Gauged supergravities: solutions with a Killing tensor

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Based on
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Contents

Motivation

- The theoretical study of ultra-compact cosmic objects is one of the most relevant areas of physics due to the emergence of **new channels of astrophysical observations**: gravitational-wave astronomy and interferometry of a very long baseline.
- The most justified **deviations from general relativity** are associated with supergravity/superstring theory, as models with deep theoretical motivation.
- In the bosonic sector, supergravity models can be considered as scalar-vector-tensor theories with a **wide range of Lagrangians**.
- Black hole solutions on gauged supergravities have **ADS asymptotic**; they are sought after in holographic models.
- Apart from the Einstein-Maxwell case, supergravity ADS black holes have **previously only been introduced by guesswork**.

Anzats for the metric

Consider the metric in the form of the restricted[1]

Benenti-Francaviglia anzats

$$ds^2 = \frac{A_2}{\Sigma}(b dt - B_{23} d\varphi)^2 - \frac{B_2}{\Sigma}(a dt - A_{23} d\varphi)^2 - \frac{\Sigma}{A_2} dr^2 - \frac{\Sigma}{B_2} dy^2, \tag{1}$$

where $A_i = A_i(r),\ B_i = B_i(y)$ and $a,\ b$ are constants. We impose the condition

$$\Sigma = \sqrt{-g} = A_{23} - aB_{23}, \tag{2}$$

leading to the separation of the variables in the Klein-Gordon equation for the scalar field

$$\nabla^\mu \nabla_\mu \phi = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g}\, g^{\mu\nu} \partial_\nu \phi) = -\mu^2 \phi. \tag{3}$$

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[1] D. Gal'tsov and A. Kulitskii, "Petrov types, separability, and generalized photon surfaces of supergravity black holes," Phys. Rev. D **110**, no.12, 124008 (2024) [arXiv:2409.13324].

Tetrad formalism

Consider tetrad one-forms e^a $(a = 1, .., 4)$, associated with our metric parametrisation

$$\begin{aligned} e^1 &= \alpha(bdt - B_{23}d\varphi), \\ e^2 &= \beta(adt - A_{23}d\varphi), \\ e^3 &= \alpha^{-1}dr, \\ e^4 &= \beta^{-1}dy, \end{aligned} \tag{4}$$

where

$$\alpha = \sqrt{A_2/\Sigma}, \qquad \beta = \sqrt{B_2/\Sigma}. \tag{5}$$

Than

$$ds^2 = \eta_{ab}e^a e^b, \qquad \eta_{ab} = diag(1, -1, -1, -1). \tag{6}$$

Ricci tensor

Tetrad components of the Ricci tensor are given by

$$R_{ab} = -\frac{1}{2}\Big(\lambda_{ab}{}^{c}{}_{,c} + \lambda_{ba}{}^{c}{}_{,c} + \lambda^{c}{}_{ca,b} + \lambda^{c}{}_{cb,a} + \lambda^{cd}{}_{b}\lambda_{cda} + \lambda^{cd}{}_{b}\lambda_{dca} - \\ - \frac{1}{2}\lambda_{b}{}^{cd}\lambda_{acd} + \lambda^{c}{}_{cd}\lambda_{ab}{}^{d} + \lambda^{c}{}_{cd}\lambda_{ba}{}^{d}\Big), \tag{7}$$

where the quantities $\lambda_{abc} = -\lambda_{acb}$ are defined by the commutators of the basis vectors as

$$\lambda_{abc} = (e_{a\mu,\nu} - e_{a\nu,\mu})e_{b}^{\mu}e_{c}^{\nu}. \tag{8}$$

The only nonvanishing components are

$$\lambda_{113} = \frac{\alpha}{2}\left(\frac{A_2'}{A_2} - \frac{bA_{23}'}{\Sigma}\right), \quad \lambda_{114} = -\frac{\beta}{2}\frac{aB_{23}'}{\Sigma}, \quad \lambda_{124} = \alpha\frac{bB_{23}'}{\Sigma}, \\ \lambda_{224} = -\frac{\beta}{2}\left(\frac{B_2'}{B_2} + \frac{aB_{23}'}{\Sigma}\right), \quad \lambda_{223} = -\frac{\alpha}{2}\frac{bA_{23}'}{\Sigma}, \quad \lambda_{213} = \beta\frac{aA_{23}'}{\Sigma}, \\ \lambda_{334} = \frac{\beta}{2}\frac{aB_{23}'}{\Sigma}, \quad \lambda_{434} = \frac{\alpha}{2}\frac{bA_{23}'}{\Sigma}. \tag{9}$$

