Restrictions on parameters of stabilized Randall Sundrum 1 model, following from the four top quark production process

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Aleshko A.M., Boos E.E.

SINP MSU

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SMEFT

SMEFT is constructed by extending the SM Lagrangian with gauge-invariant operators of higher dimension:

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum \frac{c_i^{\{6\}}}{\Lambda^2} O_i^{d=6} + \sum \frac{c_i^{\{8\}}}{\Lambda^4} O_i^{d=8} + \ldots, \quad (1)$$

where L_{SM} is the SM Lagrangian, Λ - hypothetical scale of the BSM physics, $O_i^{d=n}$ - local composite SMEFT operators of dimension n, c_i - dimensionless Wilson coefficients.

In the SMEFT framework, any observable, in particular the cross-section, can be parametrized in the following form:

$$\sigma = \sigma_{SM} + \sum_k \frac{c_i}{\Lambda^2} \sigma_k^{(1)} + \sum_{j<=k} \frac{c_i c_k}{\Lambda^4} \sigma_{k,j}^{(2)} + \ldots, \quad (2)$$

$\sigma^{(1)}$ and $\sigma^{(2)}$ - coefficients, representing linear and quadratic (in terms of EFT coupling) contributions of the SMEFT operators

Four top quark production

Available analyses:

$$\sigma_{ATLAS} = 22.5^{+6.6}_{-5.5} \text{ fb}^{\,a}$$

$$\sigma_{CMS} = 17.7^{+4.4}_{-4.0} \text{ fb}^{\,b}$$

[a] arXiv:2303.15061
[b] arXiv:2305.13439

Best available results for SM:

$$\sigma^{NLO}_{SM} = 11.97^{+2.2}_{-2.5} \text{ fb}^{\,a}$$

$$\sigma^{NLL'}_{SM} = 14.65^{+1.2}_{-2.5} \text{ fb}^{\,b}$$

[a] arXiv:1711.02116
[b] arXiv:2212.03259

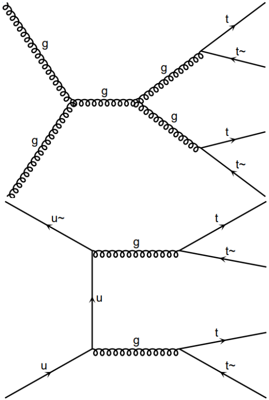


Figure 1: Representative diagrams of the four top quark hadroproduction process

Stabilized RS1 model[1]

Randall-Sundrum 1 - stabilized brane world model with two branes and massive radion.

- The fifth dimension composed of the orbifold S^1/Z_2
- $-L \le y \le L$ - the corresponding coordinate
- The metric g^{MN} and the scalar field ϕ satisfy the corresponding orbifold symmetry conditions
- The branes are located at the fixed points of the orbifold, $y = 0$ and $y = L$

The action of the stabilized brane world model

$$S = -2M^3 \int d^4x \int_{-L}^{L} dy R\sqrt{-g} + \int d^4x \int_{-L}^{L} dy (\frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - V(\phi))\sqrt{-g}$$
$$- \int_{y=0} d^4x \lambda_1(\phi)\sqrt{-\tilde{g}} + \int_{y=L} d^4x (-\lambda_2(\phi) + \mathcal{L}_{SM})\sqrt{-\tilde{g}}, \quad (3)$$

where $V(\phi)$ is a bulk scalar field potential, $\lambda_{1,2}(\phi)$ are quadratic brane scalar field potentials, $\tilde{g}_{\mu\nu}$ - the metric induced on the branes

[1] arXiv:0710.3100

Stabilized RS1 model[2]

Using the following metric representation

$$g_{MN}(x,y) = \gamma_{MN}(y) + \frac{1}{\sqrt{2M^3}} h_{MN}(x,y)$$
$$\phi(x,y) = \phi_1 e^{-u|y|} + \frac{1}{\sqrt{2M^3}} f(x,y)$$

