Beam polarization effects on Higgs boson production at the International Linear Collider

Prog. Theor. Exp. Phys. 2022 073C01(20 pages), DOI: 10.1093/ptep/ptac090

based on the calculation of Dr. Nhi My Uyen's Ph.D thesis of Sokendai (https://www.soken.ac.jp/en/) in 2022.

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QFTHEP-270, Jul 4th ,2025

Outline

1. Introduction
2. e^+e^- International Linear Collider : the ILC
- Schematic layout of the ILC
- Beam polarization of the ILC
- Recoil mass analysis at the ILC
3. Two calculation Methods
   (1) ISR structure function method
   (2) Usage of the GRACE – Loop system for the O_α corrections
4. Results and Discussions
   - O_α corrections with ILC polarization for 9 processes: $e^+e^- \to f\bar{f}H$
   - Recoil mass distribution after O_α corrections with experimental cuts
5. Conclusion

1. Introduction

- Full electroweak O_α corrections of 9 $f\bar{f}H$ production with polarization were calculated with the GRACE-loop system.
- For these processes, Z-fusion and W-fusion diagrams are interesting.
- We made comparison O_α weak corrections and ISR ones.
- For the quark processes, QCD corrections are needed, but it is well known as $\delta_{QCD} = 1 + \frac{\alpha_s}{\pi}$, thus this effect is not included in the results.

2. e+e- International Linear Collider :the ILC

- One of physics goals of the ILC is exploring physics beyond the Standard Model via Higgs boson and top quark precise measurements.
- ILC has a plan to start with CM energy at 250 GeV to get the high statistics of Higgs production(so called "the Higgs factory")
- Special feature of the ILC is the longitudinal polarization beams are available.
- In e^+e^- collision, CM energy is well known, therfore the recoil mass analysis is available to make the mode lindependent measuring the Higgs properties.

Schematic layout of the ILC [1] from the TDR

- 20 km in length: e^+e^- linear collider
- (first stage) √s = 250 GeV, $\int L\,dt$ = 2000 fb^{-1} (10 years)
- Extendablity: 30km length

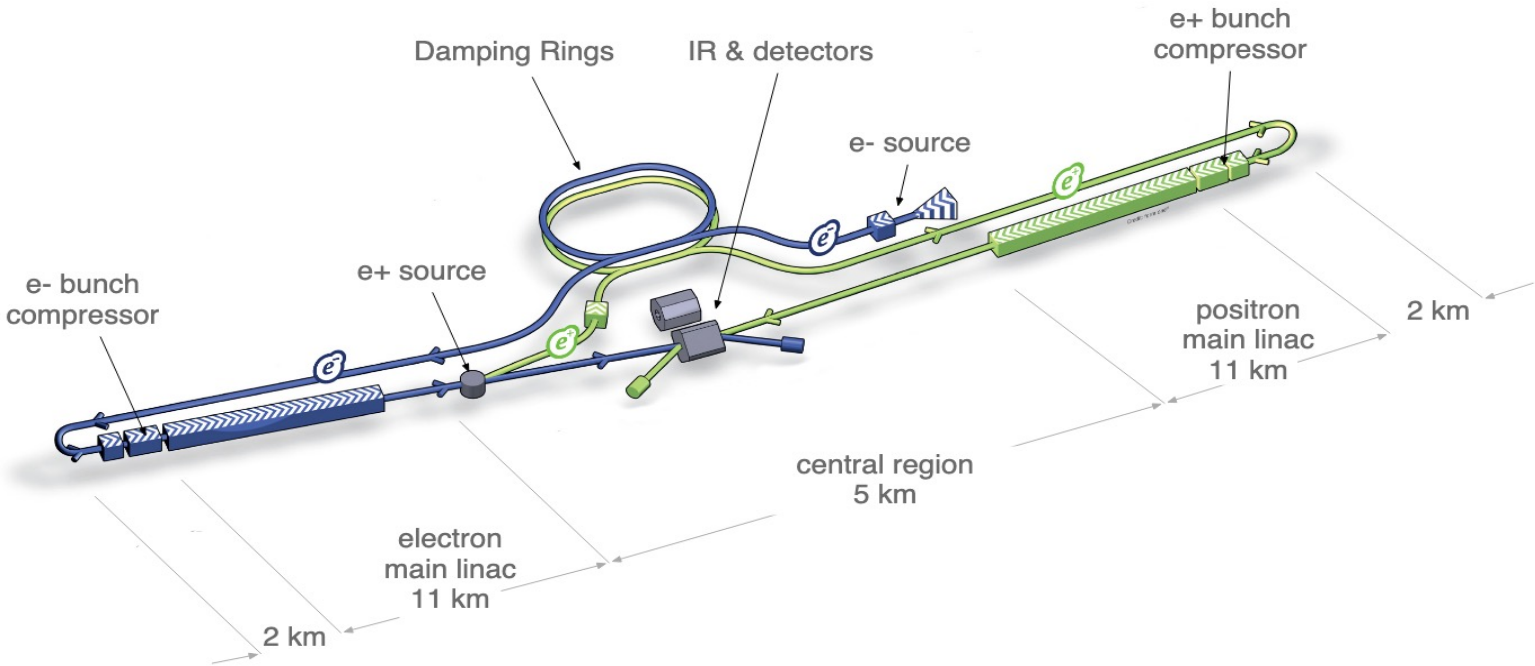


Figure 3.1. Schematic layout of the ILC, indicating all the major subsystems (not to scale).

