



RG flows in dilaton gravity models with properties similar to QCD

Phys.Rev.D 111 (2025) 4, 046013, arXiv:[2503.09444](https://arxiv.org/abs/2503.09444)

Based on a joint work with I.Ya. Arefeva, A. Hajilou, P. Slepov

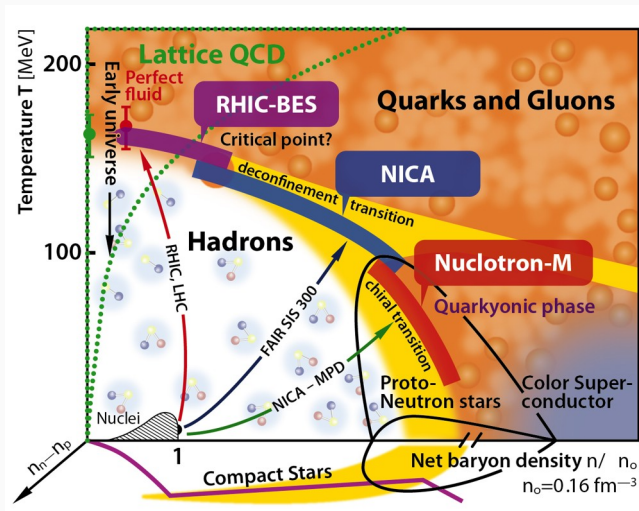
Marina Usova

Steklov Mathematical Institute of RAS

QFTHEP-270

June 30 - July 5, Moscow, Russia

Studies of QCD Phase Diagram is the main goal of new facilities



From: <https://nica.jinr.ru/physics.php>

AdS/CFT-correspondence for AdS₅

Gravity in 5-dimensional AdS₅ \Leftrightarrow 4d conformal field theory

The 5-dimensional Einstein-Maxwell-scalar action

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R - \frac{f_0(\varphi)}{4} F^2 - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \mathcal{V}(\varphi) \right],$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Ansatz for the metric:

$$ds^2 = B^2(z) \left[-g(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{g(z)} \right],$$

$$B(z) = \frac{e^{A(z)}}{z}, \quad \varphi = \varphi(z), \quad A_\mu = (A_t(z), \vec{0}, 0)$$

Light quarks

$$A_{LQ}(z) = -a \log(bz^2 + 1)$$

$$f_{LQ}(z) = e^{-c z^2 - A(z)}$$

fitting with experimental data:

$$a = 4.046, b = 0.01613 GeV^2, c = 0.227 GeV^2$$

(Li et.al.'17; Aref'eva et. al.'21, '22)

Heavy quarks

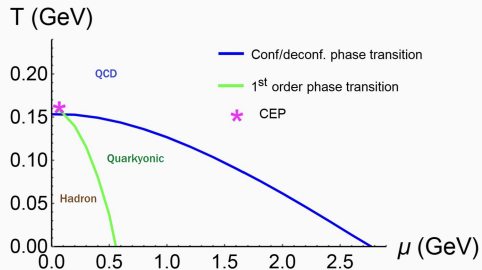
$$A_{HQ}(z) = -\frac{s}{3} z^2 - p z^4$$

