

Inclusive ψ' charmonia production at NLO within k_T -factorization approach

Andrei Prokhorov¹, Sergey Baranov², Artem Lipatov³, Maxim Malyshev³

¹DLNP JINR, Dubna

²LPI, Moscow

³SINP MSU, Moscow

**The XXV International Workshop-School
High Energy Physics and Quantum Field Theory**

June 30 - July 5 2025

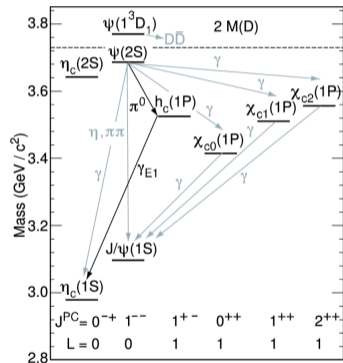
Introduction: Nonrelativistic QCD

- ▶ **Nonrelativistic QCD (NRQCD)** [*Phys.Rev.D 53,150 (1996)*],[*Phys.Rev.D 51,1125 (1995)*]:

$$\sigma(pp \rightarrow \psi + X) = \sum_n \sigma(pp \rightarrow c\bar{c}[{}^{2S+1}L_J^{(a)}] + X) \langle \mathcal{O}^\psi[n] \rangle$$

- ▶ $\sigma(pp \rightarrow c\bar{c}[{}^{2S+1}L_J^{(a)}] + X)$ cross-section of heavy quark pair production $Q\bar{Q}$ in Fock state $n = {}^{2S+1}L_J^{(a)}$ with definite spin S , orbital L и total J angular momenta and color representation a (color singlet (CS) [1] and color octet (CO) [8]) - can be calculated in the framework of perturbative QCD.
- ▶ **LDME** (nonperturbative matrix element) $\langle \mathcal{O}^\psi[n] \rangle$ describe the transition of intermediate state $c\bar{c}[n]$ into physical quarkonium via emission of soft gluons.
- ▶ Decomposition of wave functions of S-wave charmonia over the small parameter v of quark relative motion and Fock states:

$$|\psi\rangle = O(v^0)|c\bar{c}[{}^3S_1^{(1)}]\rangle + O(v^1)|c\bar{c}[{}^3P_J^{(8)}]g\rangle + O(v^2)|c\bar{c}[{}^3S_1^{(8)}]gg\rangle + \dots$$



Charmonia production: recent status and challenges

- ▶ Nonrelativistic QCD in **collinear factorization** approach at the **NLO**:
 - ↪ good description of charmonia ($J/\psi, \psi', \chi_c$) and bottomonia (Υ, χ_b) transverse momentum distributions (separately) [*Phys.Lett.B* 673, 197 (2009); *Phys.Rev.D* 84, 051501 (2011); *Phys.Rev.D* 83, 111503 (2011); *Phys.Rev.Lett.* 106, 172002 (2012); *Phys.Rev.D* 90, 074021 (2014); *Phys.Rev.D* 83, 114021 (2011)]
 - ↪ tree-level NNLO calculations to the color singlet contributions [*Phys.Rev.Lett.* 101, 152001 (2008); *Phys.Lett.B* 695, 149 (2011)]
 - ↪ **problem with polarization** [*Phys.Rev.Lett.* 108, 172002 (2012)].
 - ↪ **problem with global fit for ψ'** [*Phys. Rev. D* 107, 034003 (2023)]: a satisfactory fit can be achieved only at $p_T > 7$ GeV, polarization problem is not solved
 - ↪ **global fit for J/ψ and χ_c is absent** (possibly the same problem as for ψ')
- ▶ **Is it possible to achieve a good simultaneous description of ψ' ?**

Charmonia production: recent status and challenges

- ▶ A possible **solution of polarization problem** [*Phys.Rev.D* 93, 054037 (2016)].
- ▶ Calculation in **high-energy factorization (k_T -factorization)** approach at the **LO**:
 - ↪ good agreement with LHC data, including the polarization observables for $J/\psi, \psi', \chi_c$ and $\Upsilon(nS), \chi_b$ [*Phys.Rev.D* 100, 114021 (2019); *Eur.Phys.J. C* 79, 621 (2019); 80, 1022 (2020); 79, 830 (2019); 80, 486 (2020); 81, 1085 (2021)]
 - ↪ results for P-wave quarkonia χ_c and χ_b are in contradictions with the Heavy Quark Spin Symmetry (HQSS) relations
 - ↪ calculations of J/ψ and ψ' overestimates data at low p_T for definite sets of TMD gluon distributions
- ▶ Calculation in **k_T -factorization** approach at the **LO+NLO[†]** [*Eur. Phys. J. C* 84, 348 (2024)]:
 - ↪ χ_{cJ} production: restoration of the HQSS relations + good descriptions of polarization observables
- ▶ **Recent goals: implementation of tree-level NLO corrections for ψ' + global fit**