ILC TDR:https://linearcollider.org/files/images/pdf/Executive%20Summary.pdf

Beam polarization of the ILC

- At the ILC, it is possible to collide e^+ and e^- beams with high loditudinal spin polarization.
- This makes available many new observables that cannot be measured at hadron colliders or circular e^+e^- colliders.

Definition of the beam polarization

For the electeon beam:

$$p_e = (N_{e_L} - N_{e_R})/(N_{e_L} + N_{e_R})$$

where N_{e_L} and N_{e_R} are numbers of left-handed and right-handed electrons

On the other hand, for the positron beam:

$$p_p = (N_{p_R} - N_{p_L})/(N_{p_R} + N_{p_L})$$

where N_{p_R} and N_{p_L} are numbers of right-handed and left-handed positrons

We use the following abbreviation:

- UP : un-polarization with (p_e= 0, p_p = 0) ,
- RR : right-right polarization with (p_e= -1, p_p =+1) ,
- LL : left-left polarization with (p_e= +1, p_p = −1) ,
- RL : right-left polarization with (p_e= -1, p_p = −1) ,
- LR : left-right polarization with (p_e= +1, p_p= +1),
- ILC : proposed ILC polarization with (p_e= 0.8, p_p= 0.3)

To include beam polarization effects, we multiply the projection operators $\frac{1\pm\gamma_5}{2}$ to produce amplitudes

⬇

Program size becomes much larger!

Recoil mass analysis

The recoil mass is defined as follows:

$$m_{rec}^2 \equiv s - 2\sqrt{s}\left(E_{\mu^+} + E_{\mu^-}\right) + m_{\mu^+\mu^-}^2,$$

where $\sqrt{s}$ is the centre-of-mass energy, E_{μ^+} and E_{μ^-} are the energies of μ^+ and μ^- from Z decay, *and* $m_{\mu^+\mu^-}^2$ is the invariant mass of μ^+ and μ^- from Z decay.

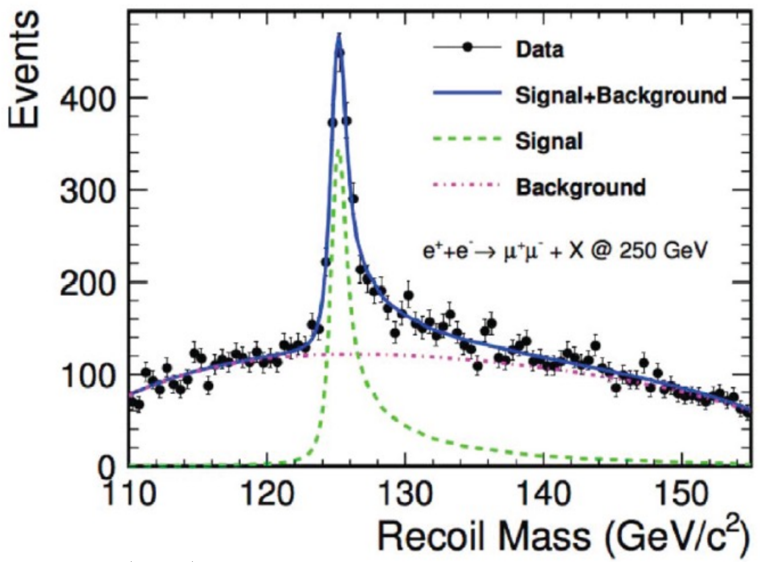
The cross sections of the Higgs production for the final-state decays H → XX can be expressed as;

$$\sigma(e^+e^- \to ZH) \times BR(H \to XX) \propto \frac{g^2_{HZZ} \times g^2_{HXX}}{\Gamma_H}$$

On the other hand, the cross sections of ZH can also be measured with the recoil mass distribution just using the kinematical information of observed μ^+ and μ^- from Z decay.

Even if the Higgs undergoes invisible or unknown decay modes, its existence and mass can still be measured.

Recoil mass distribution with the ISR effects



[2] arXiv:1710.07621v4:
Keisuke Fujii, Christophe Grojean, Michael E. Peskin, Tim Barklow et.al.,
LCC Physics Working Group,
"Physics Case for the 250 GeV Stage of the International Linear Collider".

σ (ZH) is largest at Higgs mass, and has the peak there, the resonance shape is enough narrow.
This distribution allows us to measure the Higgs mass precisely.

Note:Radiative tail is observed.

➡

Using this distribution, even if Higgs decays invisibly, we can determine the total width of the Higgs boson including its invisible decay modes.

3. Two calculation methods

(1) ISR structure function method

The effects of the photon radiation from the initial state can be factorized, when the total energy of the photons are small enough compared with the beam energy or a small angle (co-linear) emission . The calculations under such approximations referred as the “soft-colinear photon approximation, SPA.