, one can obtain the so-called second variation Lagrangian, the interaction part of which reads (arXiv:hep-th/0511185):

$$\mathcal{L}_{int} = -\frac{1}{2\sqrt{2M^3}} (h_{\mu\nu} T^{\mu\nu}) \quad (4)$$

Performing diagonalization and KK reduction, the action built with the second variation Lagrangian can be expressed as:

$$S_{eff} = \frac{1}{4}\sum_{k=0}^{\infty}\int dx(\partial^\sigma b^k_{\mu\nu}\partial_\sigma b_{k,\mu\nu} - m_k^2 b^{k,\mu\nu} b^k_{\mu\nu}) + \frac{1}{2}\sum_{k=1}^{\infty}\int dx(\partial^\mu \phi^k \partial_\mu \phi^k - \mu_k^2 \phi^k \phi^k) \quad (5)$$

[2] arXiv:0710.3100

Stabilized RS1 model[3]

Interaction with SM fields

$$\mathcal{L}_{int} = \frac{1}{2\sqrt{2}M^3}\left(\psi_0(L) b^0_{\mu\nu}(x) T^{\mu\nu} + \sum \psi_n(L) b^n_{\mu\nu}(x) T^{\mu\nu} + \frac{1}{2}\sum g_n(L)\phi_n(x) T^\mu_\mu\right), \quad (6)$$

where $\psi_n(y)$ and $g_n(y)$ - wave functions of the modes in the extra dimension

KK modes interact with $T_{\mu\nu}$ - SM energy-momentum tensor:

$$T_{\mu\nu} = 2\frac{\delta\mathcal{L}_{SM}}{\delta\eta^{\mu\nu}} - \eta_{\mu\nu}\mathcal{L}_{SM} \quad (7)$$

[3] arXiv:0710.3100

Introduction summary

Goals of the study

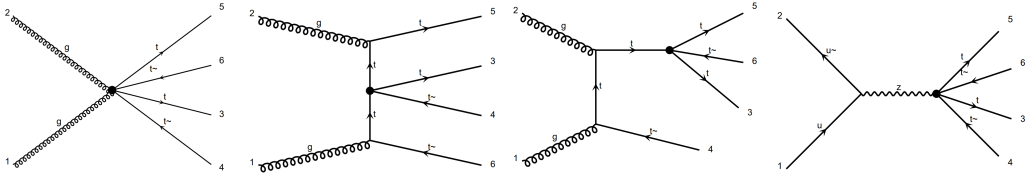
- Obtain experimental limits for Wilson Coefficients of relevant dimension 8 SMEFT operators from the four top-quark production observation
- Perform matching for the stabilized RS1 model and obtain experimental bounds on parameters of one by recalculation corresponding limits on WC

Numerical Setup

Setup

- Only operators of the following classes are considered: $\psi^4 X$, $\psi^4\phi^2$, $\psi^4\phi D$, and $\psi^4 D^2$
- Basis for operators is chosen as in arXiv:2005.00059 [hep-ph]
- Simulation parameters follow CMS analysis (arXiv:2305.13439)

Examples of diagrams with inclusion of effective vertices:



Numerical Setup

Numerical toolchain

FeynRules[a] $\longrightarrow$ MadGraph[b] $\longrightarrow$ Statistical software[c]

[a]arXiv:1310.1921
[b]arXiv:1405.0301
[c]Good examples of dedicated packages are SMEFiT (arXiv:2302.06660) and EFTfitter (arXiv:1605.05585)

C.o.m. energy, TeV	SM cros.-sect. σ_{SM}, fb	scale uncert., %	PDF uncert., %
LO			
13	9.02	73.4	7.32
14	12.0	72.6	7.28
NLO (QCD + EW)			
13	13.2	20.1	2.5
14	17.8	20.3	2.5

