Remnants and entanglement islands in linear dilaton gravity with regular black holes

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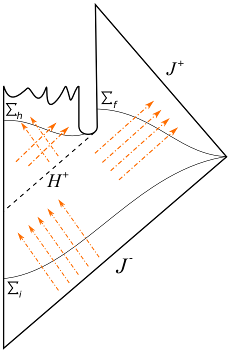
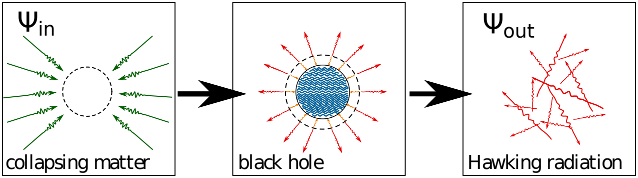
Quantum Theory vs General Relativity



Disconnected Cauchy surfaces Σ_h and Σ_f contain information about Σ_i



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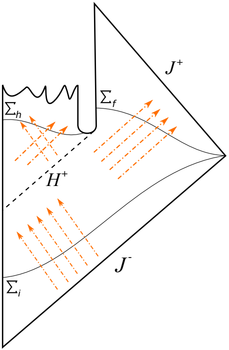

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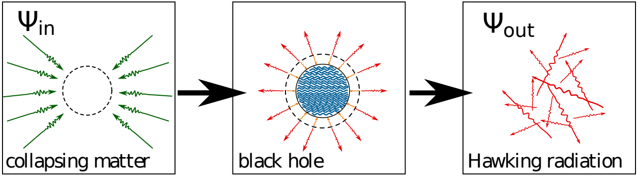

Apparent violation of unitarity:

$$\hat{\rho}_{in} = |\Psi_{in}\rangle\langle\Psi_{in}| \mapsto$$

$$\hat{\rho}_{out} = \mathrm{Tr}_{BH}\left(|\Psi_{ext}\rangle|\Psi_{BH}\rangle\langle\Psi_{BH}|\langle\Psi_{ext}|\right)$$

$\mathrm{Tr}(\hat{\rho}^2_{out}) < 1$ Danger to quantum laws!

This is *information paradox.*

S.W. Hawking, 1976

Quantum Theory vs General Relativity

- Pro-unitary arguments:
  - Black hole complementarity

    Susskind et al.
  - Remnants/baby universes
  - Holography: gauge/string duality (AdS/CFT)

    Maldacena et al.
  - Islands: unitary Page curve for entanglement entropy $S_{\rm ent}(t)$

    1911.12333 [hep-th] Almheiri, 1905.08255 [hep-th] Pennington ...

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- Problems?
  - AMPS-firewall: unitarity vs equivalence principle.

    Almheiri et al.
  - Dynamics: S-matrix derivation (path integral)

    ArXiv:gr-qc/9607022 't Hooft
  - Microscopic: fundamental dofs in Quantum Gravity.

Toy models

CGHS model

$$S = \int d^2x\sqrt{-g}\left[e^{-2\phi}\left(R+4(\nabla\phi)^2+4\lambda^2\right) - \frac{1}{2}(\nabla f)^2\right]$$

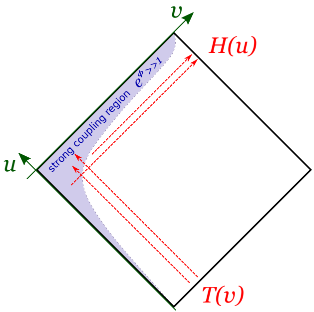
ArXiv:9111056 [hep-th] C. Callan, S. Giddings, J. Harvey, A. Strominger, 1991

In the bulk: $ds^2 = -e^{2\phi}dvdu$,
$f(v,u) = f_{out}(u) + f_{in}(v)$
$e^{-2\phi} = -\lambda^2 vu - \mathcal{T}(v) - \mathcal{H}(u)$
$\partial_v^2\mathcal{T} = (\partial_v f_{in})^2/2$, $\partial_u^2\mathcal{H} = (\partial_u f_{out})^2/2$

