The scalar-torsion gravity corrections in the models of cosmological inflation

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Inflationary cosmological models

- Inflation $\Rightarrow$ cosmological perturbations $\Rightarrow$ large-scale structure formation (by scalar perturbations) and relic gravitational waves (tensor perturbations)
- Scalar and tensor perturbations $\Rightarrow$ CMB anisotropy and polarization $\Rightarrow$ observational constraints
- For the case of GR these constraints restrict the possible types of potentials of inflaton $V(\phi)$
- The additional constraints on the inflationary parameters besides the slow-roll approach
- For the case of modified gravity theories the models with arbitrary potentials can satisfy the observational constraints
- Restriction on modified gravity theories $|c_g - 1| \leq 5 \times 10^{-16}$ (gravitational waves from neutron star merging)

Friedmann-Robertson-Walker space-time

The standard cosmological theory of the big bang is based on the geometry of the Universe, which is homogeneous and isotropic at large distances, which is determined by the Friedmann-Robertson-Walker metric in the system of units $8\pi G = m_{pl}^{-2} = c = 1$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right), \quad \mu, \nu = 0, 1, 2, 3, \tag{1}$$

where $a = a(t)$ is the scale factor, t is the cosmic time.
The case of $k = 0$ and $a(t) = \exp(\lambda t)$, where λ is some constant (exponential expansion) corresponds to the de Sitter metric

$$(ds^2)_{dS} = -dt^2 + \exp(2\lambda t)\left(dx^2 + dy^2 + dz^2\right); \tag{2}$$

Thus, a homogeneous isotropic four-dimensional Friedmann-Roberstson-Walker space is considered as a space-time model, which corresponds to the most common approach to describing the geometry of the Universe.

The action for the cosmological models based on the scalar-torsion gravity

The action for the cosmological models

$$S = \int d^4xe \left[\frac{1}{2} F(\phi)T + P(\phi, X) - \Box\phi G(\phi, X) \right]. \tag{3}$$

where $F(\phi)$ is the function of the scalar field ϕ; $P(\phi, X)$, $G(\phi, X)$ are functions of the scalar field and its kinetic energy $X = -\frac{1}{2}\phi_{,\mu}\phi^{,\mu}$ and $\phi_{,\mu} = \partial_\mu\phi = \frac{\partial\phi}{\partial x^\mu}$. T be the torsion scalar, e^A_μ are the tetrad components in a coordinate basis, $e = det(e^A_\mu) = \sqrt{-g}$ and $G(\phi, X)$ is the Galileon-type field self-interaction. Also, the function P can be considered as the function of the potential of a scalar field and its kinetic energy $P = P\left(V(\phi), X\right)$.
The torsion scalar is defined as follows

$$T = S_\gamma^{\ \alpha\beta} \, T^\gamma_{\ \alpha\beta} \,, \tag{4}$$

$$T^\gamma_{\ \alpha\beta} = e_A^{\ \gamma} \left[\partial_\alpha e^A_{\ \beta} - \partial_{\ \alpha} e^A_{\ \beta} + \omega^A_{\ B\alpha} \, e^B_{\ \beta} - \omega^A_{\ B\beta} \, e^B_{\ \alpha} \right] \,, \tag{5}$$

$$S_\gamma^{\ \alpha\beta} = \frac{1}{2} \left(K^{\alpha\beta}_{\ \ \gamma} + \delta^\alpha_{\ \gamma} \, T^{\mu\beta}_{\ \ \mu} - \delta^\beta_{\ \gamma} \, T^{\mu\alpha}_{\ \ \mu} \right) \,, \tag{6}$$

$$K^{\alpha\beta}_{\ \ \gamma} = -\frac{1}{2} \left(T^{\alpha\beta}_{\ \ \gamma} - T^{\alpha\beta}_{\ \ \gamma} - T_\gamma^{\ \alpha\beta} \right) \,, \tag{7}$$

where $T^\gamma_{\ \alpha\beta}$ is the torsion tensor, $S_\gamma^{\ \alpha\beta}$ is the super-potential, $K^{\alpha\beta}_{\ \ \gamma}$ is the contorsion tensor, and spin connection $\omega^A_{\ B\alpha} = 0$ for a special class of inertial frames.