High energy factorization (k_T -factorization) approach

- ▶ In the region of high energies (small x) parton model assumptions about collinear factorization of parton distributions and partonic cross sections are violated. Along with contributions $\sim \alpha_s^n \ln^n \mu^2 / \Lambda_{\text{QCD}}^2$ arises contributions $\sim \alpha_s^n \ln^n 1/x$, which can be summarized up to all orders of perturbative theory with evolution equations **BFKL** (Balitsky-Fadin-Kuraev-Lipatov) or **CCFM** (Catani-Ciafaloni-Fiorani-Marchesini)

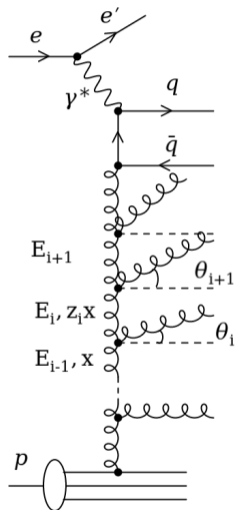
- ▶ In **k_T -factorization approach** [*Phys.Rep.* 100,1 (1983)], [*Sov.J.Nucl.Phys.* 53,657 (1991)], [*Nucl.Phys.B* 366,135 (1991)]:

$$d\sigma = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} f_1(x_1, \mathbf{k}_{1T}^2, \mu_F^2) f_2(x_2, \mathbf{k}_{2T}^2, \mu_F^2) d\hat{\sigma}^*(k_1, k_2, \mu_R^2),$$

where $f(x, \mathbf{k}_T^2, \mu_F^2)$ - TMD (transverse momentum dependent) parton distribution function in proton which obey the BFKL or CCFM evolution equation.

- ▶ LO in k_T -factorization approach can include the large piece of high-order corrections from collinear QCD approach (partially NLO + NNLO + ...) taking them into account in the form of TMD gluon distributions $f(x, \mathbf{k}_T^2, \mu_F^2)$ in the proton.
- ▶ **Gauge invariance of off-shell amplitudes** are achieved by using the effective vertex [*Nucl. Phys. B* 452,369 (1995)]

CCFM evolution equation and double counting problem



- ▶ **CCFM evolution equation** [*Nucl. Phys. B* 296, 49 (1988)], [*PLB* 234,339 (1990)] at leading logarithmic approximation:

$$f_g(x, \mathbf{k}_T^2, q^2) = f_g(x, \mathbf{k}_T^2, q_0^2) \Delta_s(q, q_0) + \int dz \int \frac{dq'^2}{q'^2} \Theta(q - zq') \times \\ \times \Delta_s(q, zq') \tilde{P}_{gg}(z, q', \mathbf{k}_T) f_g(x/z, \mathbf{k}'_T{}^2, q'^2),$$

- ▶ Emitted gluons obey the angular ordering conditions (due to color coherence effect):

$$q_{i+1} \quad q > z_n q_n > z_{n-1} q_{n-1} > \dots > q_1 > q_0,$$

↪ at $z \rightarrow 0$: no constraints on q_i - BFKL conditions

q_i ↪ at $z \rightarrow 1$: ordering on q_i - DGLAP conditions

- ▶ NLO corrections in k_T -factorization approach:

↪ more complicated calculation of off-shell amplitudes (extended set of Feynman diagrams)

↪ **Double counting problem** for LO and NLO: some contributions (gluon emissions) can be counted twice - from tree-level amplitude (NLO) and initial gluon radiation (from evolution of TMD gluon distributions) (LO)

