Primordial black holes production in inflationary models of $f(R)$ and induced gravity

Sergey Yu. Vernov

Skobeltsyn Institute of Nuclear Physics,
Lomonosov Moscow State University

based on E.O. Pozdeeva and S.Yu. Vernov,
Phys.Part.Nucl. 56 (2025) 542, arXiv:2407.00999
and current investigations

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• A black hole is called primordial (PBH) if it has been formed before the matter dominance epoch of the Universe evolution.
Large peaks in the amplitude of perturbations during inflation can lead to PBH formation in early post-inflation stage of the Universe evolution.

• The PBHs can be originated from the collapse of matter overdensities in a certain wavelength interval of inflationary perturbations.

• The abundance of PBHs is related to the amplitude of the inflaton fluctuations, the enhancement of which must be by several orders of magnitude with respect to the amplitude probed by cosmic microwave background (CMB) radiation.

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B.J. Carr, Astrophys. J. **201** *(1975) 1*

• The PBHs are the DM candidates. The hypothesis that a part of the dark matter consists of PBH has been proposed in

A. Dolgov, J. Silk, Phys. Rev. D, 1993, **47***, p. 4244.*
P. Ivanov, P. Naselsky, I. Novikov, Phys. Rev. D, 1994, **50***, p. 7173*

The possibility that a significant fraction or even the totality of the dark matter is not a new form of matter but consists of primordial black holes (PBHs) is actively discussed

M.Y. Khlopov, Res. Astron. Astrophys. 10 (2010) 495
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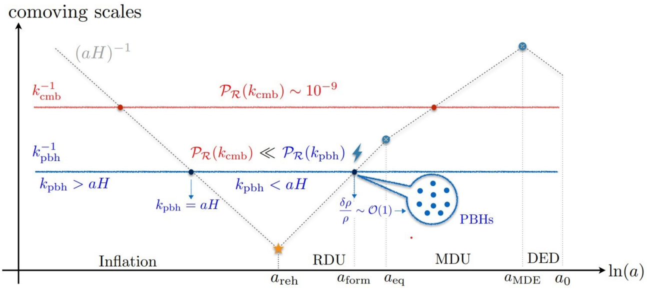
• The rapidly growing number of direct and indirect observations of black holes with masses beyond the astrophysical range, the occurrence of which is not described by models of stellar collapse, confirms an assumption about the existence of primordial black holes (PBH).
The study of the discovered black holes in the centers of galaxies has led to the assumption that it is not a black hole formed due to the accretion of galactic matter, but a galaxy formed around a previously formed black hole.



O. Özsoy and G. Tasinato, Inflation and Primordial Black Holes, Universe **9** (2023) 203 [arXiv:2301.03600].

If PBH mass belongs to the interval $10^{-17} \leq M_{pbh}[M_\odot] \leq 10^{-12}$, ($M_\odot \simeq 1.98 \cdot 10^{33}$ gr), then PBH can a part of DM.

It corresponds to $34 < N_{PBH} - N_{CMB} < 40$.

The slow-roll conditions should be violated in the models that unify inflation and PBH formation.
In inflationary models with one minimally coupled scalar field, two slow-roll parameter should be smaller than one in the slow-roll regime. When the first parameter becomes equal to one, inflation as an accelerated expansion of the Universe stops, so only the second slow-roll parameter can be more than one during inflation.
In single-field inflationary models, PBH formation corresponds to an ultra slow-roll stage of inflation.

J. Kristiano and J. Yokoyama, Constraining Primordial Black Hole Formation from Single-Field Inflation, Phys. Rev. Lett. **132**, 221003 (2024), arXiv:2211.03395

A. Riotto, The Primordial Black Hole Formation from Single-Field Inflation is Not Ruled Out, arXiv:2301.00599.

A.Yu. Kamenshchik, E.O. Pozdeeva, A. Tribolet, A. Tronconi, G. Venturi, S.Yu. Vernov, Phys. Rev. D **110** (2024) 104011, *arXiv:2406.19762* .