Under the SPA, the total cross section with QED higher order corrections from the Initial state Radiation(ISR) can be calculated using the structure function as follows;

$$\sigma_{ISR} = \int_0^1 dx_1 \int_0^1 dx_2\, D(x_1, s) D(x_2, s) \sigma_{Tree}(s x_1 x_2),$$

Here D(x, s) is the structure function, gives a probability to emit a photon with energy fraction of x at the CM energy squared s. In this method, electron and positron can emit different energies, and thus finite boost of the CM system can be treated. The structure function can be obtained as

$$D(1-x, s)^2 = \Delta\beta[x^{\beta-1} - (1 - \frac{x}{2})] + \frac{\beta^2}{8}\left[-4(2-x)\log x - \frac{1+3(1-x)2}{x} + \log(1-x) - 2x\right],$$

where, $\beta = \frac{2\alpha}{\pi}(\log\frac{s}{m_e^2} - 1)$, $\Delta = 1 + \frac{\alpha}{\pi}(\frac{3}{2}\log\frac{s}{m_e^2} + \frac{\pi^2}{3} - 2)$.

(2)Usage of the GRACE – Loop system for the O_α corrections

- GRACE is a system that automatically calculates tree-level and one-loop cross sections with beam-polarization based on SM and MSSM.
- The GRACE-Loop system has been used to calculate,

  for SM processes:

$$e^+e^- \rightarrow t\bar{t}\,H \quad [2]$$

$$e^+e^- \rightarrow e^+e^-\gamma \quad [3]$$

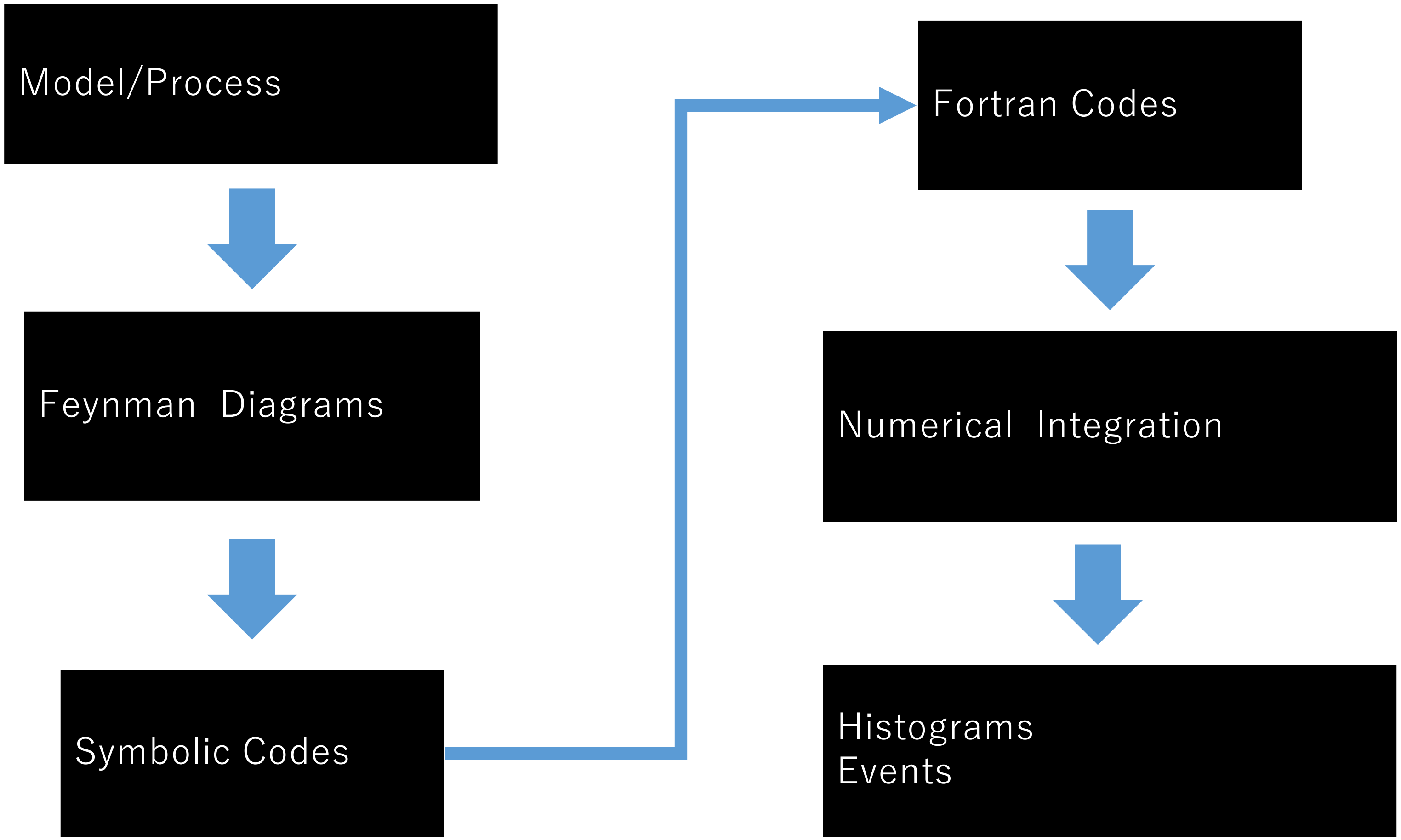
for MSSM processes:

$$e^+e^- \rightarrow ZH$$

$$e^+e^- \rightarrow \nu\bar{\nu}\,H \quad [4]$$

[2] P. H. Khiem, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Kurihara, Y. Shimizu, T. Ueda, J. A. M. Vermaseren and Y. Yasui, Eur. Phys. J. C 73, 2400 (2013),
[3] P. H. Khiem, Y. Kurihara, J. Fujimoto, T. Ishikawa, T. Kaneko, Phys. Lett. B740, 192 (2014) ,
[4] Y. Kouda, T. Kon, Y. Kurihara, T.Ishikawa, M.Jimbo, K. Kato, M. Kuroda,
One-loop effects of Minimal Supersymmetric Standard Model particles in $e^+e^- \rightarrow$ ZH and $e^+e^- \rightarrow \nu\bar{\nu}$ H at the International Linear Collider

How GRACE system works?



- For one-loop diagrams, the renormalization has been carried out with the on-shell condition of the Kyoto scheme[5].

- The non-linear gauge fixing Lagrangian is applied[6]:

$$\mathcal{L}_{GF} = -\frac{1}{\xi_W} \mid (\partial_\mu - ie\tilde{\alpha} A_\mu - igc_W \tilde{\beta} Z_\mu) W^{\mu +} + \xi_W \frac{g}{2}(v + \tilde{\delta} H + i\tilde{\kappa}\chi_3)\chi^+ \mid^2 \\ -\frac{1}{2\xi_Z}(\partial.Z + \xi_Z \frac{g}{2c_W}(v + \tilde{\epsilon} H)\chi_3)^2 - \frac{1}{2\xi_A}(\partial.A)^2.$$

- The results must be independent of non-linear gauge parameters $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\varepsilon})$.

- ξ_W, ξ_Z, ξ_A are set 1 in this study.

5 K. Aoki et al, Suppl. Prog. Theor. Phys. 73 (1982) 1.
6 G. Bélanger, F.Boudjema, J.Fujimoto, T. Ishikawa, T.Kaneko, K.Kato, Y.Shimizu, Phys, Rept. 430, 117 (2006)

Checking the amplitudes in the numerical manner

The $O\alpha$ corrected total cross sections are represented as follows:

$$\sigma_{O\alpha} = \sigma_{Tree}(\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\delta}, \tilde{\kappa}, \widetilde{\varepsilon}) + \sigma_{Loop}(C_{UV}, \lambda, \widetilde{\alpha}, \widetilde{\beta}, \widetilde{\delta}, \tilde{\kappa}, \tilde{\varepsilon})$$
$$+ \sigma_{Tree}(\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\delta}, \tilde{\kappa}, \widetilde{\varepsilon}) \cdot \delta_{Soft}(\lambda, E_\gamma < k_c)$$
$$+ \sigma_{Hard}(E_\gamma \geq k_c, \widetilde{\alpha}, \widetilde{\beta}, \widetilde{\delta}, \tilde{\kappa}, \tilde{\varepsilon}),$$

where Soft photon: $E_\gamma < k_c$

Hard photon: $E_\gamma \geq k_c$

Because every parameter above is introduced as temporal one,
So, when they are changed, the final total numerical results must be undchanged.

We have checked the following indepences :

- Ultra violet coefficient (C_{UV}) independence, $C_{UV} = 1/\varepsilon - \gamma_E + \log 4\pi,\ \varepsilon = 2 - n/2$
- Fictitious Photon mass (λ) independence,
- Non-linear Gauge parameters independence ($\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\delta}, \tilde{\kappa}, \tilde{\varepsilon}$),
- Hard photon cut-off (k_c) independence.

1. Ultra violet coefficient (C_{UV}) independence check at an arbitrary phase point with UP case

C_{UV}	Evaluation of the one-loop amplitude
0	0.127 132 019 079 521 763 060 824 923 741 166
100	0.127 132 019 079 521 763 060 824 923 741 166

Changing C_{UV} from 0 to 100 of $e^+e^- \to \mu^+\mu^- H$ process with $k_c = 10^{-1}$ GeV and $\lambda = 10^{-17}$ GeV at $\sqrt{s}$ = 250 GeV with $\Gamma_Z = 0$ to avoid the gage violation.

33 digit full agreement with quadruple precision

- We have calculated them with quadruple precision to avoid the rounding-off errors.

2. Fictitious photon mass (λ) independence at an arbitrary phase point with UP case

λ [GeV]	Evaluation of the one-loop amplitude
10^{-17}	0.127 132 019 079 521 763 060 824 923 741 166
10^{-19}	0.127 132 019 079 522 845 528 273 933 268 792

Changing λ= 10^{-17} GeV to λ= 10^{-19} GeV of $e^+e^- \rightarrow \mu^+\mu^-H$ process with k_c= 10^{-1} GeV at √s = 250 GeV with Γz=0.