LO+NLO[†] merging scheme

- ▶ **CS contributions:** $\left| \begin{array}{l} \text{LO } 2 \rightarrow 2: \quad g^*(k_1) + g^*(k_2) \rightarrow c\bar{c} \left[{}^3S_1^{[1]} \right] (p) + g(p_g) \end{array} \right.$
- ▶ **CO contributions:** $\left| \begin{array}{l} \text{LO } 2 \rightarrow 1: \quad g^*(k_1) + g^*(k_2) \rightarrow c\bar{c} \left[{}^1S_0^{[8]}, {}^3P_J^{[8]}, {}^3S_1^{[8]} \right] (p), \\ \text{NLO } 2 \rightarrow 2: \quad g^*(k_1) + g^*(k_2) \rightarrow c\bar{c} \left[{}^1S_0^{[8]}, {}^3P_J^{[8]}, {}^3S_1^{[8]} \right] (p) + g(p_g) \end{array} \right.$
- ▶ **Double counting problem:** separation of LO and NLO contributions is needed (DCE cut)
 - ↔ k_T - transverse momenta of incoming gluon can serve as characteristic momentum of emitted gluons (as a result of evolution) with a good accuracy: $\langle k_T \rangle > \langle p_{gT}^{\text{emission}} \rangle$
- ▶ Merging scheme of LO + tree-level NLO (NLO[†]) (validated for χ_{cJ} production at central rapidities [Eur. Phys. J. C 84, 348 (2024)]):
 - ★ $2 \rightarrow 1$: without cuts
 - ★ $2 \rightarrow 2$: $|\mathbf{p}_{gT}| > \max(|\mathbf{k}_{1T}|, |\mathbf{k}_{2T}|)$
- ▶ **Modified merging scheme** of LO + tree-level NLO (NLO[†]) at forward rapidities and low p_T (initial transverse momentum $\mathbf{k}_T^{\text{init}}$ of incoming gluons are important !):
 - ★ $2 \rightarrow 1$: without cuts
 - ★ $2 \rightarrow 2$: $|\mathbf{p}_{gT}| > \max(|\mathbf{k}_{1T}|, |\mathbf{k}_{2T}| + \overline{|\mathbf{k}_{2T}^{\text{init}}|})$

LDME's fitting procedure

$$\frac{d\sigma^{\psi'}}{dp_T} = \frac{d\sigma(c\bar{c}[{}^3S_1^{[1]}])}{dp_T} \langle \mathcal{O}^{\psi'}[{}^3S_1^{[1]}] \rangle + \frac{d\sigma(c\bar{c}[{}^1S_0^{[8]}])}{dp_T} \langle \mathcal{O}^{\psi'}[{}^1S_0^{[8]}] \rangle + \sum_J (2J+1) \frac{d\sigma(c\bar{c}[{}^3P_J^{[8]}])}{dp_T} \langle \mathcal{O}^{\psi'}[{}^3P_0^{[8]}] \rangle + \frac{d\sigma(c\bar{c}[{}^3S_1^{[8]}])}{dp_T} \langle \mathcal{O}^{\psi'}[{}^3S_1^{[8]}] \rangle$$

► Parameters:

- ↪ CS LDME is determined from the meson decay width: $\langle \mathcal{O}^{\psi'}[{}^3S_1^{[1]}] \rangle = 0.73 \text{ GeV}^3$
- ↪ only 3 free parameters (CO LDME's): $\langle \mathcal{O}^{\psi'}[{}^1S_0^{[8]}] \rangle$, $\langle \mathcal{O}^{\psi'}[{}^3P_0^{[8]}] \rangle$ and $\langle \mathcal{O}^{\psi'}[{}^3S_1^{[8]}] \rangle$
- ↪ HQSS: $\langle \mathcal{O}^{\psi'}[{}^3P_J^{[8]}] \rangle = (2J+1) \langle \mathcal{O}^{\psi'}[{}^3P_0^{[8]}] \rangle$ where $J = 0, 1, 2$
- ↪ two sets of TMD gluon distributions: **A0** and **JH'2013-set-2**

► Unpolarized ψ' datasets: $p_T = 0 - 360 \text{ GeV}$ and $y < 4.5$

↪ **ATLAS** 5, 7, 8, 13 TeV || **CMS** 5, 7, 13 TeV || **LHCb** 7, 13 TeV || **ALICE** 7, 8, 13 TeV

► Minimization of χ^2 : strong negative correlation between ${}^1S_0^{[8]}$ and ${}^3P_J^{[8]}$ contributions, with the correlation coefficient $|U_{cov}({}^1S_0^{[8]}, {}^3P_0^{[8]})| \simeq 0.996$ for both considered TMD densities