In many two-field models, one scalar field plays a role of inflaton in the beginning of inflation and another field plays the same role at the end. The investigations of such inflationary models with two stages of inflation show that density perturbations at the time corresponding to the transition between two inflationary stages can be so large that leads to PBH production.

J. Garcia-Bellido, A. D. Linde and D. Wands, Phys. Rev. D **54** (1996) 6040

M. Braglia, D.K. Hazra, F. Finelli, G.F. Smoot, L. Sriramkumar, A.A. Starobinsky, JCAP **08** (2020) 001.

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M. Braglia, A. Linde, R. Kallosh, F. Finelli, JCAP **04** (2023) 033.

Let us consider the induced gravity model with two scalar fields, σ and χ:

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{\xi}{2}\sigma^2\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \tilde{V}(\sigma,\chi)\right],$$

where ξ is a positive constant.

Using the conformal transformation of the metric

$$g_{\mu\nu} = \frac{\xi\sigma^2}{M_{\mathrm{Pl}}^2}\tilde{g}_{\mu\nu},$$

where M_{Pl} is the Planck mass, we obtain the action:

$$S_E = \int d^4x\sqrt{-g}\left[\frac{M_{\mathrm{Pl}}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{y}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V_E\right], \quad (1)$$

where

$$\phi = M_{\mathrm{Pl}}\sqrt{6+\frac{1}{\xi}}\,\ln\left(\frac{\sigma}{M_{\mathrm{Pl}}}\right), \quad (2)$$

$$y = \frac{M_{\mathrm{Pl}}^2}{\xi\sigma^2} = \frac{1}{\xi}\exp\left(-2\sqrt{\frac{\xi}{6\xi+1}}\frac{\phi}{M_{\mathrm{Pl}}}\right), \quad (3)$$

$$V_E = y^2(\phi)\tilde{V}(\sigma(\phi),\chi). \quad (4)$$

Evolution equation in the FLRW metric

In the spatially flat FLRW metric, the evolution equations are

$$H^2 = \frac{1}{6M_{\rm Pl}^2}\left(X^2 + 2V_E\right), \qquad \dot{H} = -\frac{X^2}{2M_{\rm Pl}^2}, \tag{5}$$

where dots denote the time derivatives, $X \equiv \sqrt{\dot{\phi}^2 + y\,\dot{\chi}^2}$ and the Hubble parameter $H(t)$ is the logarithmic derivative of the scale factor: $H = \dot{a}/a$. The field equations are as follows:

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{2}\frac{dy}{d\phi}\,\dot{\chi}^2 + \frac{\partial V_E}{\partial \phi} = 0, \tag{6}$$

$$\ddot{\chi} + 3H\dot{\chi} + \frac{1}{y}\frac{dy}{d\phi}\,\dot{\chi}\dot{\phi} + \frac{1}{y}\frac{\partial V_E}{\partial \chi} = 0. \tag{7}$$

It is suitable to consider the e-folding number $N = \ln(a/a_0)$, where a_0 is a constant, as an independent variable during inflation.
The standard slow-roll parameters ε_1 and ε_2 are

$$\varepsilon_1 = -\frac{H'}{H} = \frac{1}{2M_{\mathrm{Pl}}^2}\left[{\phi'}^2 + y{\chi'}^2\right], \tag{8}$$

$$\varepsilon_2 = \frac{\varepsilon_1'}{\varepsilon_1} = 2\varepsilon_1 + \frac{1}{HX^2}\frac{dX}{dt}, \tag{9}$$

where primes denote derivatives with respect to N.