Ricci tensor

Tetrad components of the Ricci tensor are then given by

$$
\begin{aligned}
R_{11} &= \frac{\left((A'_{23})^{2} + (B'_{23})^{2}\right)\left(\alpha^{2} b^{2} - a^{2}\beta^{2}\right)}{2\Sigma^{2}} - \frac{a\beta^{2} B''_{23}}{2\Sigma} - \frac{a B'_{2} B'_{23}}{2\Sigma^{2}} - \frac{\alpha^{2} b A''_{23}}{2\Sigma} - \frac{b A'_{2} A'_{23}}{2\Sigma^{2}} + \frac{A''_{2}}{2\Sigma}, \\
R_{12} &= \frac{\sqrt{A_{2}}\sqrt{B_{2}}}{2\Sigma^{2}}\left(a A''_{23} + b B''_{23}\right), \\
R_{22} &= R_{11} - \frac{1}{2\Sigma}(A''_{2} + B''_{2}), \\
R_{33} &= \frac{a^{2}\beta^{2}\left((A'_{23})^{2} + (B'_{23})^{2}\right)}{2\Sigma^{2}} + \frac{a\beta^{2} B''_{23}}{2\Sigma} + \frac{a B'_{2} B'_{23}}{2\Sigma^{2}} - \frac{\alpha^{2} b A''_{23}}{2\Sigma} + \frac{b A'_{2} A'_{23}}{2\Sigma^{2}} - \frac{A''_{2}}{2\Sigma}, \\
R_{44} &= \frac{b^{2}\alpha^{2}\left((A'_{23})^{2} + (B'_{23})^{2}\right)}{2\Sigma^{2}} + \frac{a\beta^{2} B''_{23}}{2\Sigma} - \frac{a B'_{2} B'_{23}}{2\Sigma^{2}} - \frac{\alpha^{2} b A''_{23}}{2\Sigma} - \frac{b A'_{2} A'_{23}}{2\Sigma^{2}} - \frac{B''_{2}}{2\Sigma}, \\
R_{13} &= R_{14} = R_{23} = R_{24} = R_{34} = 0.
\end{aligned}
\tag{10}
$$

Action of the *EMDA* theory

Consider the Einstein-Maxwell-Dilaton-Axion(EMDA) theory, which is a consistent truncation of the $\mathcal{N} = 4$ gauged supergravity theory

$$S = \frac{1}{16\pi} \int \left(-R + 2\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}e^{4\phi}\partial_\mu\kappa\partial^\mu\kappa + \frac{1}{l^2}V - e^{-2\phi}F_{\mu\nu}F^{\mu\nu} - \kappa F_{\mu\nu}\tilde{F}^{\mu\nu} \right)\sqrt{-g}\,d^4x,$$

where

$$V = 4 + e^{-2\phi} + e^{2\phi}\left(\kappa^2 + 1\right). \tag{11}$$

Ungauged EMDA theory corresponds to the limit $l \to \infty$, i.e. theory without a scalar potential.

Complex form

Consider a complex scalar field defined by

$$z = \kappa + ie^{-2\phi}. \tag{12}$$

Than written in terms of this field the action is

Complex form of the action

$$S = -\frac{1}{16\pi}\int\left(R + \frac{2\nabla z\nabla\bar{z}}{(z-\bar{z})^2} - \frac{1}{l^2}V - \left(iz\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + c.c.\right)\right)\sqrt{-g}\,d^4x, \tag{13}$$

where $\mathcal{F}^{\mu\nu} = \frac{1}{2}\left(F^{\mu\nu} + i\tilde{F}^{\mu\nu}\right)$.