► Only 2 free parameters: $\langle \mathcal{O}_{l.c.} \rangle = \langle \mathcal{O}[{}^3P_0^{[8]}] \rangle + r \times \langle \mathcal{O}[{}^1S_0^{[8]}] \rangle$ and $\langle \mathcal{O}^{\psi'}[{}^3S_1^{[8]}] \rangle$

Fit results

- ▶ Minimal $\chi^2/\text{n.d.f.}$ as function of p_T^{min} cutoff

- ▶ **A0 gluons:**

- ↪ good description without cut on p_T

- ↪ $\chi^2/\text{n.d.f.}(\text{LO+NLO}^\dagger) = 2.39$

- $\chi^2/\text{n.d.f.}(\text{LO}) = 2.65$

- ▶ **JH'2013-set-2:**

- ↪ problem with description at low p_T

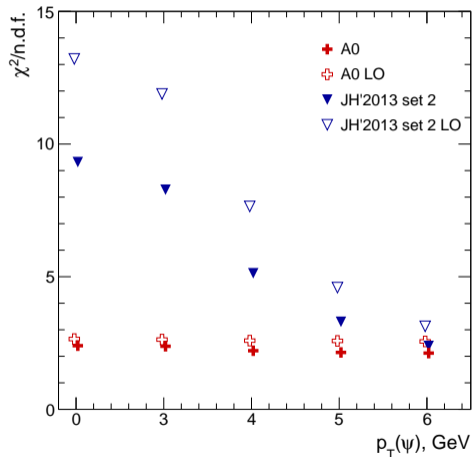
- ↪ $p_T > 0$: $\chi^2/\text{n.d.f.}(\text{LO+NLO}^\dagger) = 8.59$

- $\chi^2/\text{n.d.f.}(\text{LO}) = 13.2$

- ↪ $p_T > 6$: $\chi^2/\text{n.d.f.}(\text{LO+NLO}^\dagger) = 2.35$

- $\chi^2/\text{n.d.f.}(\text{LO}) = 3.13$

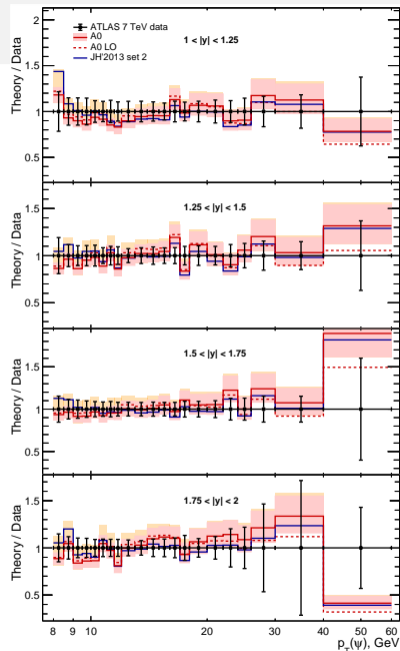
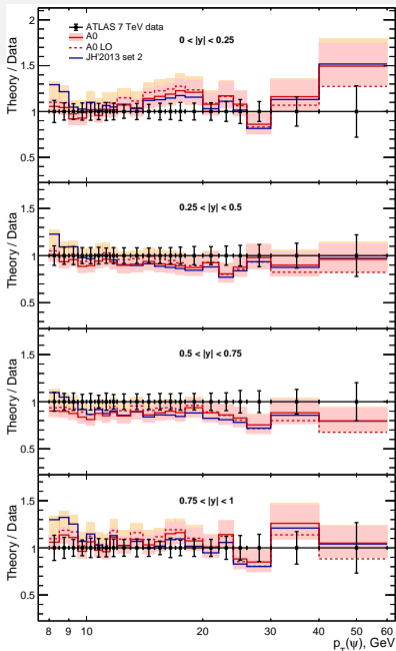
- ▶ The inclusion of NLO corrections leads to changing the shape and allow to significantly soften the discrepancy