Using the relation $\frac{d}{dt} = H\,\frac{d}{dN}$, equations (5) can be written as follows

$$H^2 = \frac{2V_E}{6M_{\mathrm{Pl}}^2 - \phi'^2 - y\chi'^2}\,, \tag{10}$$

$$H' = -\,\frac{H}{2M_{\mathrm{Pl}}^2}\left[\phi'^2 + y\chi'^2\right]. \tag{11}$$

Using Eqs. (10) and (11), we eliminate H^2 and H' from the field equations and obtain

$$\begin{aligned} \phi'' =& (\varepsilon_1 - 3)\phi' + \frac{1}{2}\,\frac{dy}{d\phi}\,\chi'^2 - \frac{6M_{\mathrm{Pl}}^2 - y\chi'^2 - \phi'^2}{2V_E}\,\frac{\partial V_E}{\partial \phi}\,, \\ \chi'' =& (\varepsilon_1 - 3)\chi' - \frac{1}{y}\,\frac{dy}{d\phi}\,\chi'\phi' - \frac{6M_{\mathrm{Pl}}^2 - \phi'^2 - y\chi'^2}{2yV_E}\,\frac{\partial V_E}{\partial \chi}\,. \end{aligned} \tag{12}$$

Induced gravity inflationary model

We consider the following potential:

$$\tilde{V}(\sigma,\chi) = \lambda\sigma^4 \left(F_1(\chi) + F_2(\chi)\mathrm{e}^{\gamma\,[\ln(\sigma/M_{\mathrm{Pl}})]^{2\alpha}} \right), \tag{13}$$

where[1]

$$F_1(\chi) = \left(1 - \frac{\chi^2}{\chi_0^2}\right)^2 - d\,\frac{\chi}{\chi_0}, \qquad F_2(\chi) = \frac{c_2\chi^2}{\chi_0^2} + c_0\,, \tag{14}$$

α, γ, λ, χ_0, c_0, c_2 and d are constants. Note that the potential $\tilde{V}$ is real even if $\ln(\sigma/M_{\mathrm{Pl}}) < 0$ and α is not an integer number.
In the Einstein frame, we get

$$V_E(\phi,\chi) = V_0 \left(F_1(\chi) + F_2(\chi)\mathrm{e}^{\beta\left(\frac{\phi^2}{M_{\mathrm{Pl}}^2}\right)^{\alpha}} \right), \tag{15}$$

where

$$V_0 = \frac{\lambda M_{\mathrm{Pl}}^4}{\chi^2}, \qquad \beta = \gamma\left(\frac{\chi}{1+6\,\chi}\right)^{\alpha}. \tag{16}$$

[1] *M. Braglia, A. Linde, R. Kallosh, F. Finelli*, JCAP **04** (2023) 033.

Note that slow-roll parameters ε_1 and ε_2 do not depend on V_0. So, inflationary parameters: the spectral index n_s and the tensor-to-scalar ratio r

$$n_s = 1 - 2\varepsilon_1 - \varepsilon_2, \qquad r \approx 16\varepsilon_1, \tag{17}$$

do not depend on V_0 as well.

The parameter V_0 is fixed by the condition the observation value of the amplitude of scalar perturbations $A_s = 2.10 \times 10^{-9}$ at $N = 0$, where

$$A_s = \frac{2H^2}{\pi^2 M_{\rm Pl}^2 r}. \tag{18}$$

$\phi = 0$ corresponds to $\sigma = M_{\rm Pl}$, so at $\chi = 1$ the induced gravity model becomes close to general relativity at the second stage of inflation.
By this reason, we fix $\chi = 1$.

Numerical solutions of the evolution equations

To analyze the evolution of scalar fields during inflation and to get values of inflationary parameters n_s, r, and A_s we solve system (12) numerically. We define the e-folding number N in such a way that $N = 0$ corresponds to the moment at which inflationary parameters are calculated.

