Thermodynamical properties of Bardeen-type black hole in AdS spacetime.

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Introduction

Possible ways of singularity problem solving:

1. String theory
2. GR modifications
3. Exotic sources of gravitation (scalar fields, «exotic matter», nonlinear electrodynamics etc.)

Nonlinear electrodynamics

Born-Infeld model(1934)

$$\mathcal{L}^{HE} = T - T\sqrt{-det\left(\eta_{ik} + T^{-1/2}F_{ik}\right)}$$

1. Coulomb singularity is absent
2. T - tension of D-brane.

Euler-Heisenberg model (1936)

$$\mathcal{L}^{BI} = \frac{1}{4}F_{ik}F^{ik} + \frac{\alpha^2}{90m^4}\left[(F_{ik}F^{ik})^2 + 7(F_{ik}F^{*ik})^2\right]$$

1. One-loop QCD corrections
2. Proofed in ATLAS detector*

*Nature Physics **13**, 852-858 (2017)

NED in regular black holes

$$\mathcal{L} = \mathcal{L}(\mathcal{F}, \mathcal{G}^2) \quad \mathcal{F} = \frac{1}{4} F_{ik} F^{ik}; \quad \mathcal{G} = \frac{1}{4} F_{ik} F^{*ik}; \quad F^{*ik} = \frac{1}{2} E^{ikmn} F_{mn}.$$

Regular NED black hole must have nonzero magnetic charge*

*K. A. Bronnikov, Regular magnetic black holes and monopoles from nonlinear electrodynamics, Phys. Rev. D **63**, 044005 (2001);

The model

Action

$$S = \int d^4x\sqrt{-g}\left\{\frac{R-2\Lambda}{2\kappa}+\mathcal{L}^{NE}\right\}$$

Lagrangian of nonlinear electrodynamics

$$\mathcal{L}^{NE} = \frac{(n+1)Mg^n\left(\frac{2\mathcal{F}}{g^2}\right)^{(n+3)/4}}{\left(1+(2\mathcal{F}g^2)^{n/4}\right)^{(2n+1/n)}}$$

Lagrangian's quantities

$$F_{\theta\varphi} = \frac{g}{r^2} \quad \mathcal{F} = \frac{1}{4}F_{ik}F^{ik} = \frac{g^2}{2r^4}.$$

New type of Bardeen-AdS metric

Metric

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

Classical type (Bardeen 1968)

$$f(r) = 1 - \frac{2Mr^2}{(r^2+g^2)^{3/2}} - \frac{\Lambda r^2}{3}$$

New type

$$\tilde{f}(r) = 1 - \frac{2Mr^n}{(r^n+g^n)^{\frac{n+1}{n}}} - \frac{\Lambda r^2}{3}$$

Approximation

$$f(r) \approx 1 - \frac{2M}{r} + \frac{3Mg^2}{r^3} - \frac{\Lambda r^2}{3} + O\left(\frac{1}{r^5}\right) \quad \tilde{f}(r) \approx 1 - \frac{2M}{r} + \frac{2(n+1)Mg^n}{nr^{n+1}} - \frac{\Lambda r^2}{3} + O\left(\frac{1}{r^{2n+1}}\right)$$

Curvature invariants

Ricci scalar

$$R = -\frac{2Mr^{n-2}}{(r^n+g^n)^{\frac{n+1}{n}}}\left[\frac{(n+4)r^n-4ng^n}{r^n+g^n}-\frac{(r^n-ng^n)((n-1)g^n-(n+2)r^n)}{(r^n+g^n)^2}-2\right]-4\Lambda$$

Ricci square

$$R_{ik}R^{ik} = \frac{4M^2r^{2n-4}}{(r^n+g^n)^{\frac{4n+2}{n}}}\left(a(r)^2+4a(r)b(r)+8b(r)^2\right)-\frac{4M\Lambda r^{n-2}}{3(r^n+g^n)^{\frac{2n+1}{n}}}\left(3a(r)+2b(r)\right)+$$

$$+\frac{8M^2r^{2n-4}}{(r^n+g^n)^{\frac{2n+2}{n}}}\left((2n+1)g^n-r^n\right)+\frac{24M\Lambda r^{n-2}}{3(r^n+g^n)^{\frac{n+1}{n}}}+4\Lambda^2$$