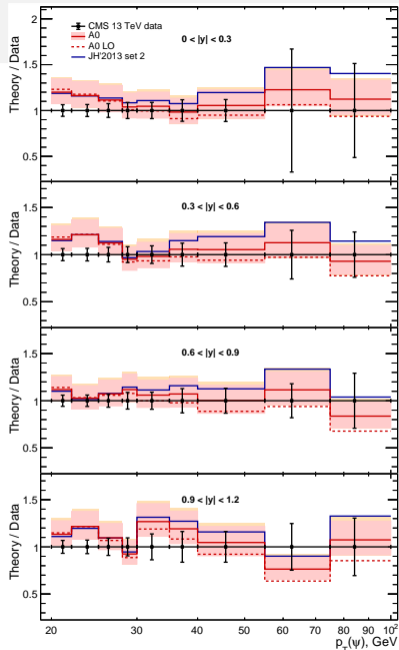
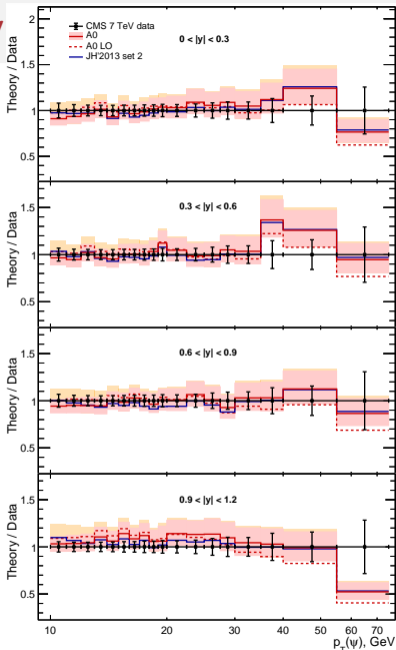


The fitted values of CO LDME's for ψ' meson

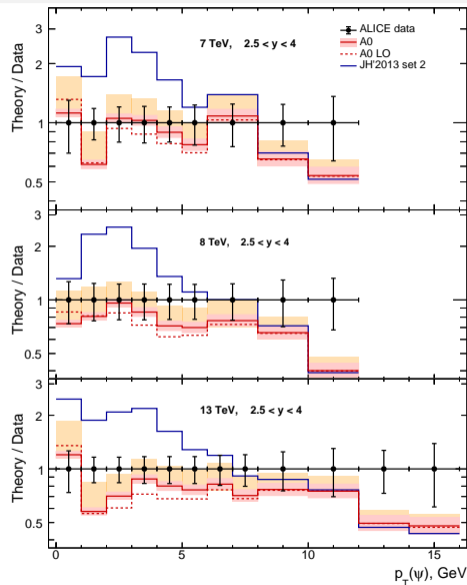
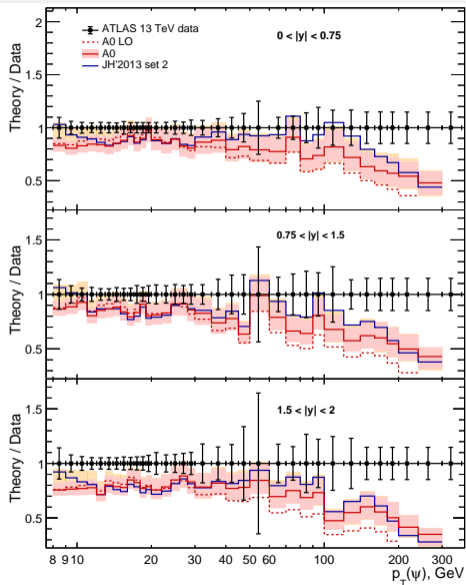
LDME	A0	JH'2013 set 2	JH'2013 set 2 ($p_T > 6$)
LO+NLO [†]			
$\langle \mathcal{O}_{l.c.} \rangle / \text{GeV}^5$	$(5.45 \pm 0.06) \times 10^{-3}$	$(5.59 \pm 0.07) \times 10^{-3}$	$(10.3 \pm 0.1) \times 10^{-3}$
$\langle \mathcal{O}[{}^3S_1^{[8]}] \rangle / \text{GeV}^3$	$(11.4 \pm 0.1) \times 10^{-4}$	$(11.3 \pm 0.1) \times 10^{-4}$	$(7.65 \pm 0.13) \times 10^{-4}$
LO			
$\langle \mathcal{O}_{l.c.} \rangle / \text{GeV}^5$	$(10.4 \pm 0.1) \times 10^{-3}$	$(8.64 \pm 0.09) \times 10^{-3}$	$(18.8 \pm 0.2) \times 10^{-3}$
$\langle \mathcal{O}[{}^3S_1^{[8]}] \rangle / \text{GeV}^3$	$(20.4 \pm 0.3) \times 10^{-4}$	$(29.1 \pm 0.2) \times 10^{-4}$	$(15.8 \pm 0.3) \times 10^{-4}$