14 digit agreement

3. Non-Linear gauge parameters ($\tilde{\alpha}, \tilde{b}, \tilde{\delta}, \tilde{\kappa}, \tilde{\varepsilon}$) independence at an arbitrary phase point with UP case

$\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\varepsilon}$	Evaluation of the one-loop amplitude
0,0,0,0,0	0.127 132 019 079 521 763 060 824 923 741 166
10,20,30,40,50	0.127 132 019 079 522 096 127 732 311 288 128

Non-linear gauge parameters independence by changing $\tilde{\alpha}, \tilde{b}, \tilde{\delta}, \tilde{\kappa}, \tilde{\varepsilon}$ from (0,0,0,0,0) to (10,20,30,40,50) of $e^+e^- \to \mu^+\mu^- \mathrm{H}$ with $k_c = 10^{-1}$ GeV and λ= 10^{-17} GeV at √s = 250 GeV with Γz=0 to avoid the gage violation.

15 digit agreement

Hard photon cut-off (kc) independence after the phase space integration

To observe k_c independence, we need to perform the phase space integration.

Here, we introduce the NOLLS model for the loop diagrams in the SM to make the phase space integration,

- NOLLS apprx. : NO Light-fermion Light-fermion Scalar coupling in the SM except for bottom- and top-quark. The size of the produced amplitudes are moderate.

- FULL model: the whole couplings are kept in the SM. The size of the produced amplitudes are too huge to make integration.

- With 2.4 million sampling points, it takes 4.5 hours to make the phase space intagration for $\mu^+\mu^-$H and other light fermon processes with NOLLS apprx, but almost 52 hours* for $b\bar{b}$H with NOLLS apprx. because the Yukawa couplings of the top- and botrom-quark are kept.

  The typical integration errors are around 0.1% for the loop diagrams.

→ We need to confirm the NOLLS apprx. has enough accuracy compared to the FULL model, at an arbitrary phase space point.

* 16 Xeon 3.20 GHz CPU with the size of memory 128GB

Numbers of Feynman diagrams

	the NOLLS approximation			the FULL model		
Graph information	1-loop	Tree	5-point	1-loop	Tree	5-point
$\mu^+\mu^-H$	208	1	10	2235	21	170
e^+e^-H	416	2	20	4470	42	340
$\tau^+\tau^-H$	208	1	28	2235	21	188
$\nu_\mu\bar{\nu}_\mu H$	122	1	6	604	4	36
$\nu_e\bar{\nu}_e H$	219	2	15	1350	12	113
$u\bar{u}H$	209	1	10	2327	21	174
$d\bar{d}H$	209	1	10	2327	21	174
$c\bar{c}H$	209	1	10	2327	21	174
$b\bar{b}H$	651	6	29	2327	21	193

The number of tree and one-loop diagrams with the FULL model are more than 10 times of the NOLLS approx.

Therefore, the phase space integration of the amplitudes with the FULL model is unrealistic.

Comparison with the previous work for $e^+e^- \to \nu\bar{\nu}H$ at $\sqrt{s}$ = 500 GeV

M_H (GeV)	M_W (GeV)	σ_{Tree} (pb)	$\sigma_{\mathcal{O}_\alpha}$ (pb)	$\delta_{Total}\%$	
150	80.3767	6.1072(9)E-02	6.075(6)E-02	-0.51	Current
		6.1076(5)E-02	6.080(2)E-02	-0.44	Ref.[23]
200	80.3571	3.7302(5)E-02	3.703(4)E-02	-0.71	Current
		3.7293(3)E-02	3.709(2)E-02	-0.56	Ref.[23]
250	80.3411	2.110(2)E-02	2.059(2)E-02	-2.42	Current
		2.1134(1)E-02	2.060(1)E-02	-2.53	Ref.[23]
300	80.3275	1.0744(7)E-02	1.0258(7)E-02	-4.51	Current
		1.07552(7)E-02	1.0282(4)E-02	-4.40	Ref.[23]
350	80.3158	4.6077(4)E-03	4.172(2)E-03	-9.46	Current
		4.6077(2)E-03	4.181(1)E-03	-9.27	Ref.[23]

Comparison between the unpolarized cross sections of $e^+e^- \to \nu\bar{\nu}$ H between the current results and those of German group at center of mass energy $\sqrt{s}$ = 500 GeV.

The correction factors (δ_{Total}) agreed in 0.3%.

[23] A. Denner, S. Dittmaier, M. Roth, M. Weber, Electroweak radiative corrections to e+ e- —> nu anti-nu H,
Nucl. Phys. B 660 (2003) 289–321. arXiv:hep-ph/0302198.

Parameters summary of the calculation

m_e	$0.510\,999\,06 \times 10^{-3}$ GeV[15]	m_u	0.0063 GeV [18]	m_Z	91.1876 GeV[15]
m_μ	$105.658\,3389 \times 10^{-3}$ GeV [15]	m_d	0.0063 GeV [18]	m_W	80.366 GeV [18]
m_τ	1.771 GeV[15]	m_c	1.5 GeV [18]	m_H	125.1 GeV[15]
$m_{\nu e}$	0 GeV	m_s	0.0094 GeV [18]	α	1/137.035 999 084[15]
$m_{\nu\mu}$	0 GeV	m_b	4.7 GeV [18]	$\sin^2\theta_W$	$1-\frac{m_W^2}{m_Z^2}$
$m_{\nu\tau}$	0 GeV	m_t	172.9 GeV [15]	Γ_Z	2.49 GeV [15]

used m_e, m_μ, m_τ, m_t, m_Z, m_W, m_H, α, Γ_Z in [15]
and m_u, m_d, m_c, m_s, m_b in [18]

[15] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update.
[18] Recommended values to reproduce Δr by Z. Hioki, Progr. Theor. Phys. 68 (1982), 2134.