Table 1: SM cross-sections for process $pp \to t\bar{t}t\bar{t}$

Obtaining statistical limits for WC

The cross-section with inclusion of dimension 8 SMEFT operators is parametrized as follows:

$$\sigma = \sigma_{SM} + \sum_k \frac{c_i}{\Lambda^4}\sigma_k^{(1)} + \sum_{j<=k} \frac{c_i c_k}{\Lambda^8}\sigma_{k,j}^{(2)}, \qquad (8)$$

- The value for σ is taken from CMS the measurement
- The SM cross-section σ_{SM} is taken at next-to-leading logarithmic accuracy (NLL′)
- Values for coefficients $\sigma^{(1)}$ and $\sigma^{(2)}$ are obtained by simulation of corresponding contributions in Madgraph
- **All variables are modeled by normal distribution with their errors corresponding to a Gaussian 1σ interval**

Given above, limits are obtained using MC sampling technique

Matching

Using the special parametrization given in arXiv:0710.3100, "Integrated-out" Lagrangian can be written in the form:

$$\mathcal{L}_{int} \simeq \frac{1.82}{\Lambda_\pi^2 m_1^2}(T_{\mu\nu}T^{\mu\nu} + (\frac{1}{3} - \frac{\delta}{2})T^\mu_\mu T^\nu_\nu), \quad (9)$$

where Λ_π^2 and m_1^2 - the coupling constant and the mass of the first KK resonance, respectively; δ - the contribution of the scalar modes.

In the following, convenience constants are introduced for matching coefficients:

$$C_T \equiv 1.82(\Lambda_\pi^2 m_1^2)^{-1}$$
$$C_S \equiv (\frac{1}{3} - \frac{\delta}{2})$$

After the contraction there are many redundant operators, which must be reduced to the basis of choice. Simplification techniques (IBP, EOM, FI) are used to reduce operators to the basis presented in arXiv:2005.00059

Matching

Example

Fermion part of the energy-momentum tensor has the following form:

$$T_F^{\mu\nu} = \frac{i}{2}\bar{\psi}\gamma^\mu D^\nu \psi + \frac{i}{2}\bar{\psi}\gamma^\nu D^\mu \psi - \frac{i}{4}\partial^\mu(\bar{\psi}\gamma^\nu\psi) - \frac{i}{4}\partial^\nu(\bar{\psi}\gamma^\mu\psi) + ig^{\mu\nu}(\frac{1}{2}\partial_\rho(\bar{\psi}\gamma^\rho\psi) - \bar{\psi}\gamma^\rho D_\rho\psi), \quad (10)$$

Its contractions can be simplified using the following techniques

- All EOMs should be eliminated in a physical basis, e.g.

$$(\dots)(\bar{q}\gamma^\mu D_\mu q) \longrightarrow (\dots)(Y_u u\tilde{H} + Y_d dH)$$

- Mixed contractions are resolved using corresponding Fierz identities, e.g.

$$(\bar{u}^\alpha\gamma^\mu u^\beta)(\bar{u}^\beta\gamma_\mu u^\alpha) \longrightarrow 2(\bar{u}^\alpha\gamma^\mu T^A u^\alpha)(\bar{u}^\beta\gamma_\mu T^A u^\beta) + \frac{1}{3}(\bar{u}^\alpha\gamma^\mu u^\alpha)(\bar{u}^\beta\gamma_\mu u^\beta) \quad (11)$$

- The ordering of the derivatives can be changed using integration-by-part, e.g.

$$(\dots)^\nu D_\mu D_\nu(\bar{u}\gamma^\mu u) \longrightarrow D_\nu(\dots)^\nu D_\mu(\bar{u}\gamma^\mu u) + (\text{total derivative}) \quad (12)$$

Matching

Example

Contractions of the Fermion part lead to operators of the following classes:

$$
\begin{aligned}
Fermion - Fermion &: \underline{\psi^4 D^2}(\text{with different Lorentz structure}),\\
Fermion - Higgs &: \psi^2\phi^5, \psi^2\phi^4 D, \psi^2\phi^3 D^2, \psi^2\phi^2 D^3, \psi^2 X\phi^2 D, \underline{\psi^4\phi D}, \underline{\psi^4\phi^2}\\
Fermion - Yukawa &: X^3\phi^2, X^2\phi^4, X^2\phi^2 D^2, \psi^2 X^2\phi, \psi^2 X\phi^2 D
\end{aligned}
$$

In the current work, matching is performed with the help of Matchete[a] package.