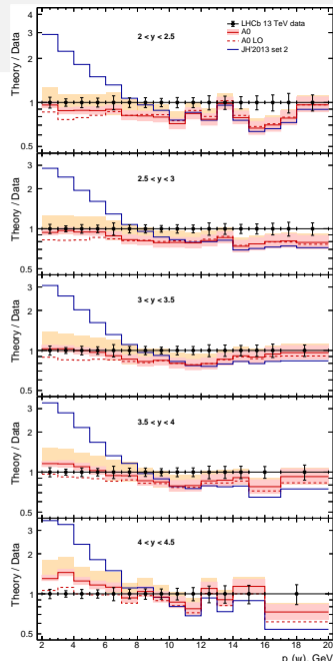
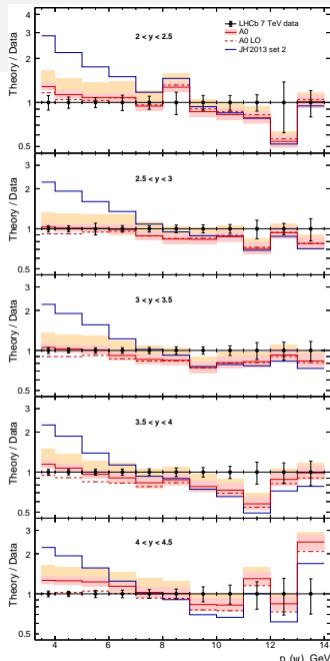
	n.d.f.	A0	JH'2013 set 2	JH'2013 set 2 ($p_T > 6$)
ATLAS 7 TeV [Eur. Phys. J. C 76, 283 (2016)]	168	1.12 1.12	3.77 5.92	1.21 1.63
ATLAS 8 TeV [Eur. Phys. J. C 76, 283 (2016)]	172	1.39 2.23	3.12 4.23	1.39 1.67
ATLAS 13 TeV [Eur. Phys. J. C 84, 169 (2024)]	90	5.62 6.01	10.2 15.7	4.76 4.41
CMS 7 TeV [Phys.Rev.Lett. 114, 191802 (2015)]	72	0.99 1.41	5.31 7.14	0.75 0.85
CMS 13 TeV [Phys. Lett. B 780, 251 (2018)]	36	1.91 2.12	11.3 19.1	2.38 3.57
LHCb 7 TeV [Eur. Phys. J. C 80, 185 (2020)]	55(40)	4.35 3.53	18.6 28.2	5.27 8.38
LHCb 13 TeV [Eur. Phys. J. C 80, 185 (2020)]	79(59)	4.56 4.43	27.1 42.6	5.17 8.49
ALICE 7 TeV [Eur. Phys. J. C 74, 2974 (2014)]	9(3)	1.09 1.39	2.79 6.26	1.96 2.38
ALICE 8 TeV [Eur. Phys. J. C 76, 184 (2016)]	9(3)	1.27 1.65	1.92 4.35	1.51 1.79
ALICE 13 TeV [Eur. Phys. J. C 77, 392 (2017)]	12(6)	2.01 2.95	4.06 8.56	1.33 1.71
ATLAS 5.02 TeV [Eur. Phys. J. C 78, 171 (2018)]	15	0.82 0.95	2.02 3.14	0.99 1.52
CMS 5.02 TeV [Phys. Lett. B 790, 509 (2019)]	10(8)	0.69 0.58	6.46 10.5	4.17 14.5
Summary	727(673)	2.39 2.65	8.59 13.2	2.35 3.13