Eternal black hole metric: linear $\phi = -\lambda r$,

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 ,$$

$$f(r) = 1 - \frac{M}{2\lambda\exp(2\lambda r)} .$$



Models

Sinh-CGHS model

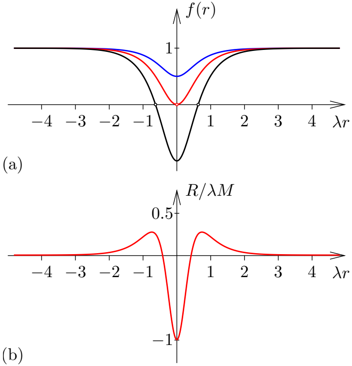
$$S_{\mathrm{sinh}} = -\frac{M_{\mathrm{ext}}}{2\lambda}\int d^2x\,\sqrt{-g}\,\sinh(2\phi)\,\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right)$$

ArXiv:2202.00023 [gr-qc] M.F.

Regular black holes: linear $\phi = -\lambda r$,

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2\ ,$$

$$f(r) = 1 - \frac{M}{M_{\mathrm{ext}}\cosh(2\lambda r)}\ .$$



Motivation:

- Limiting curvature $R_{\mu\nu\rho\sigma}{}^2 < \Lambda^2$.

  Markov, 2111.14318 [gr-qc] Frolov ...

- Other models: Bardeen's black hole, black bounces, planck stars...

  1812.07114 Visser, 1802.04264 Rovelli...

Thermodynamic properties

Euclidean solution

$$ds_E{}^2 = f(r)dt_E{}^2 + \frac{dr^2}{f(r)}\,, \qquad 0 \le t_E < \beta_H\,,$$

has imaginary time period

$$\beta_H = T_H{}^{-1} = 4\pi/f'(r_{\rm h})$$

$\Leftarrow$ no conifold singularity at $r = r_{\rm h}$.

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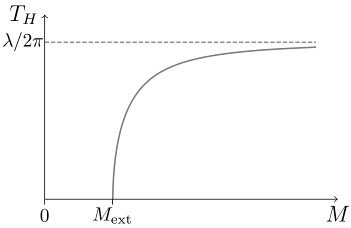
Regular black holes temperature and entropy

$$S_{BH} = \frac{2\pi}{\lambda} M \sqrt{1 - \frac{M_\mathrm{ext}^2}{M^2}}$$

$$T_H = \frac{\lambda}{2\pi}\sqrt{1 - \frac{M_\mathrm{ext}^2}{M^2}}$$



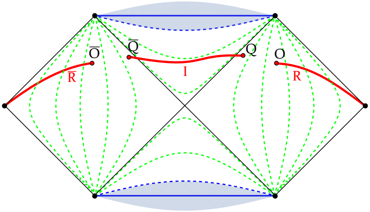
Sinh-CGHS reduces to CGHS in $M_\mathrm{ext} \to 0$.

Entropy from entanglement island

Hawking's semiclassical answer (for $R \cup R^*$):

$$\dot{S}_{\rm ent} \simeq 2\pi c T/3 \,, \qquad \text{at } r_O \to +\infty \,, \quad \Rightarrow \quad S_{\rm ent} \leq 2 S_{BH}$$

- in violation of the Bekenstein bound.



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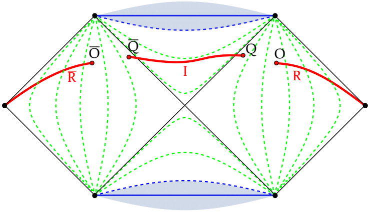
Island formula for black hole entropy

$$\boxed{S_{\rm gen}[R] = \min_{I} \operatorname*{ext}_{\partial I} \left(S_{\rm grav}[\partial I] + S_{\rm ent}[R \cup I]\right)}$$



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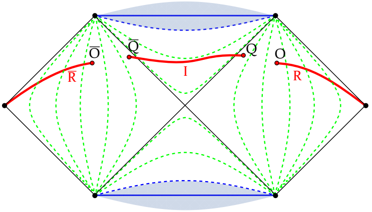


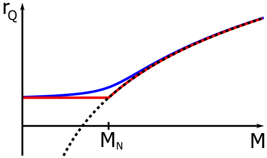
For linear dilaton CGHS $W(\phi) = e^{-2\phi}$ and sinh-CHGS $W(\phi) = -\frac{M_{\rm ext}}{2\lambda}\sinh(2\phi)$

$$S_{\rm gen} = 8\pi W(-\lambda r_Q) + \frac{c}{3}\log(\epsilon^{-2}(v_O - v_Q)(u_Q - u_O)) + \frac{c}{3}(\rho_O + \rho_Q)$$