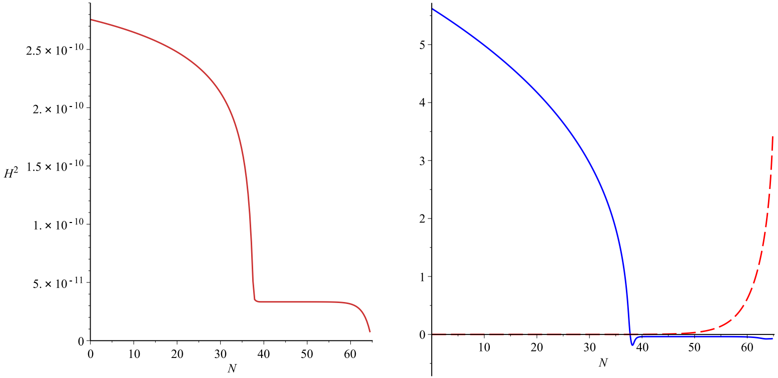

Figure: The behaviour of $H^2(N)$ and scalar fields $\phi(N)$ (blue solid curve) and $\chi(N)$ (red dash curve) during inflation. Values of the model parameters are given in (19). $H(N)$ and the fields are shown in units of $M_{\rm Pl}$.

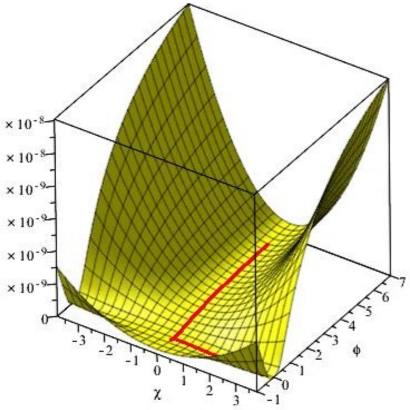

Figure: The potential $V(\phi, \chi)$ and the trajectory. The fields are shown in units of $M_{\rm Pl}$.

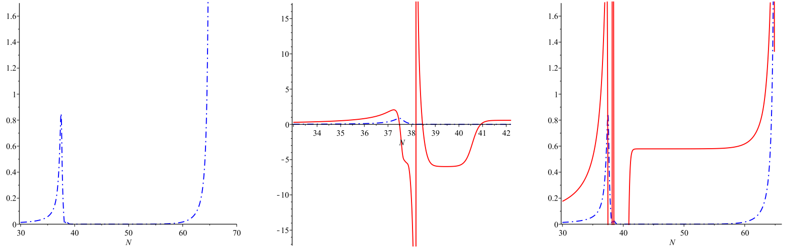

Figure: The behavior of slow-roll parameters ε_1 (blue dash-dot curve) and ε_2 (red solid curve) during inflation. Values of the model parameters are given in (19).

The slow-roll parameter $\varepsilon_1 < 1$ during inflation.
In the slow-roll regime, $|\varepsilon_2| < 1$ as well.
The slow-roll regime is violated at the interval $37 < N < 41$.

For the values of models parameters

$$V_0 = 10^{-10} M_{\rm Pl}^4, \quad \alpha = -0.37, \quad \beta = -1.8, \\ \chi_0 = 3.5\, M_{\rm Pl}, \quad c_0 = 12, \quad c_2 = 147, \quad d = 10^{-3}, \tag{19}$$

we get inflationary parameters

$$n_s = 0.9622, \quad r = 0.0266, \quad A_s = 2.10 \cdot 10^{-9}\,, \tag{20}$$

which values are in agreement with **the Planck observation data**:

$$n_s = 0.9649 \pm 0.0048, \quad r < 0.028, \quad A_s = (2.10 \pm 0.03) \cdot 10^{-9}. \tag{21}$$

We suppose that the second stage of inflation leading to the possible generation of PBH begins at the point N_*. We estimate N_* by the relation:

$$2\varepsilon_1(N_*) - \frac{\varepsilon_2(N_*)}{2} \simeq 3\,. \tag{22}$$

To get the duration of inflation N_{tot} we use expression $\varepsilon_1(N_{tot}) = 1$.

The choice of parameters of the inflation scenario under consideration specified by formula (19) is not the only possible one. T
The value of the field ϕ_0 is chosen such that $A_s(\phi_0) = 2.1 \cdot 10^{-9}$.