NOLLS approximation check at an arbitrary phase point of $\mu^+\mu^- H$

FULL model vs. NOLLS apprx.	Evaluation of the one-loop amplitude
FULL model	0.127 132 019 079 521 763 060 824 923 741 166
NOLLS apprx.	0.127 180 115 426 583 029 147 877 823 561 430

Evaluation of the 1 loop amplitude at a phase space point for $e^+e^- \to \mu^+\mu^- H$ with $k_c = 10^{-1}$ GeV at √s = 250 GeV with Γz=0 to avoid the gage violation.

4 digit agreement

- This agreement is enough because the phase space integration by MC has only 3 digit accuracy.
- We need to add the hard photon contributions, of which contribution is performed with the FULL model, not approximated.

4. Hard photon cut-off (kc) independence with UP

$$\delta_{Total} \equiv \left(\frac{\sigma_{O_\alpha}}{\sigma_{Tree}} - 1\right) \times 100$$

$$\Delta(\%) \equiv \delta_{Total_{k_C=10^{-3}\text{GeV}}} - \delta_{Total_{k_C=10^{-5}\text{GeV}}}$$

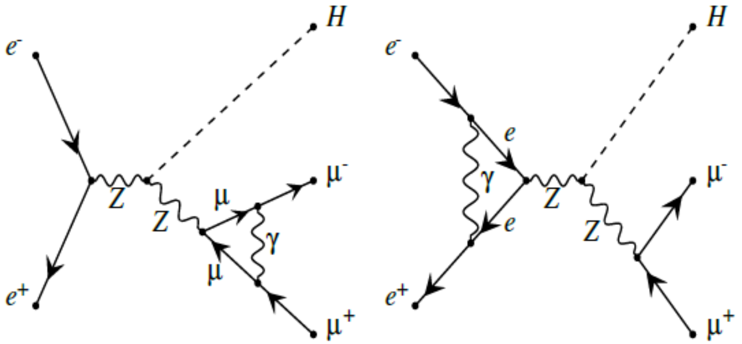
The worst accurate cases are both of $\mu^+\mu^-\mathrm{H}$ and $e^+e^-\mathrm{H}$ of 0.09%

➡ It guarantees the accuracy of the O_α calculations about 0.1% for all processes.

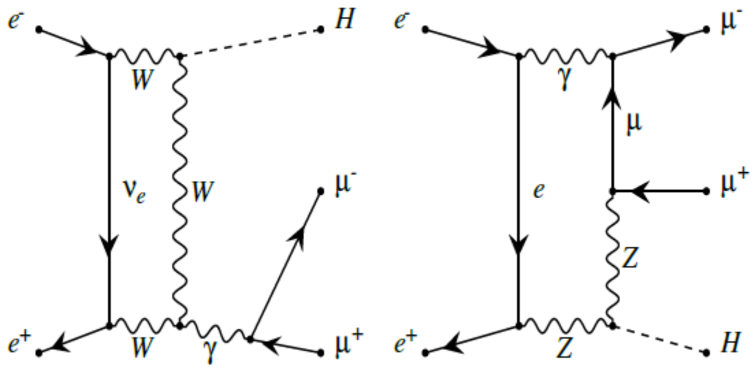
Processes	Δ(%)
$\mu^+\mu^-H$	0.09
e^+e^-H	- 0.09
$\tau^+\tau^-H$	- 0.03
$\nu_\mu\bar{\nu}_\mu H$	0.05
$\nu_e\bar{\nu}_e H$	0.00
$u\bar{u}H$	- 0.05
$d\bar{d}H$	0.06
$c\bar{c}H$	0.07
$b\bar{b}H$	- 0.05

The difference in percentage of the total ratio δ_{Total} with $k_c = 10^{-3}$ GeV and $k_c = 10^{-5}$ GeV at $\sqrt{s}$ = 250 GeV with UP for all $e^+e^- \to f\bar{f}H$ processes with Γz=2.49 GeV[15].

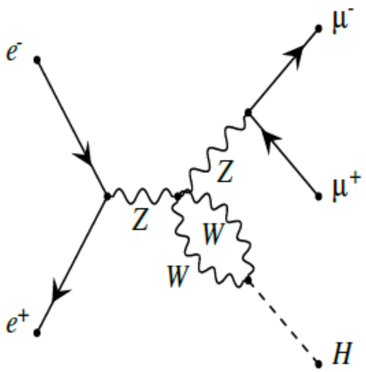
Typical Feynman diagrams to estimate



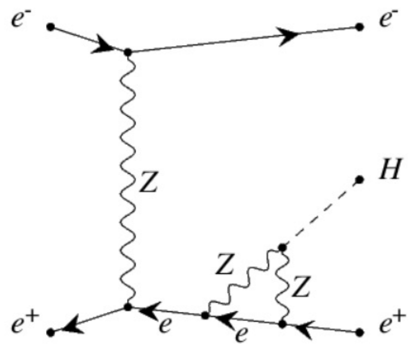
Typical final and initial vertex correction Feynman diagrams of $e^+e^- \to \mu^+\mu^- H$