Axidilaton equation of motion

$$\Box z - \frac{2\partial z\partial z}{(z-\bar{z})} + \frac{i}{l^2}(z^2+1) - \frac{(z-\bar{z})^2}{4}\left(iF_{\mu\nu}F^{\mu\nu} + F_{\mu\nu}\tilde{F}^{\mu\nu}\right) = 0. \tag{14}$$

Einstein Equations

Varying the action with respect to the metric one obtains the

Einstein equations

$$R_{ab} = T^{sc}_{ab} + \frac{(z-\bar{z})}{2i} T^{em}_{ab} + \frac{2}{l^2}\left(1 + \frac{i}{2}\frac{1+z\bar{z}}{(z-\bar{z})}\right)\eta_{ab}, \tag{15}$$

where

$$T^{sc}_{ab} = -\frac{1}{(z-\bar{z})^2}(z_{,a}\bar{z}_{,b} + z_{,b}\bar{z}_{a}), \tag{16}$$

and

$$T^{em}_{ab} = 2\left(F_{ac}F^{c}{}_{b} + \frac{\eta_{ab}}{4}F_{cd}F^{cd}\right). \tag{17}$$

Einstein Equations

R_{12} component of the Einstein equations gives

$$aA''_{23} + bB''_{23} = 0, \tag{18}$$

thus

$$\begin{aligned} A_{23}(r) &= \alpha_0 + 2\alpha_1 r + cr^2, \\ B_{23}(y) &= -\beta_0 - 2\beta_1 y - \frac{a}{b} cy^2, \end{aligned} \tag{19}$$

where $\alpha_0, \beta_0, \alpha_1, \beta_1, c$ - are arbitrary constants. We perform coordinate shifts $r \to r + r_0$, $y \to y + y_0$, to put $\alpha_1 = 0$, $\beta_1 = 0$. We can also put $c = 1$ by rescaling of the coordinate φ and put $b = 1$ by rescaling of t. Than

$$\begin{aligned} A_{23}(r) &= \alpha_0 + r^2, \\ B_{23}(y) &= -\beta_0 - ay^2, \end{aligned} \tag{20}$$

and we obtain

$$\Sigma = r^2 + a^2 y^2 + \alpha_0 + a\beta_0. \tag{21}$$

Einstein Equations

One more simple nonlinear equation(becoming linear in the ungauged limit) can be obtained from the difference of R_{11} and R_{22} components of the Einstein equation

$$A_2'' + B_2'' = \frac{8\Sigma}{l^2}\left(1 + \frac{i}{2}\frac{1 + z\bar{z}}{z - \bar{z}}\right). \tag{22}$$

The sum of R_{11} and R_{33} together with the difference of R_{22} and R_{44} components of the Einstein's equations give correspondingly

$$\frac{1}{2\Sigma^2}(4\Sigma - ((A_{23}')^2 + (B_{23}')^2)) = \frac{2}{(z - \bar{z})^2} z_{,r}\bar{z}_{,r}, \tag{23}$$

$$\frac{a^2}{2\Sigma^2}(4\Sigma - ((A_{23}')^2 + (B_{23}')^2)) = \frac{2}{(z - \bar{z})^2} z_{,y}\bar{z}_{,y}. \tag{24}$$

R_{34} component of the Einstein's equations includes only the axidilaton and gives

$$z_{,r}\bar{z}_{,y} + z_{,y}\bar{z}_{,r} = 0. \tag{25}$$

Anzats for the axidilaton

The last three equations lead to the fact that the axidilaton field must be a holomorphic function

$$z_{,y} = iaz_{,r}, \tag{26}$$

and must satisfy the Laplace equation

$$z_{,yy} + a^2 z_{,rr} = 0. \tag{27}$$

But than the axidilaton function is single valued and thus must be represented as a fractional-linear transformation:

$$z = z_\infty \cdot \frac{r + iay + c_1}{r + iay + c_2} \rightarrow //z_\infty = i// \rightarrow z = i\frac{r + iay + c_1}{r + iay + c_2}. \tag{28}$$

Maxwell field

An ansatz for the vector one-form compatible with our metric parameterization was first found by Carter[2] and is given by

Carter's anzats for the potential one-form

$$A = \frac{R}{\alpha\Sigma} e^1 + \frac{Y}{\beta\Sigma} e^2. \tag{29}$$

This one-form leads to separation of the Hamilton-Jacoby and Klein-Gordon equation for the charged particles.