[a] arXiv:2212.04510

Summary tables

$\psi^4 \phi D$

Name	Definition	Matching coefficient	Limits on WC $C_k/\Lambda^4[\mathrm{TeV}^{-4}]$
$Q^{(1)}_{q^3uHD}$ + h.c.	$i(\bar{q}\gamma^\mu q)[(\bar{q}^j u)D_\mu H^{\dagger k}\epsilon_{jk}]$ + h.c.	$C_T Y_t$	[-8.637, 8.637]
$Q^{(1)}_{qu^3HD}$ + h.c.	$i(\bar{u}\gamma^\mu u)[(\bar{q}^j u)D_\mu H^{\dagger k}\epsilon_{jk}]$ + h.c.	$C_T Y_t$	[-, -]
$Q^{(1)}_{qud^2HD}$ + h.c.	$i(\bar{d}\gamma^\mu d)[(\bar{q}^j u)D_\mu H^{\dagger k}\epsilon_{jk}]$ + h.c.	$C_T Y_t$	[-, -]
$Q^{(1)}_{q^3dHD}$ + h.c.	$i(\bar{q}\gamma^\mu q)[(\bar{q}^j d)D_\mu H]$ + h.c.	$C_T(-Y_b)$	[-, -]
$Q^{(1)}_{qu^2dHD}$ + h.c.	$i(\bar{u}\gamma^\mu u)[(\bar{q}^j d)D_\mu H]$ + h.c.	$C_T(-Y_b)$	[-, -]
$Q^{(1)}_{qd^3HD}$ + h.c.	$i(\bar{d}\gamma^\mu d)[(\bar{q}^j d)D_\mu H]$ + h.c.	$C_T(-Y_b)$	[-, -]

Table 2: Definitions, matching coefficients and 1D statistical limits for corresponding WC of operators of the $\psi^4 \phi D$ class, obtained from matching of stabilized RS1 model to dimension 8 SMEFT

Summary tables

$\psi^4\phi^2$: $(\bar{L}L)(\bar{L}L)$, $(\bar{R}R)(\bar{R}R)$ and $(\bar{L}R)(\bar{L}R)$

Name	Definition	Matching coefficient	Limits on WC $C_k/\Lambda^4[\mathrm{TeV}^{-4}]$
		$(\bar{L}L)(\bar{L}L)$	
$Q^{(1)}_{q^4H^2}$	$(\bar{q}\gamma^\mu q)(\bar{q}\gamma_\mu q)(H^\dagger H)$	$C_T(\frac{1}{6}g_S^2 - \frac{5}{36}g_Y^2)$	[-, -]
$Q^{(3)}_{q^4H^2}$	$(\bar{q}\gamma^\mu\tau^I q)(\bar{q}\gamma_\mu\tau^I q)(H^\dagger H)$	$C_T\frac{1}{2}(g_S^2 + g_L^2)$	[-, -]
		$(\bar{R}R)(\bar{R}R)$	
$Q_{u^4H^2}$	$(\bar{u}\gamma^\mu u)(\bar{u}\gamma_\mu u)(H^\dagger H)$	$C_T(\frac{2}{3}g_S^2 + \frac{11}{9}g_Y^2)$	[-, -]
$Q_{q^4H^2}$	$(\bar{d}\gamma^\mu d)(\bar{d}\gamma_\mu d)(H^\dagger H)$	$C_T(\frac{2}{3}g_S^2 + \frac{1}{18}g_Y^2)$	[-, -]
$Q^{(1)}_{u^2d^2H^2}$	$(\bar{u}\gamma^\mu u)(\bar{d}\gamma_\mu d)(H^\dagger H)$	$C_T(-\frac{13}{18}g_Y^2)$	[-, -]
$Q^{(2)}_{u^2d^2H^2}$	$(\bar{u}\gamma^\mu T^A u)(\bar{d}\gamma_\mu T^A d)(H^\dagger H)$	$C_T 4g_S^2$	[-, -]
		$(\bar{L}R)(\bar{L}R)$	
$Q^{(1)}_{q^2udH^2}$ + h.c.	$(\bar{q}^j u)\epsilon_{jk}(\bar{q}^k d)(H^\dagger H)$ + h.c.	$C_T Y_t Y_b(-\frac{5}{2} + 2C_S)$	[-, -]
$Q^{(2)}_{q^2udH^2}$ + h.c.	$(\bar{q}^j u)(\tau^I\epsilon)_{jk}(\bar{q}^k d)(H^\dagger\tau^I H)$ + h.c.	$C_T(-\frac{3}{2}Y_t Y_b)$	[-, -]