Typical 4 point and 5 point function Feynman diagrams of $e^+e^- \to \mu^+\mu^- H$



Typical Fish Feynman diagram of $e^+e^- \to \mu^+\mu^- H$



Typical t-channel diagram of $e^+e^- \to e^+e^-$H

4. Results and Discussions with the beampolarization of $\mu^+\mu^- H$

- At √s = 250 GeV
- $\sigma_{O_\alpha} = \sigma_{Tree} + \sigma_{Loop} + \sigma_{Soft} + \sigma_{Hard}$
- $\delta_{Total} = (\frac{\sigma_{O_\alpha}}{\sigma_{Tree}} - 1) \times 100$

	σ_{Tree} (pb)	σ_{O_α} (pb)	δ_{Total}%	σ_{ISR} (pb)	δ_{ISR}%
UP	$7.021(2) \times 10^{-3}$	$6.724(1) \times 10^{-3}$	−4.2	$6.312(4) \times 10^{-3}$	−9.9
RR	$2.16(1) \times 10^{-11}$	$8.4(2) \times 10^{-7}$	3.91×10^{6}	$9.62(3) \times 10^{-13}$	−95.6
LL	$2.16(1) \times 10^{-11}$	$8.45(2) \times 10^{-7}$	3.91×10^{6}	$7(2) \times 10^{-11}$	224.1
RL	$1.108(2) \times 10^{-2}$	$1.194(1) \times 10^{-2}$	7.7	$1.534(3) \times 10^{-3}$	−19.3
LR	$1.709(2) \times 10^{-2}$	$1.497(1) \times 10^{-2}$	−12.1	$9.944(2) \times 10^{-2}$	−9.9
ILC	$1.035(2) \times 10^{-2}$	$9.182(1) \times 10^{-3}$	−11.3	$9.321(4) \times 10^{-3}$	−9.9

$$\left|\frac{m_e^2}{s}\right| \propto \left(\frac{10^{-3}}{10^2}\right)^2 \sim 10^{-10} \rightarrow \frac{LL+RR}{LR+RL} \sim 10^{-11}$$

$\sigma_{TreeILC} = 1.035(2)\times10^{-2}$(pb) $> \sigma_{TreeUP} = 7.021(2)\times10^{-3}$(pb)

$\delta_{ISR} \sim 10\%$ for UP, LR, RL and ILC ➡ ISR is independent of the polarization as expected

The pure weak corrections can be estimated from δ_{Total}-δ_{ISR}

$\delta_{Total\ RL} = +7.7\%$, $\delta_{ISRRL} = -19.3\ \%$
$\delta_{Total\ LR} = -12\%$, $\delta_{ISRLR} = -9.9\ \%$
$\delta_{Total\ ILC} = -11.3\%$, $\delta_{ISRILC} = -9.9\%$

➡ for RL pure weak corrections is about 27 %.
for LR, pure weak corrections is about 2 %.
for ILC, pure weak corrections is about 1 %.

Summary of the lepton processes

Process	σ_{Tree} (pb)	$\sigma_{\mathcal{O}_\alpha}$ (pb)	$\delta_{\text{Total}}\%$	σ_{ISR} (pb)	$\delta_{\text{ISR}}\%$
$\mu^+\mu^- H$	$1.035(2) \times 10^{-2}$	$9.182(1) \times 10^{-3}$	-11.3	$9.321(4) \times 10^{-3}$	-9.9
$e^+e^- H$	$1.119(1) \times 10^{-2}$	$9.841(2) \times 10^{-3}$	-12.0	$9.159(4) \times 10^{-3}$	-18.1
$\tau^+\tau^- H$	$1.034(1) \times 10^{-2}$	$9.145(2) \times 10^{-3}$	-11.6	$9.261(4) \times 10^{-3}$	-10.5
$\nu_\mu \bar{\nu}_\mu H$	$2.045(1) \times 10^{-2}$	$1.782(1) \times 10^{-2}$	-12.9	$1.837(1) \times 10^{-2}$	-10.2
$\nu_e \bar{\nu}_e H$	$3.632(1) \times 10^{-2}$	$3.357(3) \times 10^{-2}$	-7.6	$3.044(2) \times 10^{-2}$	-16.2

Lepton processes with the ILC proposed beam polarization with kc = 10^{-3}GeV at √s = 250 GeV and without experimental cuts.

- δ_{ISR} for s-channel processes are around -10%, for t-channel processes around -18%.
- The size of the pure weak corrections , δ_{Total}-δ_{ISR} reaches several % level.

