORE PRECISION How we may do this if EXISTING THEORY EFFECTS OF QCD ARE MISSED ?