Table 3: Definitions, matching coefficients and 1D statistical limits for corresponding WC of operators of the $\psi^4\phi^2$ class, obtained from matching of stabilized RS1 model to dimension 8 SMEFT

Summary tables

$\psi^4\phi^2$: $(\bar{L}L)(\bar{R}R)$

Name	Definition	Matching coefficient $(\bar{L}L)(\bar{R}R)$	Limits on WC $C_k/\Lambda^4[\mathrm{TeV}^{-4}]$
$Q^{(1)}_{q^2u^2H^2}$	$(\bar{q}\gamma^\mu q)(\bar{u}\gamma_\mu u)(H^\dagger H)$	$C_T(\frac{31}{36}g_Y^2 + Y_t^2(\frac{5}{12} - \frac{1}{3}C_S))$	[-, -]
$Q^{(2)}_{q^2u^2H^2}$	$(\bar{q}\gamma^\mu \tau^I q)(\bar{u}\gamma_\mu u)(H^\dagger \tau^I H)$	$C_T(-\frac{1}{4}Y_t^2)$	[-, -]
$Q^{(3)}_{q^2u^2H^2}$	$(\bar{q}\gamma^\mu T^A q)(\bar{u}\gamma_\mu T^A u)(H^\dagger H)$	$C_T(4g_S^2 + Y_t^2(\frac{5}{2} - 2C_S))$	[-, -]
$Q^{(4)}_{q^2u^2H^2}$	$(\bar{q}\gamma^\mu T^A \tau^I q)(\bar{u}\gamma_\mu T^A u)(H^\dagger \tau^I H)$	$C_T(-\frac{3}{2}Y_t^2)$	[-, -]
$Q^{(1)}_{q^2d^2H^2}$	$(\bar{q}\gamma^\mu q)(\bar{d}\gamma_\mu d)(H^\dagger H)$	$C_T(-\frac{11}{36}g_Y^2 + Y_t^2(\frac{5}{12} - \frac{1}{3}C_S))$	[-, -]
$Q^{(2)}_{q^2d^2H^2}$	$(\bar{q}\gamma^\mu \tau^I q)(\bar{d}\gamma_\mu d)(H^\dagger \tau^I H)$	$C_T(\frac{1}{4}Y_t^2)$	[-, -]
$Q^{(3)}_{q^2d^2H^2}$	$(\bar{q}\gamma^\mu T^A q)(\bar{d}\gamma_\mu T^A d)(H^\dagger H)$	$C_T(4g_S^2 + Y_t^2(\frac{5}{2} - 2C_S))$	[-, -]
$Q^{(4)}_{q^2d^2H^2}$	$(\bar{q}\gamma^\mu T^A \tau^I q)(\bar{d}\gamma_\mu T^A d)(H^\dagger \tau^I H)$	$C_T(\frac{3}{2}Y_t^2)$	[-, -]

Table 4: Definitions, matching coefficients and 1D statistical limits for corresponding WC of operators of the $\psi^4\phi^2$ class, obtained from matching of stabilized RS1 model to dimension 8 SMEFT