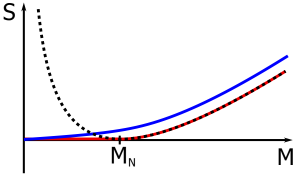
where ρ is metric factor: $ds^2 = -e^{2\rho} dv du$. Vary $S_{\rm gen}$ with respect to t_Q and r_Q.

Quantum Extremal Point and $S_{\rm ent}$ in CGHS model

Position of QEP



Entanglement entropy



Horizon at $r_{\rm hor}(M) = \frac{1}{2\lambda}\ln(M/2\lambda)$ (black dashed)
Position of QEP at finite r_O (blue line)
Position of QEP at $r_O \to +\infty$ (red line)

$$1. \qquad r_Q \simeq r_{\rm hor}(M)\,, \qquad M > M_c\,,$$

$$\text{where} \quad M_c = \frac{c\lambda}{24\pi}$$

$$2. \qquad r_Q \simeq r_{\rm hor}(M_c)\,, \qquad M < M_c$$

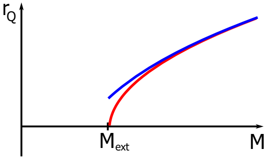
Zero entropy for lightest black hole: $S^0_{\rm ent}(0) = 0$
Analytic answer at $r_O \to +\infty$

$$1. \quad S_{\rm ent} = \frac{2\pi}{\lambda}M - \frac{c}{12}\left(\ln(M/M_c) + 1\right)\,, \quad M > M_c$$

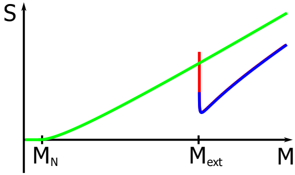
$$2. \qquad S^0_{\rm ent}(M) = 0\,, \qquad M < M_c$$

Quantum Extremal Point and $S_{\rm ent}$ in sh-CGHS model

Position of QEP



Entanglement entropy



Position of QEP at finite r_O (blue line)
QEP at $r_O \to +\infty$ and horizon at (red line)

$$r_{\rm hor}(M) = \frac{1}{2\lambda}\ln\left(M/M_{\rm ext} + \sqrt{(M/M_{\rm ext})^2 - 1}\right)$$

Extremal hole is heavy $M_{\rm ext} > M_c$. At finite r_A its entropy diverges!

$$S_{\rm ent} \simeq \frac{4\pi}{\lambda}\frac{M_{\rm ext}}{\lambda r_O}\,, \quad M \to M_{\rm ext}$$

What if far observer can not distinguish CGHS and sh-CGHS, $S_{\rm ent}(M)/S^0_{\rm ent}(M) \simeq 1$ as $M \to +\infty$,

$$S_{\rm ent} \simeq \frac{c}{6}\ln\left(\frac{M_c}{M - M_{\rm ext}}\right) + \ldots\,,$$

Minimal entropy at $M_{\rm qb} \simeq M_{\rm ext} + 2M_c^2/M_{\rm ext}$,

$$S_{\rm min} \simeq \frac{c}{6}\left(1 + \ln(M_{\rm ext}/M_c) - 2M_c/M_{\rm ext}\right)$$