ATLAS 13 TeV and ALICE



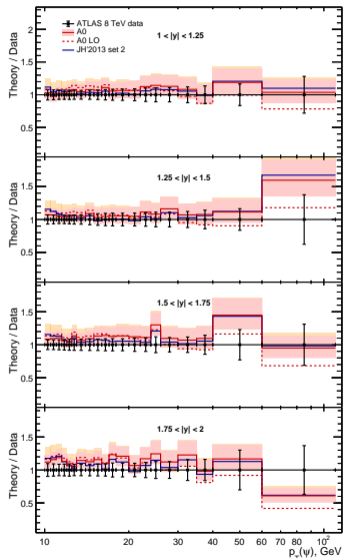
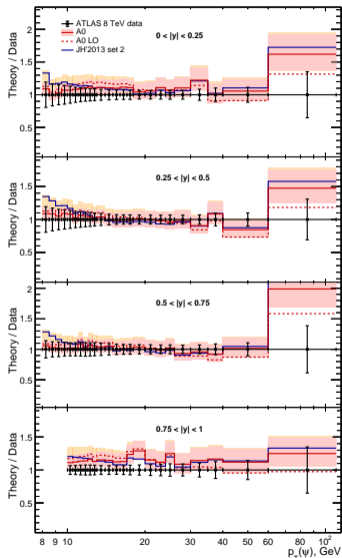


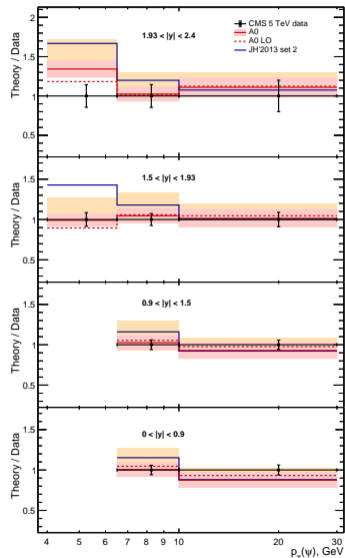
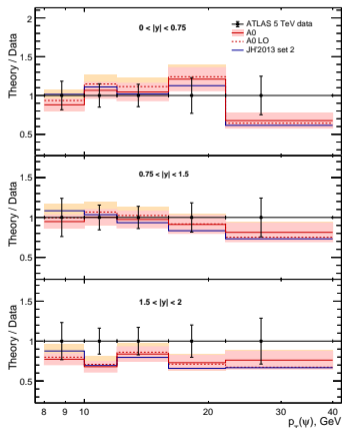
Summary

- ▶ Our work represents the first attempt to carry out a simultaneous description of all available unpolarized data on ψ' meson production in the entire kinematic region within high-energy factorization approach
- ▶ Tree-level NLO corrections allow to achieve a reasonable agreement with data in the whole kinematic range
- ▶ The modification for merging scheme of LO+NLO[†] are developed for the low p_T region
- ▶ Results with A0 and JH'2013-set-2 gluon densities shows no significant difference at $p_T > 6$ GeV
- ▶ Results with JH'2013-set-2 overestimate data at low $p_T < 6$ GeV

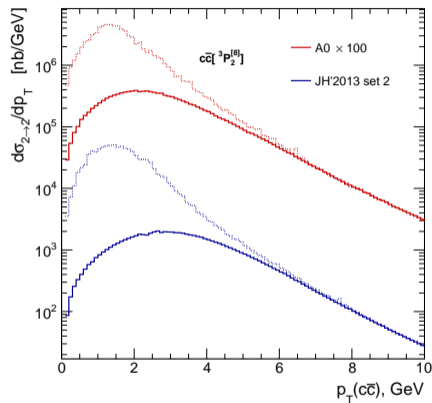
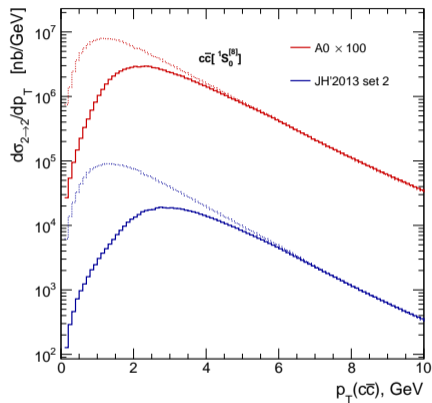
Our study was supported by the Russian Science Foundation under grant No. 25-22- 00066,
<https://rscf.ru/en/project/25-22-00066/>