α	β	$\phi_0/M_{\rm Pl}$	n_s	r	N_*	N_{tot}
−0.40	−2	5.867	0.962	0.027	38.2	61.9
−0.40	−1.8	5.502	0.959	0.027	35.4	58.9
−0.40	−1.5	4.936	0.954	0.027	31.1	53.5
−0.37	−2	6.008	0.965	0.026	40.8	64.2
−0.37	−1.8	5.623	0.962	0.026	37.6	60.7
−0.37	−1.5	5.018	0.957	0.028	32.8	54.9
−0.35	−2	6.104	0.967	0.025	42.7	65.4
−0.35	−1.8	5.701	0.964	0.026	39.2	61.9
−0.35	−1.5	5.072	0.959	0.027	34.1	56.2

Table: Dependence of inflation parameters, duration of the first stage of inflation N_* and total duration of inflation N_{tot} on the model parameters α and β. Other model parameters are chosen as follows:
$V_0 = 10^{-10} M_{\rm Pl}^4, \quad \chi_0 = 3.5\, M_{\rm Pl}, \quad c_0 = 12, \quad c_2 = 147, \quad d = 0.003.$

The parameter d does not influence to values of inflationary parameters, but influences to the length of inflation, namely, to the length of the second stage.

d	N_{tot}	$N_{tot} - N_*$	$M_{PBH}/M_{\rm Pl}$	$M_{PBH}/M_\odot$	M_{PBH}/g
0.001	64.5	26.9	$8.57 \cdot 10^{28}$	$1.87 \cdot 10^{-10}$	$3.72 \cdot 10^{23}$
0.002	62.1	24.5	$7.07 \cdot 10^{26}$	$1.54 \cdot 10^{-12}$	$3.07 \cdot 10^{21}$
0.003	60.7	23.1	$4.31 \cdot 10^{25}$	$9.40 \cdot 10^{-14}$	$1.87 \cdot 10^{20}$
0.007	57.7	20.1	$1.18 \cdot 10^{23}$	$2.57 \cdot 10^{-16}$	$5.14 \cdot 10^{17}$
0.01	56.5	18.9	$9.92 \cdot 10^{21}$	$2.16 \cdot 10^{-17}$	$4.30 \cdot 10^{16}$

Table: The dependence of duration of inflation N_{tot} and the PBH mass M_{PBH} from the model parameter d. Other model parameters are given by (19). The end of the first stage of inflation is at $N_* = 37.6$ independent on d.

We are interesting to estimate mass of PBHs which could be formed during radiation dominate stage.
The current estimation of the mass region of PBHs considered as candidates for dark matter is $10^{-17}\, M_\odot \leqslant M_{PBH} \leqslant 10^{-12} M_\odot$, where $M_\odot \approx 1.98 \cdot 10^{33} g$ is the Solar mass.
The proposed model with $0.002 \leqslant d \leqslant 0.01$ allows us to reproduce the masses of the PBH from this interval.

$F(R)$ gravity models

Let us consider a generic $F(R)$ gravity model with the following action

$$S_F = \int d^4x\sqrt{-g}F(R), \qquad (23)$$

with a differentiable function F.
The $F(R)$ gravity action can be rewritten as

$$S_J = \int d^4x\sqrt{-g}\left[F_{,\sigma}(R-\sigma)+F\right] , \qquad (24)$$

where the new scalar field σ has been introduced, and $F_{,\sigma}(\sigma) = \frac{dF(\sigma)}{d\sigma}$.
To avoid graviton as a ghost one should put the following condition

$$F_{,\sigma}(\sigma) > 0 \qquad (25)$$

that restricts possible values of parameters and $\sigma = R$.