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[2] B. Carter, "Hamilton-Jacobi and Schrodinger separable solutions of Einstein's equations," Commun. Math. Phys. **10** (1968) no.4, 280-310.

Maxwell field

Now we can obtain that the only nonvanishing components of the Maxwell tensor $F = \frac{1}{2} F_{ab} e^a \wedge e^b = dA$ of the electromagnetic field are given by

$$F_{13} = -\tilde{F}_{24} = \frac{A'_{23}(R + aY) - \Sigma R'}{\Sigma^2}, \quad F_{24} = \tilde{F}_{13} = -\frac{B'_{23}(R + aY) + \Sigma Y'}{\Sigma^2}, \tag{30}$$

which leads to the fact that only two equations of the full system of Maxwell equations and Bianchi identities aren't satisfied by default, namely:

Maxwell equations

$$\alpha F_{13}(z - \bar{z})_{,r} - (z - \bar{z}) \left[F_{13}(\lambda_{223} - \lambda_{434}) - F_{24}\lambda_{124} - \alpha F_{13,r} \right] + i\alpha F_{24}(z + \bar{z})_{,r} = 0, \tag{31}$$

$$\beta F_{24}(z - \bar{z})_{,y} + (z - \bar{z}) \left[F_{24}(\lambda_{114} - \lambda_{334}) - F_{13}\lambda_{213} + \beta F_{24,y} \right] - i\beta F_{13}(z + \bar{z})_{,y} = 0. \tag{32}$$

Maxwell field

Components of the Maxwell tensor $F_{13},\ F_{24}$ can be represented as:

$$F_{13} = -\frac{\partial}{\partial r}\left(\frac{R + aY}{\Sigma}\right), \quad F_{24} = -\frac{1}{a}\frac{\partial}{\partial y}\left(\frac{R + aY}{\Sigma}\right). \tag{33}$$

Than we have a possibility to make a gauge transformations of the form

$$\frac{R + aY}{\Sigma} \to \frac{R + aY}{\Sigma} + C. \tag{34}$$

From the difference of Maxwell equations one can obtain the linear equation

$$aR'' - Y'' = 0. \tag{35}$$

Using it together with the transformation (34) one can remove the quadratic terms in the functions $R,\ Y$ and thus take

$$R = R_0 + qr, \qquad Y = Y_0 - py. \tag{36}$$

System closure

It turns out that from the whole system of the equations of motion there is one more independent Einstein equation, which we choose to be the sum of R_{11} and R_{22} components

remaining Einstein equation

$$P_{,rr} + \frac{1}{a^2} P_{,yy} = -2i(z - \bar{z})(F_{13}^2 + F_{24}^2), \tag{37}$$

where

$$P = \frac{A_2 - a^2 B_2}{\Sigma}. \tag{38}$$

Thus we have rewritten all the independent equations of motion in a quite a simple form, which, as we will see, allows the separation of variables.

Ungauged EMDA theory

Expressions for the polynomials $A_2,\ B_2$

In the limit $l \to \infty$ the equation $R_{11} - R_{22}$ is simplified to

$$A_2'' + B_2'' = \frac{8\Sigma}{l^2}\left(1 + \frac{i}{2}\frac{1+z\bar{z}}{z-\bar{z}}\right) \quad \longrightarrow \quad A_2'' + B_2'' = 0, \tag{39}$$

so the functions $A_2,\ B_2$ must be represented as

$$A_2 = a_0 - 2a_1 r + \lambda r^2, \qquad\qquad B_2 = b_0 + 2b_1 y - \lambda y^2, \tag{40}$$

where a_i, b_i, λ - are arbitrary real constants. We see that $g_{tt} = \lambda - 2a_1/r$ when $r \to \infty$, assuming that the spacetime is localy flat at spatial infinity, we must put $\lambda = 1$, and $a_1 = m$ - Schwarzschild mass.