$$f_{HQ}(z) = e^{-s z^2 - A(z)}$$

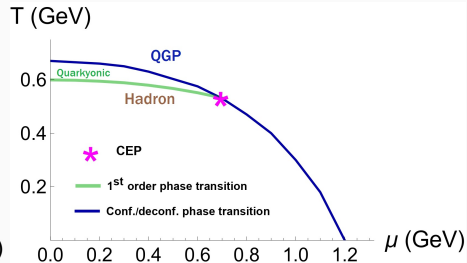
$$s = 1.16 GeV^2, p = 0.273 GeV^4$$

(Yang & Yuan, '16; Aref'eva et. al.'20, '23) ³

Light quarks



Heavy quarks



EOM

$$\begin{aligned} \varphi'' + \left(\frac{g'}{g} + 3A' - \frac{3}{z} \right) \varphi' + \left(\frac{z^2 e^{-2A} A'_t f_{0,\varphi}}{2g} - \frac{e^{2A} V_\varphi}{z^2 g} \right) &= 0, \\ A''_t + \left(\frac{f'_0}{f_0} + A' - \frac{1}{z} \right) A'_t &= 0, \\ A'' - A'^2 + \frac{2}{z} A' + \frac{\varphi'^2}{6} &= 0, \\ g'' + \left(3A' - \frac{3}{z} \right) g' - e^{-2A} z^2 f_0 A'^2_t &= 0, \\ A'' + 3A'^2 + \left(\frac{3g'}{2g} - \frac{6}{z} \right) A' - \frac{1}{z} \left(\frac{3g'}{2g} - \frac{4}{z} \right) + \frac{g''}{6g} + \frac{e^{2A} V}{3z^2 g} &= 0 \end{aligned}$$

Applied boundary conditions: $A_t(0) = \mu$, $A_t(z_h) = 0$, $g(0) = 1$, $g(z_h) = 0$,

$$\boxed{\varphi(z) \Big|_{z=z_0} = 0, \quad \text{where } z_0 = \mathfrak{z}(z_h)} \begin{cases} \mathfrak{z}_{LQ}(z_h) = 10 \exp[-z_h/4] + 0.1 \\ \mathfrak{z}_{HQ}(z_h) = \exp[-z_h/4] + 0.1 \end{cases}$$

(I. Aref'eva, K.Rannu, P.Slepov'21)

General EOM solutions

$$\begin{aligned} \varphi'(z) &= \sqrt{-6 \left(A'' - A'^2 + \frac{2}{z} A' \right)}, \quad A_t(z) = \mu \frac{e^{cz^2} - e^{cz_h^2}}{1 - e^{cz_h^2}} \\ V(z) &= -3z^2 g e^{-2A} \left[A'' + 3A'^2 + \left(\frac{3g'}{2g} - \frac{6}{z} \right) A' - \frac{1}{z} \left(\frac{3g'}{2g} - \frac{4}{z} \right) + \frac{g''}{6g} \right] \end{aligned}$$

Domain wall solution:

$$ds^2 = e^{2A(w)} \eta_{ij} dx^i dx^j + dw^2, \quad \varphi = \varphi(w)$$

E. Akhmedov'98; de Boer, Verlinde, Verlinde'98, Skenderis et. al.'98; Skenderis'99

Solution under consideration:

$$ds^2 = B(z)^2 \left(\eta_{ij} dx^i dx^j + dz^2 \right), \quad B(z) = \frac{e^{A(z)}}{z}, \quad \varphi = \varphi(z)$$

The holographic dictionary

- $\alpha = e^{\varphi(z)}$ is identified as running coupling of the field theory
- the energy scale E of the dual field theory (Galow et. al. '09)

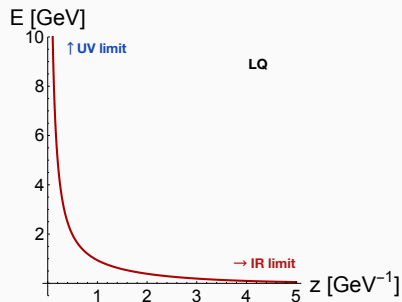
$$E = B(z) \quad E_{LQ} = \frac{1}{z (1 + bz^2)^a}, \quad E_{HQ} = \frac{1}{z e^{\frac{c}{3}z^2 + pz^4}}$$

- β -function (DeWolfe et. al. '14, Kiritsis et.al.'14)

$$\beta = \left. \frac{d\alpha}{d \log E} \right|_{QFT} = \alpha \left. \frac{d\varphi}{d \log B} \right|_{Holo}$$