The Einstein frame

After the conformal transformation of the metric

$$g_{E\,\mu\nu} = \frac{2F_{,\sigma}(\sigma)}{M_{Pl}^2} g_{\mu\nu} \tag{26}$$

one gets the following action in the Einstein frame:

$$S_E = \int d^4x \sqrt{-g_E} \left[\frac{M_{Pl}^2}{2} R_E - \frac{h(\sigma)}{2} g_E{}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V_E \right], \tag{27}$$

where we have introduced the functions

$$h(\sigma) = \frac{3M_{Pl}^2}{2F_{,\sigma}^2} F_{,\sigma\sigma}^2 \quad \text{and} \quad V_E(\sigma) = M_{Pl}^4 \frac{F_{,\sigma}\sigma - F}{4F_{,\sigma}^2} \, . \tag{28}$$

Introducing the canonical scalar field

$$\phi = \sqrt{\frac{3}{2}} M_{Pl} \ln \left[\frac{2}{M_{Pl}^2} F_{,\sigma} \right] , \qquad (29)$$

we get the action S_E in the standard form:

$$S_E = \int d^4x \sqrt{-g_E} \left[\frac{M_{Pl}^2}{2} R_E - \frac{1}{2} g_E{}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_E(\phi) \right] . \qquad (30)$$

Introducing the canonical scalar field

$$\phi = \sqrt{\frac{3}{2}} M_{Pl} \ln\left[\frac{2}{M_{Pl}^2} F_{,\sigma}\right], \qquad (29)$$

we get the action S_E in the standard form:

$$S_E = \int d^4x \sqrt{-g_E}\left[\frac{M_{Pl}^2}{2} R_E - \frac{1}{2} {g_E}^{\mu\nu} \partial_\mu\phi\partial_\nu\phi - V_E(\phi)\right]. \qquad (30)$$

The inverse transformation is as follows:

$$R = \left[\frac{\sqrt{6}}{M_{Pl}} V_{E,\phi} + \frac{4V_E}{M_{Pl}^2}\right] \exp\left(\sqrt{\frac{2}{3}} \frac{\phi}{M_{Pl}}\right), \qquad (31)$$

$$F = \frac{M_{Pl}^2}{2}\left[\frac{\sqrt{6}}{M_{Pl}} V_{E,\phi} + \frac{2V_E}{M_{Pl}^2}\right] \exp\left(2\sqrt{\frac{2}{3}} \frac{\phi}{M_{Pl}}\right), \qquad (32)$$

where $V_{E,\phi} = \frac{dV_E}{d\phi}$, defining the function $F(R)$ in the parametric form with the parameter ϕ.

The Starobinsky R^2 inflationary model

The action of the Starobinsky model of inflation,
A.A. Starobinsky, *Phys. Lett. B* **91** (1980) 99.

$$S_{\rm Star.} = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left(R+\frac{1}{6m^2}R^2\right)\ , \tag{33}$$

includes the inflaton mass m.
The action (33) is dual to the Einstein frame action

$$S_E = \int d^4x\sqrt{-g_E}\left[\frac{M_{Pl}^2}{2}R_E-\frac{1}{2}{g_E}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V_{\rm Star.}(\phi)\right], \tag{34}$$

where

$$V_{\rm Star.}(\phi)=\frac{3}{4}M_{Pl}^2m^2\left[1-\exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{Pl}}\right)\right]^2\ . \tag{35}$$

$F(R)$ models with scalar fields

Let us consider $F(R)$ model with a scalar field χ, described by the following action:

$$S_R = \int d^4x\sqrt{-g}\left[F(R,\chi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi\right]. \qquad (36)$$

The model (36) is equivalent to following two-field model

$$S_J = \int d^4x\sqrt{-g}\left[F_{,\sigma}R + \left(F - F'_{,\sigma}\sigma\right) - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi\right]. \qquad (37)$$

Using the conformal transformation of the metric, we obtain a chiral cosmological model with two scalar fields, described by the following action

$$S_E = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{g^{\mu\nu}}{2}\partial_\mu\phi\partial_\nu\phi - \frac{M_{\rm Pl}^2}{4F_{,\sigma}}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V_E\right],$$

where ϕ, χ are scalar fields and

$$V_E = \frac{M_{\rm Pl}^4}{4F_{,\sigma}^2}\left(F_{,\sigma}\sigma - F\right), \qquad \phi = \sqrt{\frac{3}{2}}M_{\rm Pl}\ln\left(\frac{2F_{,\sigma}}{M_{\rm Pl}^2}\right). \qquad (38)$$

To get the potential $V_E(\phi,\chi)$ in the explicit form we should find the function $\sigma(\phi,\chi)$.