The action for the cosmological models based on the scalar-torsion gravity

We consider a special case of negligible self-interaction of a scalar field as,

$$\frac{1}{2}F(\phi)T \gg \Box\phi G(\phi, X). \qquad (8)$$

Initial action is reduced to the following form

$$S \approx \int d^4xe\left[\frac{1}{2}F(\phi)T + P(\phi, X)\right]. \qquad (9)$$

The tetrad field

$$e^A_\mu = diag\,(1, a, a, a)\,, \qquad (10)$$

corresponds to isotropic and homogeneous Friedmann-Lemaitre-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + a^2\delta_{ij}dx^i dx^j, \qquad (11)$$

where $a = a(t)$ is the scale factor that dependents on cosmic time t only.
For FLRW metric expression for torsion scalar is

$$T = 6H^2. \qquad (12)$$

Background dynamic equations

We consider

$$P(\phi, X) = -\omega X + V(\phi) = -\frac{\omega}{2}\dot{\phi}^2 + V(\phi), \tag{13}$$

where ω is a constant.
Background dynamic equations are

$$V(\phi) = 3H^2F + \dot{H}F + \dot{F}H, \tag{14}$$

$$\omega\dot{\phi}^2 = -2H\dot{F} - 2F\dot{H}, \tag{15}$$

$$\omega\ddot{\phi} + 3\omega H\dot{\phi} + V_{,\phi} + 3H^2F_{,\phi} = 0. \tag{16}$$

For the case of minimal coupling ($F = 1$), these equations are reduced to ones for the case of inflationary models based on TEGR.

Power-law parametrization of coupling function

$$F(\phi(t)) = \left(\frac{H(t)}{\lambda}\right)^n, \quad (17)$$

where λ is a positive constant and n is the parameter that defines the influence of the non-minimal coupling between the scalar field and torsion.
Background dynamic equations are

$$V(\phi) = \frac{H^n}{\lambda^n}\left(3H^2 + \dot{H}(1+n)\right) = F(\phi)\left(3H^2 + \dot{H}(1+n)\right), \quad (18)$$

$$\omega\dot{\phi}^2 = -2\left(\frac{H}{\lambda}\right)^n \dot{H}(n+1) = -\frac{2}{\lambda^n}\frac{d}{dt}\left(H^{n+1}\right), \quad (19)$$

where

$$\omega = \lambda^{-n} = constant. \quad (20)$$

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Relation with TEGR

We introduce the connection between H and h as

$$H = h^{\frac{1}{1+n}}, \tag{21}$$

$$\dot{H} = \left(\frac{1}{1+n}\right)\dot{h}h^{-\frac{n}{1+n}}, \tag{22}$$

where h is the Hubble parameter for the case of the minimal coupling (TEGR) for $n = 0$.

Background dynamic equations are

$$V(\phi) = \lambda^{-n}\left[3h^{\frac{2+n}{1+n}} - \frac{1}{2}\dot{\phi}^2\right], \tag{23}$$

$$\dot{\phi}^2 = -2\dot{h}. \tag{24}$$

A model of chaotic inflation based on scalar-torsion gravity

The only possible type of cosmological dynamics with an exit from inflation

$$H(t) = \lambda - \mu t, \tag{25}$$

$$a(t) = a_s \exp\left(\lambda t - \frac{\mu}{2} t^2\right). \tag{26}$$

Solutions of the background equations are

$$\phi(t) = \frac{2}{2+n}\sqrt{\frac{2}{\mu}(1+n)}(\lambda - \mu t)^{1+\frac{n}{2}}, \tag{27}$$

$$V(\phi) \simeq V_0 \phi^2, \tag{28}$$

$$F(\phi) = \frac{V_0}{3}\phi^{\frac{2n}{2+n}}, \tag{29}$$

where

$$V_0 = \frac{3\mu(2+n)^2}{8\lambda^n(1+n)}, \tag{30}$$

$$0 < \mu < 8 \times 10^{-16}. \tag{31}$$

The energy scale of inflation is

$$V_*^{1/4} < 2.6 \times 10^{14}\ \text{GeV} < \Lambda_{GUT}, \tag{32}$$

where $\Lambda_{GUT} \sim 10^{16}$ GeV.

Cosmological perturbations

The parameters of cosmological perturbations at the crossing of the Hubble radius $k = aH$ can be written as

$$\mathcal{P}_S = \frac{H^2}{8\pi^2 Q_S c_S^3} = \frac{H^2}{8\pi^2 F c_S \epsilon_s}, \tag{33}$$

$$\mathcal{P}_T = \frac{H^2}{2\pi^2 Q_T} = \frac{2H^2}{\pi^2 F}, \tag{34}$$

$$n_S - 1 = -2\epsilon - \delta_F - \eta_s - s, \tag{35}$$

$$n_T = -2\epsilon - \delta_F, \tag{36}$$

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_S} = 16 c_s \epsilon_s, \tag{37}$$

where the stability parameters are

$$Q_S = \frac{w_1}{3w_2^2}\left(9w_2^2 + 4w_1 w_3\right), \qquad Q_T = \frac{1}{4}F, \tag{38}$$

and slow-roll parameters are

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \delta = -\frac{\ddot{H}}{2H\dot{H}}, \qquad \delta_F = \frac{\dot{F}}{HF}, \qquad \epsilon_s = \frac{Q_S c_S^2}{F}, \qquad \eta_s = \frac{\dot{\epsilon}_s}{H\epsilon_s}, \qquad s = \frac{\dot{c}_S}{Hc_S}. \tag{39}$$

The slow-roll parameters satisfy the conditions $\epsilon \ll 1, \delta \ll 1, \delta_F \ll 1, \epsilon_s \ll 1, \eta_s \ll 1, s \ll 1$ during inflation.