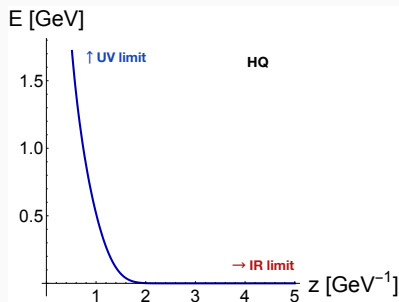
I.Ya. Aref'eva, A.Hajilou, P.S. Slepov, MU, TMF (2024), PRD (2025)

Light quarks



$$E_{LQ} = \frac{1}{z(1+bz^2)^a}$$

Heavy quarks



$$E_{HQ} = \frac{1}{z e^{\frac{c}{3}z^2 + pz^4}}$$

New dynamical variable:

$$X(z) = \frac{\dot{\varphi} B}{3\dot{B}}$$

Kiritsis et.al. '08

Physical meaning of the dynamical system

- Variable X is associated with overstretched β -function:

$$\beta = 3\alpha X$$

- The scale factor A (therefore, the energy scale) decreases along the flow

Holographic RG flows equation:

$$\frac{d\mathcal{X}}{d\varphi} = -\frac{4}{3} \left(1 - \frac{3}{8}\mathcal{X}^2\right) \left(1 + \frac{1}{\mathcal{X}} \frac{\partial_{\varphi}\mathcal{V}(\varphi)}{\mathcal{V}(\varphi)}\right)$$

I. Ya. Aref'eva, K. Rannu '18

where we consider $X(z) = \mathcal{X}(\varphi(z))$

! HRG flows equation is invariant with respect to change of b.c. !

Scalar fields and potentials

After subtraction of the scale factors into EOM:

$$\varphi_{LQ}(z, \varphi_0) = \varphi_0 + 2\sqrt{3a} \left[\sqrt{2a+1} \operatorname{arcsinh} \left(\sqrt{\frac{b(2a+1)}{3}} z \right) - \sqrt{2(a-1)} \operatorname{arctanh} \left(\sqrt{\frac{2b(a-1)}{(2a+1)bz^2+3}} z \right) \right]$$

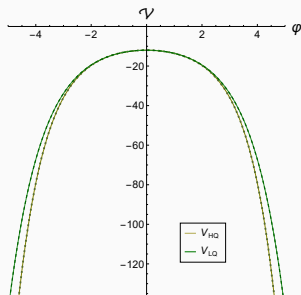
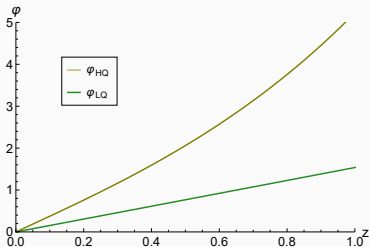
$$\varphi_{HQ}(z, \varphi_0) = \varphi_0 + \sqrt{6} \int_0^z d\xi \sqrt{\left(-4p\xi^3 - \frac{2s\xi}{3} \right)^2 + 2 \left(4p\xi^2 + \frac{2s}{3} \right) + 12p\xi^2 + \frac{2s}{3}}$$

$$V_{LQ}(z) = -6 (bz^2 + 1)^{2a-2} \left[bz^2 \left((a(6a+7) + 2)bz^2 + 5a + 4 \right) + 2 \right],$$

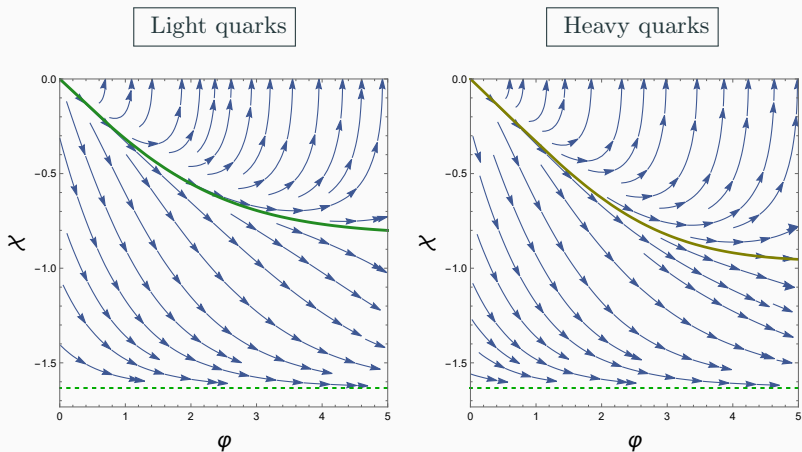
$$V_{HQ}(z) = -2 e^{\frac{1}{3}(6pz^4 + 2sz^2)} \left[6 + 5sz^2 + 2(9p + s^2)z^4 + 24psz^6 + 72p^2z^8 \right]$$