Coordinate shift

Solving the equations of motion of the system by extracting the conditions on the coefficients of the functions one can obtain the general form of the solution:

$$\begin{aligned}
A_{23} &= r^2 - |d|^2 - a\delta_0, \\
B_{23} &= -ay^2 - \delta_0, \\
A_2 &= r^2 - 2mr + (q^2 + p^2) - |d|^2 + a^2\delta_1, \\
B_2 &= -y^2 + 2b_1 y + \delta_1, \\
\Sigma &= r^2 + a^2y^2 - |d|^2, \quad d = \frac{(q - ip)^2}{2(m + iab_1)}.
\end{aligned} \tag{41}$$

In order to obtain the nonsingular static limit one has to make a coordinate transformation $y \to y + b_1$ and redefinitions $\delta_1 \to \delta_1 + b_1^2$, $\delta_0 \to \delta_0 + ab_1^2$.

The meaning of the parameter δ_1

In order to demonstrate the physical meaning of the parameter δ_1, consider the equatorial plane $y = 0$ at the spatial infinity ($r \to \infty$). Two-dimensional line element than is given by

$$dl^2_{(r,\varphi)} = \frac{\Sigma}{A_2}\left(dr^2 + \frac{A_2}{\Sigma^2}\left[B_2 A_{23}^2 - A_2 B_{23}^2\right] d\varphi^2\right) \to (dr^2 + \delta_1 r^2 d\varphi^2). \tag{42}$$

Thus the parameter δ_1 introduce the conical singularity (**cosmic string**[3]).
In order to remove the cosmic string one has to put $\delta_1 = 1$.

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[3] M. Aryal, L. H. Ford and A. Vilenkin, "Cosmic Strings and Black Holes," Phys. Rev. D **34**, 2263 (1986)

The meaning of the parameter δ_0

In order to determine the physical meaning of the constants b_1, δ_0, consider the asymptotic behavior $r \to \infty$ of the rotation function ω

$$\omega = -\frac{g_{t\varphi}}{g_{tt}} = \frac{A_2 B_{23} - a B_2 A_{23}}{A_2 - a^2 B_2}. \tag{43}$$

We obtain that the NUT-parameter n corresponds to the choice $b_1 = n/a$:

$$\omega = B_{23} - a B_2 = -2ny - \delta_0 - a\delta_1, \tag{44}$$

and the expression $\delta_0 + a\delta_1$ plays the role of the parameter, chainging **the configuration of the Misner strings**. South and North strings will be symmetric if $\delta_0 = -a\delta_1$. In this case, removing also the cosmic string we have to put $\delta_1 = 1, \delta_0 = -a$.

Final form of the solution

General solution of the ungauged theory

$$
ds^2 = \frac{A_2}{\Sigma}\left(dt + (a\cos^2\theta + 2n\cos\theta - a)d\varphi\right)^2 - \frac{B_2}{\Sigma}\left(adt - (r^2 - D^2 + a^2 + n^2)d\varphi\right)^2 - \\ - \Sigma\left(\frac{dr^2}{A_2} + d\theta^2\right),
$$

where

$$
\begin{aligned}
A_2 &= r^2 - 2mr + e^2 - D^2 + a^2 - n^2, \\
B_2 &= \sin^2\theta, \\
\Sigma &= r^2 + (a\cos\theta + n)^2 - D^2,
\end{aligned} \tag{45}
$$

and we introduced $e^2 = q^2 + p^2,\ \mu^2 = m^2 + n^2,\ D = e^2/2\mu$. The solution coincides with the solution of the *EMDA* theory, obtained previously using the Ehlers-Harrison transformations.