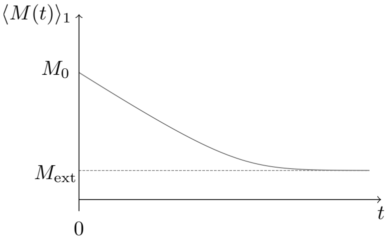
Remnants formation

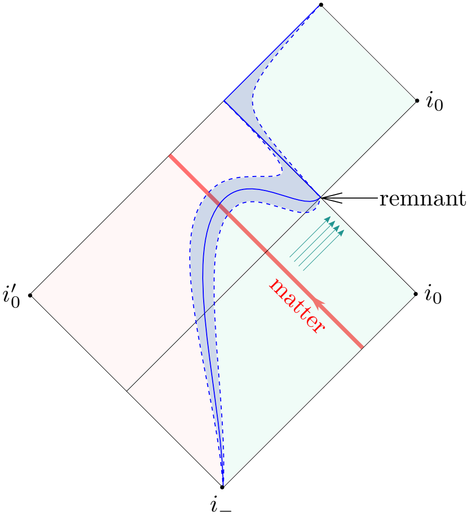
2D Stefan–Boltzmann law

$$\frac{dM}{dt} = -\frac{\pi}{12} T_H^2(M)$$

$\Rightarrow$ asymptotically

$$M \simeq M_{\rm ext}\left(1 + \exp\left(-\frac{\lambda^2 t}{24\pi M_{\rm ext}}\right)\right)$$





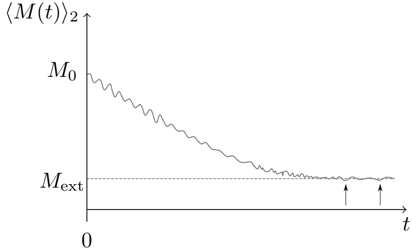
Remnants decay

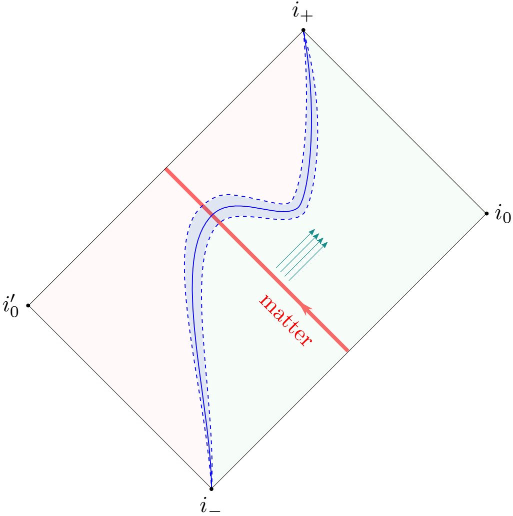
Fluctuations theory estimate (naive)

$$\langle (\Delta M)^2 \rangle = -\frac{\partial \langle E \rangle}{\partial \beta} \simeq \frac{\lambda^2}{M_{\text{ext}}} O(1)$$

$$t_{\text{dec}} \simeq 48\pi \frac{M_{\text{ext}}}{\lambda^2} \log \left(\frac{M_{\text{ext}}}{\lambda} \right)$$

assuming $\Delta M \ll M_{\text{ext}}$.

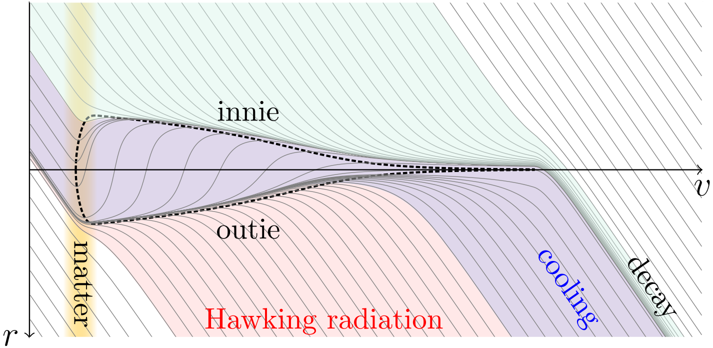




Horizonless spacetime

Vaydia metric: $ds^2 = -\left(1 - \frac{M(v)}{M_{\text{ext}}\cosh(2\lambda r)}\right)dv^2 + 2dvdr$,

$M(v) = \int dv(\partial_v f(v))^2$ - Bondi mass.



Necessity of quantum treatment

Adiabaticity condition: change in mass is negligible

$$T\frac{\partial T}{\partial M} \ll T \;\Rightarrow\; T\frac{\partial S}{\partial T} \gg 1 \;\Rightarrow\; T \gg \frac{\lambda^2}{4\pi^2 M_{ext}}$$

Breakdown on the same scale as decay expected.