Reconstruction of the potentials:

$$V(\varphi) = \sum_{i=0}^{18} c_i \varphi^i \quad -13.5 \leq \varphi \leq 13.5$$



Comparison of the flows for $T = 0$, $\mu = 0$



Green dashed lines are fixed (attractor) lines: $\mathcal{X} = -\sqrt{8/3}$.
Solid green and olive lines depict an exact solution \mathcal{X} .

- simple hRG flows for light and heavy quarks models are almost the same!

Change of variables

- The overstretched β -function analogue: $X = \frac{\dot{\phi}}{3} \frac{B}{B'}$,
- The temperature $T(z_h) = \frac{|g'|}{4\pi} \Big|_{z=z_h}$ analogue: $Y = \frac{1}{4} \frac{\dot{g}}{g} \frac{B}{B'}$,
- The chemical potential μ analogue: $H = \frac{\dot{A}_t}{B^2}$

Thus, holographic RG flows equations:

$$\begin{aligned} \frac{d\mathcal{X}}{d\varphi} &= -\frac{4}{3} \left(1 - \frac{3}{8}\mathcal{X}^2 + \mathcal{Y}\right) \left(1 + \frac{1}{\mathcal{X}} \frac{2\partial_\varphi \mathcal{V}(\varphi) - \mathcal{H}^2 \partial_\varphi f_0}{2\mathcal{V}(\varphi) + \mathcal{H}^2 f_0}\right), \\ \frac{d\mathcal{Y}}{d\varphi} &= -\frac{4\mathcal{Y}}{3\mathcal{X}} \left(1 - \frac{3}{8}\mathcal{X}^2 + \mathcal{Y}\right) \left(1 + \frac{3}{2\mathcal{Y}} \frac{\mathcal{H}^2 f_0}{2\mathcal{V}(\varphi) + \mathcal{H}^2 f_0}\right), \\ \frac{d\mathcal{H}}{d\varphi} &= -\left(\frac{1}{\mathcal{X}} + \frac{\partial_\varphi f_0}{f_0}\right) \mathcal{H}, \end{aligned}$$

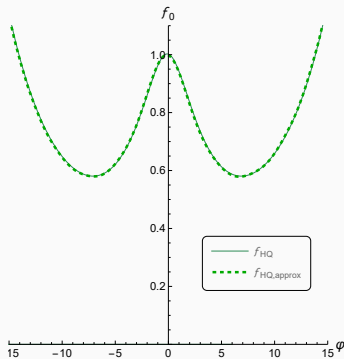
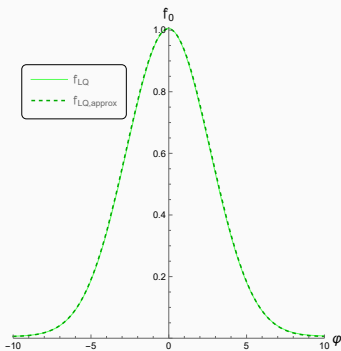
where $X = \mathcal{X}(\varphi(z)), \quad Y = \mathcal{Y}(\varphi(z)), \quad H = \mathcal{H}(\varphi(z))$