We assume that $F(\sigma,\chi)$ has the following form:

$$F(\sigma,\chi)=\frac{M_{\rm Pl}^2}{2}\left[X_0(\chi)F_{\rm Star.}(\sigma)+X_1(\chi)\sigma-U(\chi)\right], \quad F_{\rm Star.}=\sigma+\frac{\sigma^2}{6m^2},$$

$X_0(\chi)$, $X_1(\chi)$ and $U(\chi)$ are differentiable functions, m is the parameter. In this case,

$$\sigma=-\frac{3\,m^2\left[\left(X_0(\chi)+X_1(\chi)\right)y-1\right]}{X_0(\chi)y}, \quad \text{where} \quad y=\exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{Pl}}\right).$$

and

$$F_{\rm Star.}(\sigma(y,\chi))=\frac{3\,m^2\left(X_1\left(\chi\right)y-1\right)^2}{2X_0(\chi)y^2}-\frac{3m^2}{2}X_0\left(\chi\right). \qquad (39)$$

The corresponding expression for potential is:

$$
\begin{aligned}
V_E &= \frac{3M_{\rm Pl}^2 m^2 \left(X_0\left(\chi\right)\sigma^2 + 6m^2 U\left(\chi\right)\right)}{4\left(3m^2 X_0\left(\chi\right) + 3m^2 X_1\left(\chi\right) + X_0\left(\chi\right)\sigma\right)^2} \\
&= \frac{M_{\rm Pl}^2}{4X_0(\chi)}\,\Big[3m^2 y^2 X_0^2(\chi) + 3\,m^2\left(X_1(\chi)y - 1\right)^2 \\
&\quad + 2yX_0(\chi)\left(3\,m^2 yX_1(\chi) - 3\,m^2 + yU(\chi)\right)\Big]\,.
\end{aligned}
\tag{40}
$$

The $V_E(y,\chi)$ is a quadratic polynomial in y.
The choice of the functions $X_0(\chi)$, $X_1(\chi)$, and $U(\chi)$ allows us to get the potential $V_E(y,\chi)$ in the explicit form.

Let us choose the potential in the Einstein frame as follows:

$$V=\frac{\lambda}{4}\left(\chi^2-\chi_0^2\right)^2+\left(C_0+C_1\chi^2\right)(1-y)^2+d\,\chi, \qquad (41)$$

where λ, χ_0, C_0, C_1, and d are constants.
Similar modification of the hybrid inflation has been proposed in *M. Braglia, A. Linde, R. Kallosh, F. Finelli*, JCAP **04** (2023) 033..
The potential (41) corresponds to the following functions:

$$U = \frac{2\left(\lambda\left(\chi^2-\chi_0^2\right)^2+4\,d\,\chi\right)\left(C_1\,\chi^2+C_0\right)}{M_{\rm Pl}^2\left(\lambda\left(\chi^2-\chi_0^2\right)^2+4\,C_1\,\chi^2+4\,d\,\chi+4\,C_0\right)}, \qquad (42)$$

$$X_0 = \frac{3\,m^2 M_{\rm Pl}^2}{\lambda\left(\chi^2-\chi_0^2\right)^2+4\,C_1\,\chi^2+4\,d\,\chi+4\,C_0}, \qquad (43)$$

$$X_1 = \frac{4\,C_1\,\chi^2+4\,C_0-3\,m^2 M_{\rm Pl}^2}{\lambda\left(\chi^2-\chi_0^2\right)^2+4\,C_1\chi^2+4\,d\,\chi+4\,C_0}, \qquad (44)$$

During inflation the both fields play a role of the inflaton: ϕ at the beginning and χ at the end of inflation.
Inflation is too long for such a model: $N>70$.