* skipped $e^+e^- \to \nu_\tau\overline{\nu_\tau}\text{H}$, because it is quite similar to $e^+e^- \to \nu_\mu\overline{\nu_\mu}\text{H}$

Summary of the quark processes

$\delta_{QCD} = 1 + \frac{\alpha_s}{\pi}$ not included

Process	σ_{Tree} (pb)	$\sigma_{\mathcal{O}_\alpha}$ (pb)	$\delta_{\text{Total}}\%$	σ_{ISR} (pb)	$\delta_{\text{ISR}}\%$
$u\bar{u}H$	$3.570(1) \times 10^{-2}$	$2.934(1) \times 10^{-2}$	-17.8	$3.206(2) \times 10^{-2}$	-10.0
$d\bar{d}H$	$4.581(1) \times 10^{-2}$	$4.070(1) \times 10^{-2}$	-11.5	$4.115(2) \times 10^{-2}$	-10.2
$c\bar{c}H$	$3.566(1) \times 10^{-2}$	$2.926(1) \times 10^{-2}$	-18.0	$3.215(2) \times 10^{-2}$	-10.2
$b\bar{b}H$	$4.533(1) \times 10^{-2}$	$3.969(4) \times 10^{-2}$	-12.5	$4.068(2) \times 10^{-2}$	-10.3

* skipped $e^+e^- \to s\bar{s}$H because it has the same characteristic of $e^+e^- \to d\bar{d}$H.

- δ_{ISR} is about -10% for all cases.
- 1 % defference between ddH and bbH comes from the top- and bottom-Yukawa couplings kept in the NOLLS model.

Recoil mass distribution after O_α corrections with experimental cuts and beam polarization

Three experimental cuts are applied to see the recoil mass distribution

1. The angular cut $\theta_{\mu^+}, \theta_{\mu^-}$ is

$$10^o < \theta_{\mu^+}, \theta_{\mu^-} < 170^o,$$

where θ_{μ^+} and θ_{μ^-} are the scattering angles of anti-muon and muon.

2. The energy cut E_{μ^+}, E_{μ^-} is

$$E_{\mu^+}, E_{\mu^-} > 10 \text{ GeV},$$

where E_{μ^+} and E_{μ^-} are the energies of anti-muon and muon.

3. The invariant mass cut $(M_{\mu^+\mu^-})$ is

$$m_Z - 3\Gamma_Z < M_{\mu^+\mu^-} < m_Z + 3\Gamma_Z.$$

where $M_{\mu^+\mu^-}$ is the invariant mass of μ^+ and μ^- ; Γ_Z = 2.49 GeV [17] is the width of Z-boson.

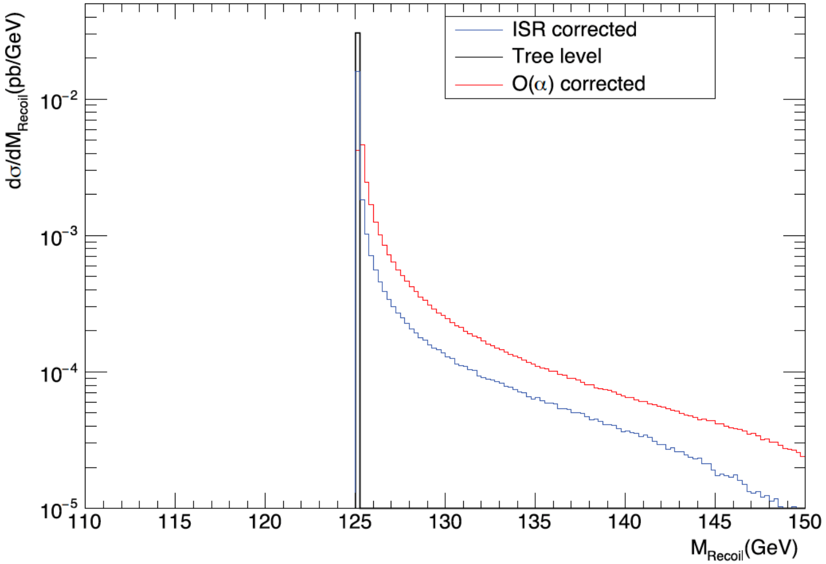
Recoil mass distribution after O_α corrections with three experimental cuts with the beam polarization

- $\sqrt{s}$ = 250 GeV
- proposed ILC beam polarization and
- 3 experimental cuts are applied

Table 18. Summary table of cross sections and total ratios of $e^+e^- \to \mu^+\mu^- H$ with experimental cuts and with the proposed ILC polarization with $p_e = -0.8$, $p_p = 0.3$.

$\sigma_{\rm Tree}$ (pb)	$\sigma_{\mathcal{O}_\alpha}$ (pb)	$\sigma_{\rm ISR}$ (pb)	$\delta_{\rm Total}\%$	$\delta_{\rm ISR}\%$
$9.143(1) \times 10^{-3}$	$7.910(1) \times 10^{-3}$	$8.236(4) \times 10^{-3}$	-13.2	-9.9

pure weak correction reaches 3% !



5. Conclusion

- We have calculated O_{α} corrections to the 9 Higgs production processes with beam polarization conditions at the ILC.
- We have confirmed 0.1 % precision through all numerical checks.
- We found 1 % difference of the pure weak corrections between ddH and bbH comes from the top- and bottom-Yukawa couplings.
- For the recoil mass distribution, the pure weak corrections reach 3%.
- Above two results show the O_{α} corrections should be taken into account seriously for the ILC high precise measurements.

Thank you very much for your attention!