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Either regularize solutions or modify field equations to make black hole decay:

- Real EFT solutions:

$$S \;\mapsto\; S + \frac{N}{96\pi}\int R\frac{1}{\Box}R\,, \qquad\qquad \text{(1−loop effective action)}$$

Example: Bardeen black hole, arXiv: 2405.13373

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- Complex saddle points:

$$S \;\mapsto\; S + i\epsilon T \qquad\qquad T - \text{“interaction time”}$$

$$\text{Amplitude}: \qquad \mathcal{A} = \lim_{\epsilon\to+0}\mathcal{A}_\epsilon$$

Levkov, Panin et al (ϵ–regularization)

Transition amplitude

Shock wave: $t = \int^r dr\, f^{-1}(r)$, energy $E = M_i - M_f$.

Find amplitude

$$\mathcal{A}_{fi} = \lim_{\epsilon \to +0} \mathcal{A}_{fi}(M + i\epsilon) \simeq \exp(iS_{tot}[\Phi]) \ ,$$

where total action $S_{tot} = S_{sw} + S_{gr} + S_{GH} + S_{free} - i \log(\Psi_f^* \Psi_i)$, semiclassical wave functions $\Psi_{f,i} = \exp(iEr_{f,i})$.

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The answer: $\mathcal{A}_{fi} = \exp(i\varphi(M_i + i\epsilon) - i\varphi(M_f + i\epsilon'))$,

$$\varphi(M) = \frac{M}{\lambda} - \frac{2M_{ext}}{\lambda}\sqrt{1 - \frac{M^2}{M_{ext}^2}} \arctan \sqrt{\frac{M_{ext} + M}{M_{ext} - M}}$$

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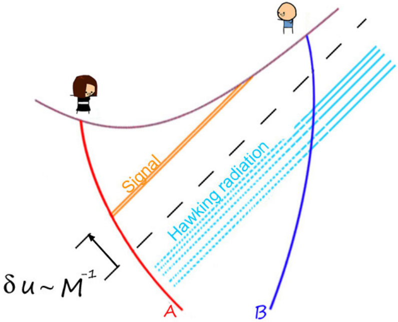
Transition probability: $\boxed{\mathcal{P}_{fi} = |\mathcal{A}_{fi}|^2 \simeq \exp(-\Delta S_{BH})}$

For a statistical ensemble: $\langle M_f \rangle = M_i - T_H(M_i)$ for large M_i,
if M_i becomes comparable with M_{ext} black hole decays completely: $\langle M_f \rangle < M_{\text{ext}}$.

Quantum xerox

Alice and Bob perform quantum cloning...



...but singularity prevents their attempts

Quantum xerox/cloner paradox:

$|\psi\rangle \mapsto |\psi\rangle|\psi\rangle$ - forbidden by unitarity

Wooters, Zurek (1982)

but black holes seems to do exactly this

Complementarity as solution (?)

with scrambling time $t_{\rm scr} \sim T_H^{-1} \ln S_{BH}$

Page, Preskill et al.

Complex saddle–point solutions relating $t_{\rm scr}$ to tunneling time - to investigate.

Far from conclusion

- Black hole information puzzle:
  - Progress: holography (AdS/CFT), replica wormholes (island formula, Page curve).
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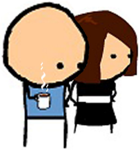
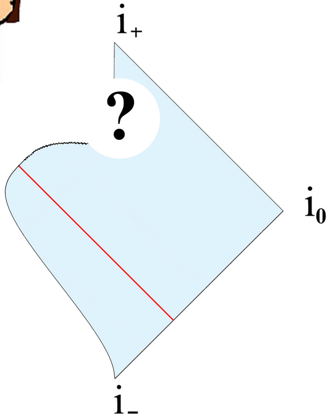
- Toy model with regular black holes was investigated:
  - Islands do not reproduce unitary Page curve in quasi–stationary situations.
  - Singularity removal does not automatically save unitarity.
- What next:
  - Model with one–loop corrections: numerical solutions for black hole decay.
  - Regularization: tunneling solutions.
  - Use them both to test unitarity

$$\int \mathcal{D}c_k^* \mathcal{D}c_k \; e^{-\int dk \; c_k^* c_k} \langle b|\hat{S}^\dagger|c\rangle\langle c|\hat{S}|a\rangle = \langle b|a\rangle \; ,$$

where path integrals for the S–matrix are evaluated using saddle points method.





Thank you!