$$f_{LQ}(z) = e^{-cz^2 + a \log(bz^2 + 1)},$$

$$f_{HQ}(z) = e^{-\frac{2}{3}s z^2 + p z^4}$$

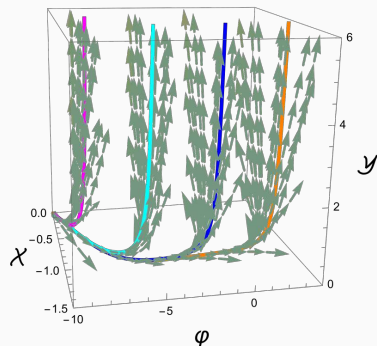
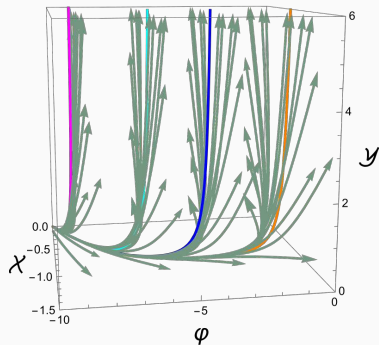
Reconstruction: $f_{LQ,approx}(\varphi) = \frac{1}{0.39931\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\varphi - \varphi_0}{2.70895}\right)^2\right]$

$$f_{HQ,approx}(\varphi) = \sum_{i=0}^{30} w_i \varphi^i$$



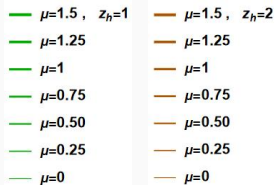
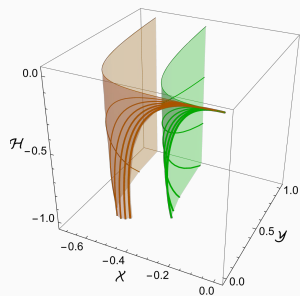
Light quarks

Heavy quarks

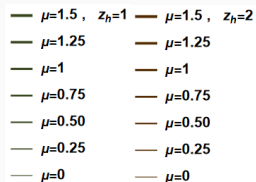
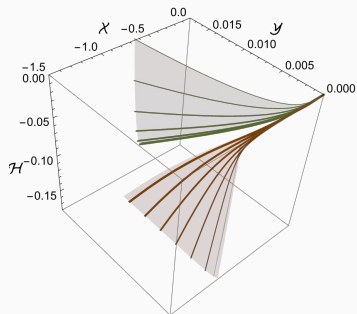


- differently colored branches of solutions correspond to different z_h

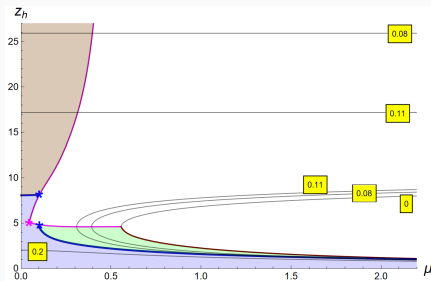
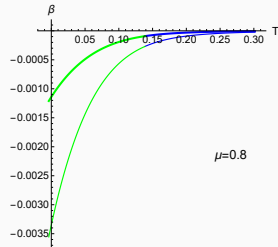
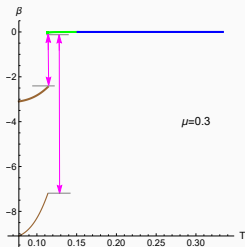
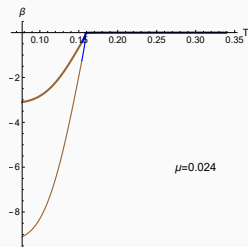
Light quarks