Cosmological perturbations

For the case $F = (H/\lambda)^n$, from equation (38) follow the expressions

$$Q_S = -(1+n)\frac{\dot{H}}{H^2}\left(\frac{H}{\lambda}\right)^n = (1+n)\left(\frac{H}{\lambda}\right)^n \epsilon > 0, \tag{40}$$

$$Q_T = \frac{1}{4}F = \frac{1}{4}\left(\frac{H}{\lambda}\right)^n > 0, \tag{41}$$

which indicate that these models are free of instabilities and ghosts.
For the case of scalar-torsion gravity, the violation of local Lorentz invariance induces additional gauge degrees of freedom. The gauge degrees of freedom correspond to an additional scalar mode, a transverse vector mode, and a (pseudo)vector mode.
However, the violation of Lorentz invariance occurs deep below the horizon, and these additional modes decay rapidly with the expansion of the early universe.

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Velocities of cosmological perturbations

Relation $F = \left(\frac{H}{\lambda}\right)^n$ leads to the constant velocity of scalar perturbations $c_S^2 = \frac{1-n}{1+n} = constant$, in general case $c_S = c_S(t)$.
The velocity of tensor perturbations $c_T = 1$ in these inflationary models (with or without non-minimal coupling the scalar field and torsion).

The difference in propagation velocities of scalar and tensor perturbations leads to the fact that the Hubble radius was crossed by them in different wavelengths, namely

$$c_S k_S = aH, \qquad k_S = \frac{2\pi a}{\lambda_S}, \tag{42}$$

$$c_T k_T = aH, \qquad k_T = \frac{2\pi a}{\lambda_T}, \tag{43}$$

$$\frac{c_S}{c_T} = \frac{k_T}{k_S} = \frac{\lambda_S}{\lambda_T} = \sqrt{\frac{1-n}{1+n}}. \tag{44}$$

In case of the minimal coupling $c_S = c_T = 1$, the wavelengths of scalar and tensor perturbations are equal ($\lambda_S = \lambda_T$).

Power-law connection between coupling function and the Hubble parameter can be redefined in terms of the velocity of propagation of scalar perturbations as

$$F(\phi(t)) = \left(\frac{4}{1+3c_S^2}\right)(H(t))^{\frac{1-c_S^2}{1+c_S^2}}. \tag{45}$$

Parameters of cosmological perturbations

Based on modern observational data on the measurement of the anisotropy and polarization of the relic radiation, the following constraints on the values of the parameters of cosmological perturbations can be determined

$$\mathcal{P}_S = 2.1 \times 10^{-9}, \quad (46)$$

$$n_S = 0.9663 \pm 0.0041, \quad (47)$$

$$r < 0.028, \quad (48)$$

where P_S is the power spectrum of scalar perturbations, n_S and r are the spectral index of scalar perturbations and the tensor-to-scalar ratio.
For $50 < \Delta N < 57$ relation between tensor-to-scalar ratio and spectral index of scalar perturbations is

$$r = \frac{32c_S^3(1-n_S)}{3+5c_S^2}. \quad (49)$$

Chaotic inflation with quadratic potential based on the scalar-torsion gravity corresponds to observational constraints on the values of the parameters of cosmological perturbations under condition $0 < c_S < 1/2$.

Equation of state (EoS) parameter at the end of the inflationary stage

Equation of state (EoS) parameter is

$$w = \frac{-3 + (\lambda^n + 1)(1+n)\epsilon}{3 + (\lambda^n - 1)(1+n)\epsilon}. \quad (50)$$

At the inflationary stage $\epsilon \ll 1$, and the state parameter $w \simeq -1$.
The speed of propagation of scalar perturbations

$$c_S^2 = \frac{1-n}{1+n} = constant, \quad (51)$$

and for the parameters

$$n = \frac{1-c_S^2}{1+c_S^2}, \quad (52)$$

$$\lambda^n = \frac{1}{4}\left(1 + 3c_S^2\right), \quad (53)$$

the end of inflation $\epsilon = 1$ corresponds to the state parameter $w = -1/3$ similar to the case of minimal coupling (with $c_S^2 = 1$).

The influence of the non-minimal coupling of the scalar field and torsion on the cosmological parameters in this case can be parameterized by the velocity of scalar perturbations only.

Conclusion

- Models of cosmological inflation based on scalar-torsion gravity can be verified using observational data without taking into account the nonlinear self-action of the scalar field.
- In contrast to the TEGR or Einstein gravity, the model of chaotic inflation based on scalar-torsion gravity can be considered as verifiable by observational constraints on the values of the parameters of cosmological perturbations.
- The influence of the non-minimal coupling of the scalar field and torsion on the cosmological parameters for chaotic inflation can be parameterized by the velocity of scalar perturbations only.