Summary tables

$\psi^4 D^2$

Name	Definition	Matching coefficient	Limits on WC $C_k/\Lambda^4[\mathrm{TeV}^{-4}]$
$Q^{(1)}_{q^4 D^2}$	$(\bar{q}\gamma^\mu \overleftrightarrow{D}^\nu q)(\bar{q}\gamma_\mu \overleftrightarrow{D}_\nu q)$	$C_T\frac{1}{8}$	[-0.30, 0.31]
$Q^{(1)}_{u^4 D^2}$	$(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u)(\bar{u}\gamma_\mu \overleftrightarrow{D}_\nu u)$	$C_T\frac{1}{8}$	[-0.55, 0.52]
$Q^{(1)}_{d^4 D^2}$	$(\bar{d}\gamma^\mu \overleftrightarrow{D}^\nu d)(\bar{d}\gamma_\mu \overleftrightarrow{D}_\nu d)$	$C_T\frac{1}{8}$	[-, -]
$Q^{(1)}_{q^2 u^2 D^2}$	$(\bar{q}\gamma^\mu \overleftrightarrow{D}^\nu q)(\bar{u}\gamma_\mu \overleftrightarrow{D}_\nu u)$	$C_T\frac{1}{4}$	[-0.58, 0.55]
$Q^{(1)}_{q^2 d^2 D^2}$	$(\bar{q}\gamma^\mu \overleftrightarrow{D}^\nu q)(\bar{d}\gamma_\mu \overleftrightarrow{D}_\nu d)$	$C_T\frac{1}{4}$	[-, -]
$Q^{(1)}_{u^4 D^2}$	$(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u)(\bar{d}\gamma_\mu \overleftrightarrow{D}_\nu d)$	$C_T\frac{1}{4}$	[-, -]

Table 5: Definitions, matching coefficients and 1D statistical limits for corresponding WC of operators of the $\psi^4 D^2$ class, obtained from matching of stabilized RS1 model to dimension 8 SMEFT

Projection of the WC bounds on parameters of RS1 model

Obtained bounds for $\Lambda_\pi^2 m_1^2$

$$\frac{1}{\Lambda_\pi^2 m_1^2} < 1.25[\text{TeV}^{-4}]$$

Reference values available in the literature[4]

- Theoretical bound from DY production on Tevatron:

$$\frac{1}{\Lambda_\pi^2 m_1^2} < 1.185[\text{TeV}^{-4}]$$

- Theoretical bound from DY production on LHC, $\sqrt{s} = 14\text{TeV}$:

$$\frac{1}{\Lambda_\pi^2 m_1^2} < 0.238 \cdot 10^{-2}[\text{TeV}^{-4}]$$

[4] arXiv:0710.3100

Projection of the WC bounds on parameters of RS1 model

One can also introduce parameter M_S^{GRW}[5], which can be related to the corresponding analyses:

$$M_S^{GRW} = (\frac{1}{2\pi}\frac{1}{\Lambda_\pi^2 m_1^2})^{-\frac{1}{4}} \quad (13)$$

Current work

Four top-quark production observation:
$M_S^{GRW} > 1.5$ TeV

Literature

- CMS, high mass diphoton events:[a] $M_S^{GRW} > 7.8$ TeV
- CMS, dijet angular distributions:[b] $M_S^{GRW} > 10.1$ TeV

[a] arXiv:1809.00327
[b] arXiv:1803.08030

[5] arXiv:hep-ph/9811291

Summary

- Experimental limits for WC of dimension 8 four-fermion SMEFT operators were obtained from the observation of the four top-quark production process
- Matching of the Stabilized RS1 model to dimension 8 SMEFT was performed
- Experimental limits for parameters of the Stabilized RS1 model were obtained from corresponding bounds on WC of dimension 8 SMEFT operators

Unfortunately, the available accuracy for WC limits lead to restrictions, which are worse than ones already present in the literature. Main contribution to the error is due to theoretical uncertainty. Results can potentially be greatly improved by performing simulation and matching at NLO