In *E.O. Pozdeeva, S.Yu. Vernov*, Phys. Scr. **98** (2023) 055001, we propose

$$F(R) = \frac{M_{Pl}^2}{2}\left[\left(1-\frac{3}{2}\beta\delta\right)R + \frac{R^2}{6m^2} + \frac{\delta}{m}\left(R+\beta^2m^2\right)^{3/2} - m^2\beta^3\delta\right],$$

with two dimensionless parameters δ and β.
At $\beta \neq 0$ and $\delta \geqslant 0$, this model is well-defined for all $R > R_{min}$, where $-\beta^2m^2 \leqslant R_{min} < 0$.
In the case of $\beta \neq 0$, the function $F(R)$ has a correct GR limit at $R \ll m^2$:

$$F = \frac{M_{Pl}^2}{2}R\left[1 + \left(1+\frac{9\delta}{4\beta}\right)\frac{R}{6m^2} + \mathcal{O}\left(\frac{R^2}{m^4}\right)\right], \qquad (45)$$

To get V_E in the analytic form we choose

$$\beta = \sqrt{3} - \frac{9}{4}\delta, \qquad (46)$$

Adding the scalar field χ, we take the same function $X_0(\chi)$, $X_1(\chi)$, and $U(\chi)$ as for the generalization of the Starobinsky model.

The standard slow-roll parameters ε_1 and ε_2 are

$$\varepsilon_1 = -\frac{H'}{H} = \frac{1}{2M_{\rm Pl}^2}\left[\phi'^2 + y\chi'^2\right],$$

$$\varepsilon_2 = \frac{\varepsilon_1'}{\varepsilon_1} = 2\varepsilon_1 + \frac{1}{HX^2}\frac{dX}{dt}.$$

In the spatially flat FLRW metric, we get the following system:

$$\phi'' = (\varepsilon_1 - 3)\phi' + \frac{1}{2}\frac{dy}{d\phi}\chi'^2 - \frac{6M_{\rm Pl}^2 - y\chi'^2 - \phi'^2}{2V_E}\frac{\partial V_E}{\partial \phi},$$

$$\chi'' = (\varepsilon_1 - 3)\chi' - \frac{1}{y}\frac{dy}{d\phi}\chi'\phi' - \frac{6M_{\rm Pl}^2 - \phi'^2 - y\chi'^2}{2yV_E}\frac{\partial V_E}{\partial \chi}.$$

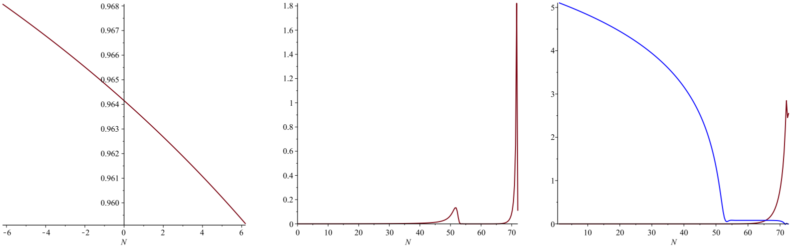

Figure: The behavior of $n_s(N)$, $\varepsilon_1(N)$ and the fields $\phi(N)$ and $\chi(N)$ in units of $M_{\rm Pl}$ during inflation.

Conclusions

1. We have proposed an induced gravity inflationary model with two scalar fields.
2. Due to the conformal transformation we get the chiral cosmological model.
3. The choice of the model parameters allows us to get the black hole masses suitable for consideration of the obtained PBHs as dark matter candidates:

$$10^{-17}\, M_{\odot} \leqslant M_{PBH} \leqslant 10^{-12} M_{\odot}.$$

4. We have proposed an $F(R,\chi)$ inflationary model with possible PBH generation.
5. If the latest Atacama Cosmology Telescope (ACT) observation data [arXiv:2503.14452] are correct, then the proposed models should be modified.

This study was conducted within the scientific program of the National Center for Physics and Mathematics, section 5 'Particle Physics and Cosmology'. Stage 2023–2025.

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Thank for your attention