Ergosphere and horizon

The Killing vector ∂_t becomes null at the surfaces where $A_2 - a^2 B_2 = 0$, which corresponds to boundaries of the egoregion

$$r_e^{\pm} = m \pm \sqrt{\mu^2 - e^2 + D^2 - a^2 y^2}. \tag{46}$$

The radii of the horizons $r_H^{\pm}$, satisfying the equation $A_2 = 0$, are

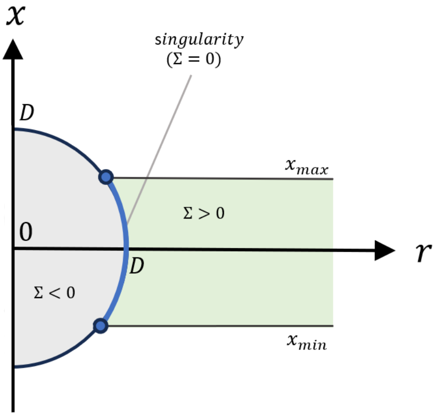
$$r_H^{\pm} = m \pm \sqrt{\mu^2 - e^2 + D^2 - a^2}. \tag{47}$$

Consider the Killing vector $\xi = \partial_t + \Omega \partial_\varphi$ with some constant Ω which can still be timelike in the region $r_H^+ < r < r_e^+$. The value of Ω on the surface $r = r_H^+$, where ξ becomes null, will be equal to

$$\Omega_H = \frac{a}{2\mu^2 - e^2 + 2m\sqrt{\mu^2 - e^2 + D^2 - a^2}}. \tag{48}$$

Singularity

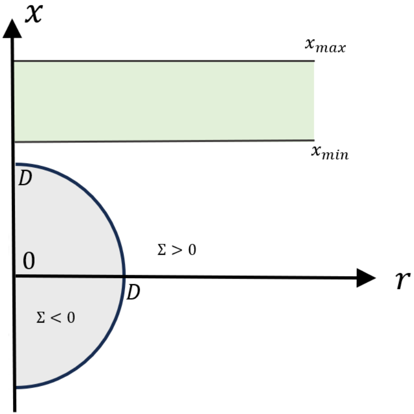
The singularity is defined by the equation $\Sigma = 0 \quad \Rightarrow \quad \boxed{r^2 + x^2 = D^2}$ where $x = a\cos\theta + n$



$$(x_{min} = n - a, \ x_{max} = n + a)$$

Singularity

If $[x_{min}, x_{max}] \cap [-D, D] = \varnothing$ than the solution doesn't have singular points



$$(x_{min} = n - a, \ x_{max} = n + a)$$

Singularity

- In the case $|n| > a$, one can obtain a condition

  $$e^2 < 2\mu(|n| - a), \tag{49}$$

  that guarantee the absence of a singularity at all.
- In the second case $|n| < a$ the singularity is represented by a part of the circle such that the point with $r_s = D$ must be included. Thus in order to not have a naked singularity, we must require $r_H^+ > D$:

  $$\sqrt{(e^2 - 2\mu^2)^2 - 4a^2\mu^2} > e^2 - 2m\mu. \tag{50}$$

  One can obtain that it is solved by requiring:

  $$e^2 < 2\mu(\mu - a), \quad m > \sqrt{a^2 - n^2}, \tag{51}$$

  which will guarantee the absence of a naked singularity.

Static limit

One can provide more detailed investigation in the static case:

- $2\mu^2 < e^2$. Then we find that

  $$(r_H^{\pm})^2 - r_s^2 = -\frac{1}{\mu}(\mu \mp m)(e^2 - 2\mu^2), \tag{52}$$

  so we immediately conclude that the solution is a naked singularity, since r_s is always larger than r_H^{+}.
- $2\mu^2 > e^2$. Then we have the relation

  $$(r_H^{\pm})^2 - r_s^2 = \frac{1}{\mu}(\mu \pm m)(2\mu^2 - e^2), \tag{53}$$

  which tells us that for $n \neq 0$ the singularity is located under both horizons and coincides with the position of the inner horizon in the case of the vanishing NUT parameter $n = 0$.