Heavy quarks

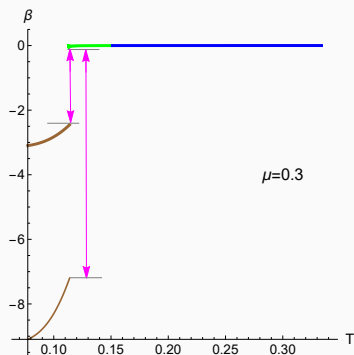


$$\text{b.c.: } z_0 = \mathfrak{z}_{LQ}(z_h) = 10 e^{(-\frac{z_h}{4})} + 0.1$$

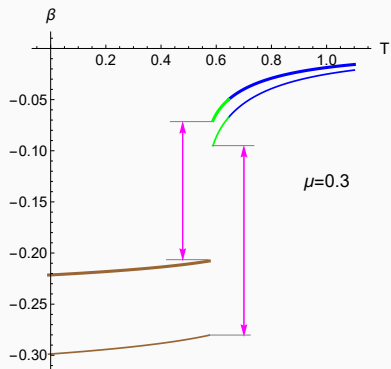


Comparison of β -functions

Light quarks



Heavy quarks



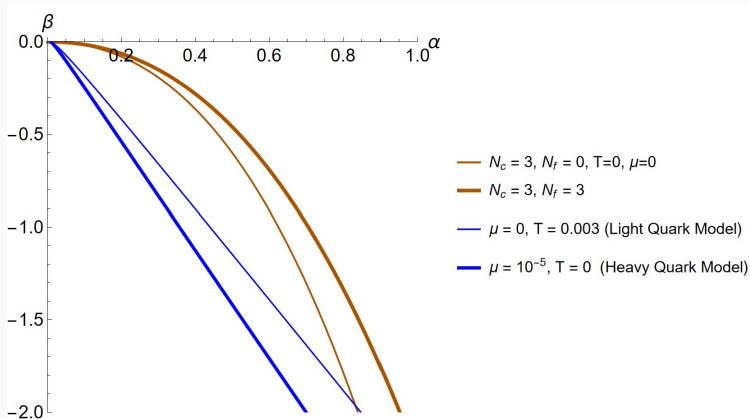
Agreement of the holographic β -function with 2-loop QCD

S. He et. al.'11; T. van Ritbergen et.al.'97

The QCD β -function at 2-loop level has the following form

$$\beta(\alpha) = -b_0 \alpha^2 - b_1 \alpha^3$$

where $b_0 = \frac{1}{2\pi} \left(\frac{11}{3} N_c - \frac{2}{3} N_f \right)$ and $b_1 = \frac{1}{8\pi^2} \left(\frac{34}{3} N_c^2 - \left(\frac{13}{3} N_c - \frac{1}{N_f} \right) \right)$.



Results for hRG

- HRG flows for LQ and HQ are approximately the same although the associated warp factor is completely different
- The choice of different boundary conditions simply shifts the RG both for heavy and light quarks, without leading to significant changes

Results for β -function

- in all regions β is negative
- on the 1-st order phase transition line, β has a jump
 - magnitude of the jump is zero at the CEP
 - this magnitude increases by decreasing the probe energy scale E
- both for hadronic and QGP regimes at fixed μ and energy scale z , β increases with increasing T - the speed of growth is different for LQ and HQ

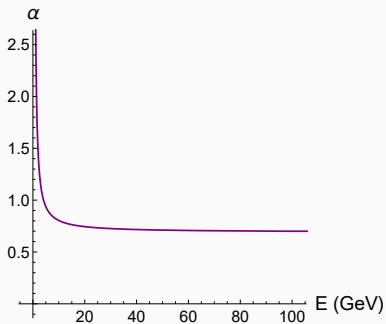
Prospective questions

- Exact numerical correspondence of β_{holo} with β_{QCD}

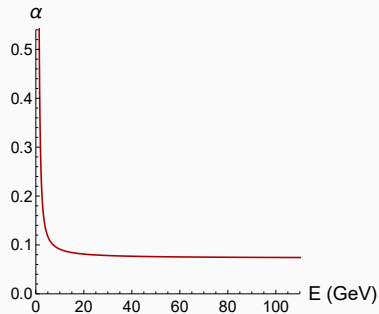
Thank you for your attention!

Extra: On the relevance of the holographic isotropic models

Light quarks



Heavy quarks



ultra-UV regime cannot be reached

BUT strongly coupled and
a small part of weakly coupled regimes are covered