Curvative invariants

Kretchmann scalar

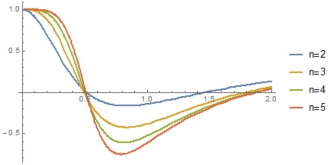
$$R_{iklm}R^{iklm} = \frac{4M^2 r^{2n-4}}{(r^n+g^n)^{\frac{4n+2}{n}}}\left(a(r)^2+4b(r)^2\right) - \frac{8M\Lambda r^{n-2}}{3(r^n+g^n)^{\frac{2n+1}{n}}}\left(a(r)+4b(r)\right) + \frac{16M^2 r^{2n-4}}{(r^n+g^n)^{\frac{2n+2}{n}}} +$$

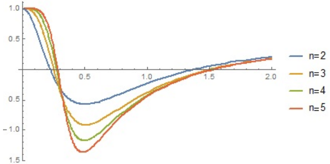
$$+\frac{16M\Lambda r^{n-2}}{3(r^n+g^n)^{\frac{n+1}{n}}} + \frac{24}{9}\Lambda^2.$$

$$a(r) = \left[nr^n - \frac{(r^n-ng^n)((n-1)g^n-(n+2)r^n)}{r^n+g^n}\right]; \quad b(r) = r^n - ng^n.$$

All invariants are regular for $n \geq 2$

Comparison between metric functions $\tilde{f}(r)$ for the different n





$g = 0.57, \Lambda = 0.086, M = 0.85$

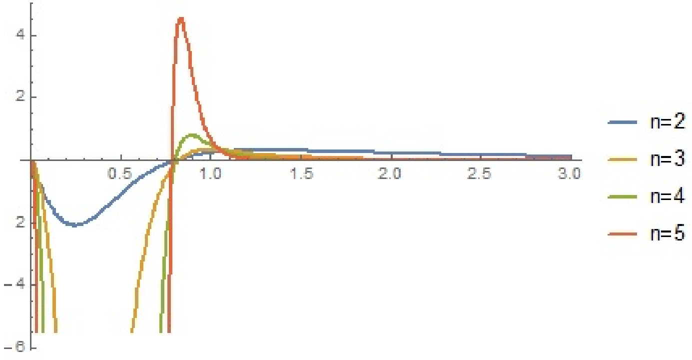
$g = 0.35, \Lambda = 0.09, M = 0.71$

Hawking temperature

$$T_H(r_+) = \frac{1}{4\pi}\tilde{f}'(r_+)$$

r_+ -event horizon radius $f(r_+) = 0$

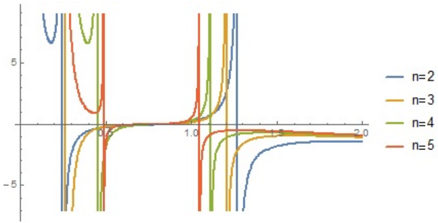
$$T_H(r_+) = \frac{M}{2\pi}\frac{1}{\left(r_+^n + g^n\right)^{(2n+1)/n}}\left(r_+^n - ng^n\right) - \frac{\Lambda r_+}{6\pi}$$



Specific heat capacity

$$C(r_+) = \frac{\partial M}{\partial r_+}\left(\frac{\partial T}{\partial r_+}\right)^{-1}$$

$$C(r_+) = \frac{\frac{(r_+^n+g^n)^{1/n}}{2r_+^n}(r_+^n - ng^n)(1-\frac{\Lambda r_+^2}{3}) - \frac{\Lambda(r_+^n+g^n)^{n+1/n}}{3r_+^{n+1}}}{\frac{(M/2\pi)}{(r_+^n+g^n)^{(3n+1)/n}}\left(-2r_+^{3n-2}+(n^2+4n-1)g^n r_+^(2n-2) - n(n-1)g^{2n}r_+^{n-2}\right) - \Lambda/6\pi}$$



Hawking-Page phase transition

Conclusion

1. In this work a new model of nonlinear electrodynamics is constructed. This model provides a charged black hole regularity.
2. Basic thermodynamical properties of Bardeen-type black holes is calculated.
3. Hawking-Page phase transition phenomena is found for the different n.

Thanks for your attention!