Wormhole

If we want our solution to be horizonless, than the following inequality must be satisfied

$$\mu^2 - e^2 + D^2 - a^2 < 0, \tag{54}$$

which can be solved as

$$2\mu(\mu - a) < e^2 < 2\mu(\mu + a) \tag{55}$$

For convenience we can write

$$e^2 = 2\mu(\mu + \gamma a), \qquad \gamma \in (-1, 1). \tag{56}$$

For a solution to be a wormhole, we must also impose the condition $\Sigma > 0$. It is seen that in the case where $|n| < a$, there is always a ring singularity $r = D$ and $y = -n/a$, so we must consider $|n| > a$. In this case, a sufficient condition for Σ to remain nonzero is

$$|a \cos\theta - n| > D. \tag{57}$$

Wormhole

Let us consider separately the following two cases:

- $n > 0$. Then, given the connection between D and e^2, we will have
  $$2\mu(n-a) > e^2. \tag{58}$$
  Using the no-horizon condition, we obtain
  $$(n-a) > (\mu + \gamma a), \tag{59}$$
  which cannot be satisfied for any γ in the range $(-1, 1)$.
- $n < 0$. In this case we obatin
  $$2\mu(n+a) < -e^2, \tag{60}$$
  while the no horizon condition now reduces to
  $$(|n| - a) > (\mu + \gamma a), \tag{61}$$
  which also cannot be satisfied.

So we cannot satisfy the no-singularity condition together with the no-horizon condition. Thus, there is no parameter region where the solution can be a traversable wormhole.

Gauged EMDA theory

Expressions for the polynomials $A_2,\ B_2$

Now the functions $A_2,\ B_2$ must be polynomials of the 4th degree of the corresponding variables:

$$\begin{aligned} A_2 &= a_0 - 2a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4, \\ B_2 &= b_0 + 2b_1 y + b_2 y^2 + b_3 y^3 + b_4 y^4. \end{aligned} \tag{62}$$

Further by solving the equations in the same order as in ungauged theory and extracting the conditions for the coefficients of the polynomials, we can obtain the general solution of the gauged EMDA theory.

Final form of the solution

General solution of the gauged theory

$$ds^2 = \frac{A_2}{\Sigma}\left(dt+(ay^2+2ny+\delta_0)d\varphi\right)^2-\frac{B_2}{\Sigma}\left(adt-(r^2-|\mathcal{D}|^2-a\delta_0+n^2)d\varphi\right)^2-\Sigma\left(\frac{dr^2}{A_2}+\frac{dy^2}{B_2}\right),$$

where

$$\begin{aligned}
A_2 &= (r^2-|\mathcal{D}|^2)\left(\lambda+\frac{r^2-|\mathcal{D}|^2+a^2+6n^2}{l^2}\right)-2mr+(q^2+p^2)+a^2\delta_1-\lambda n^2+\\
&\quad+\frac{3n^2}{l^2}(a^2-n^2),\\
B_2 &= (1-y^2)\left(\lambda-\frac{4an}{l^2}y-\frac{a^2}{l^2}y^2\right)+\delta_1-\lambda,\\
\Sigma &= r^2+(ay+n)^2-|\mathcal{D}|^2, \quad \mathcal{D}=-\frac{(q-ip)^2}{2\left(m+in\left[\lambda-\frac{a^2}{l^2}+\frac{4n^2}{l^2}\right]\right)}.
\end{aligned} \tag{63}$$

Topological solutions

Let us rewrite the metric in the form

$$ds^2 = \frac{A_2 - a^2 B_2}{\Sigma}(dt - \omega d\varphi)^2 - \frac{\Sigma}{A_2} dr^2 - \Sigma d\sigma^2, \tag{64}$$

where $d\sigma^2$ is the metric of the surface, spanned by coordinates y и φ, which in the case $a = 0$ and $r \to \infty$ takes the form

$$d\sigma^2 = \frac{dy^2}{B_2} + B_2 d\varphi^2. \tag{65}$$

Than calculating the gaussian curvature of this surface one can obtain $R_\sigma = 2\lambda$ and we obtain three different topologies

$$d\sigma^2 = \begin{cases} d\theta^2 + \sin^2\theta d\varphi^2, & \lambda = 1, \quad y = \cos\theta, \\ d\theta^2 + d\varphi^2, & \lambda = 0, \quad y = \theta, \\ d\theta^2 + \sinh^2\theta d\varphi^2, & \lambda = -1, \; y = \cosh\theta. \end{cases} \tag{66}$$

Extremal limit

Consider the equation $A_2 = 0$ on the positions of the horizons:

$$r^4 + a_2 r^2 + a_1 r + a_0 = 0, \tag{67}$$

where

$$\begin{aligned} a_2 &= l^2 + a^2 - 2|\mathcal{D}|^2 + 6n^2, \quad a_1 = -2ml^2, \\ a_0 &= l^2|\mathcal{Q}|^2 + |\mathcal{D}|^4 + (3n^2 + l^2)(a^2 - n^2) - (l^2 + a^2 + 6n^2)|\mathcal{D}|^2. \end{aligned} \tag{68}$$

The roots can be written as

$$r_\pm = \frac{1}{2}\left(\sqrt{y_1 - a_2} \pm \sqrt{2\sqrt{y_1^2 - 4a_0} - (y_1 + a_2)}\right), \tag{69}$$

where y_1 is a real root of the resolvent cubic equation

$$y^3 - a_2 y^2 - 4a_0 y - (a_1^2 - 4a_0 a_2) = 0. \tag{70}$$

Extremality condition on y_1 gives

$$y_1 = \frac{1}{3}(a_2 + 2\sqrt{(a_2)^2 + 12a_0}). \tag{71}$$

Extremal limit

It was shows that the value of the radial coordinate corresponding to extremal horizon is given by

$$r_H^{\mathrm{ext}} = \frac{l}{\sqrt{6}}\left(\eta - 1 - \frac{a^2}{l^2} + 2|\mathcal{D}|^2 - 6n^2\right)^{1/2}, \tag{72}$$

where

$$\eta = \sqrt{\left(1+\frac{a^2}{l^2}\right)^2 + \frac{12}{l^2}\left(a^2 + |\mathcal{Q}|^2 - \frac{4}{3}|\mathcal{D}|^2\right) - \frac{16}{l^4}\left[a^2\left(|\mathcal{D}|^2 - 3n^2\right) - |\mathcal{D}|^2\left(|\mathcal{D}|^2 - 6n^2\right)\right]}. \tag{73}$$

Extremal mass is given by

$$m_{\mathrm{ext}} = \frac{l}{3\sqrt{6}}\left(\eta + 2\left(1+\frac{a^2}{l^2}\right) - \frac{4|\mathcal{D}|^2}{l^2} + \frac{12n^2}{l^2}\right)\cdot\left(\eta - \left(1+\frac{a^2}{l^2}\right) + \frac{2|\mathcal{D}|^2}{l^2} - \frac{6n^2}{l^2}\right)^{1/2}, \tag{74}$$

which in particular cases coincide with r_H^{ext}, m_{ext} for previously known solutions Kerr-Sen-AdS and Kerr-Newman-NUT-AdS.

Conclusions

- A method for constructing the solutions with Killing tensor by integration of the equations of motion in gauged supergravity theories has been developed.
- For the first time, the choice of an ansatz for the axilaton field in the form of a fractional linear function is justified.
- For ungauged *EMDA*:
  - ☐ The most general solution with the Killing tensor was constructed.
  - ☐ The conditions for the absence of naked singularities are obtained.
  - ☐ It was shown that the solution cannot describe a traversable wormhole.
- For gauged *EMDA*:
  - ☐ For the first time, an analytical solution was obtained by integrating the equations of motion.
  - ☐ It was shown that the obtained solution is a generalization of previously guessed solutions of gauged EMDA theory.
  - ☐ It was shown that the solution allows for three different topologies.

Thank you!