Backup





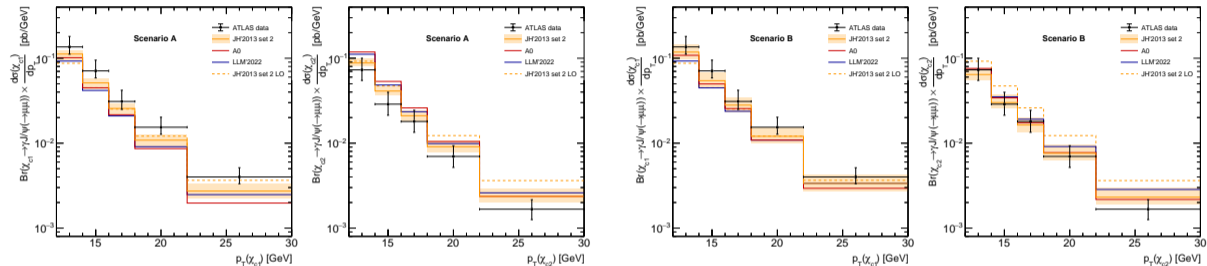
Comparison of merging LO + NLO[†] merging schemes



Comparison of $d\sigma_{2 \rightarrow 2}/dp_T$ contributions for color octet $c\bar{c} [^1S_0^{[8]}]$ (left panel) and $c\bar{c} [^3P_2^{[8]}]$ (right panel) with standard (dashed) and enhanced DCE cuts (solid) in the forward rapidity region $2 < y < 4.5$ at $\sqrt{s} = 13$ TeV

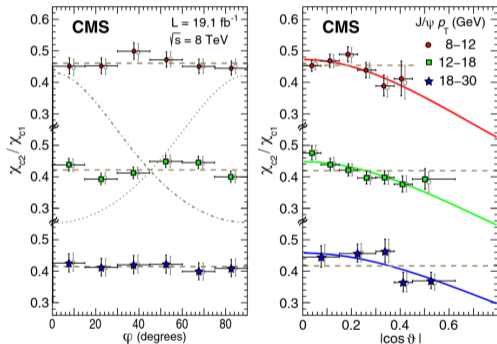
χ_{cJ} production: determination of $\langle \mathcal{O}^{\chi_{c0}} [{}^3S_1^{[8]}] \rangle$ (see [Eur.Phys.J.C 84, 348 (2024)])

Fit: ATLAS data $\sqrt{s} = 7$ TeV [JHEP 07, 154 (2014)] and CDF data $\sqrt{s} = 1.8$ TeV [Phys. Rev. Lett. 79, 578 (1997)]



The fitted values of LDME $\langle \mathcal{O}^{\chi_{c0}} [{}^3S_1^{[8]}] \rangle / \text{GeV}^3$

	Scenario A	$\chi^2/\text{n.d.f.}$	Scenario B	$\chi^2/\text{n.d.f.}$
JH'2013 set 2	$(3.07 \pm 0.89) \times 10^{-4}$	0.78	$(1.74 \pm 0.62) \times 10^{-4}$	0.39
A0	$(1.91 \pm 1.91) \times 10^{-4}$	1.8	$(1.32 \pm 0.49) \times 10^{-4}$	0.65
LLM'2022	$(4.84 \pm 0.87) \times 10^{-4}$	1.09	$(3.86 \pm 0.76) \times 10^{-4}$	1.18



- ▶ Parametrization of muon angular distributions:

$$\frac{d\sigma(J/\psi \rightarrow \mu\mu)}{d\cos\theta^* d\phi^*} \sim \frac{1}{3+\lambda_\theta} (1 + \lambda_\theta \cos^2\theta^* + \lambda_\phi \sin^2\theta^* \cos 2\phi^* + \lambda_{\theta\phi} \sin 2\theta^* \cos \phi^*)$$

- ▶ Correlation between the $\lambda_\theta^{\chi_{c1}}$ and $\lambda_\theta^{\chi_{c2}}$ parameters from CMS analysis:

$$\lambda_\theta^{\chi_{c2}} = (-0.94 + 0.90\lambda_\theta^{\chi_{c1}}) \pm (0.51 + 0.05\lambda_\theta^{\chi_{c1}}),$$

$$8 < p_T^{J/\psi} < 12 \text{ GeV},$$

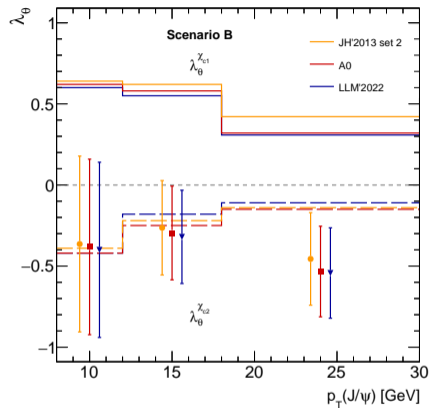
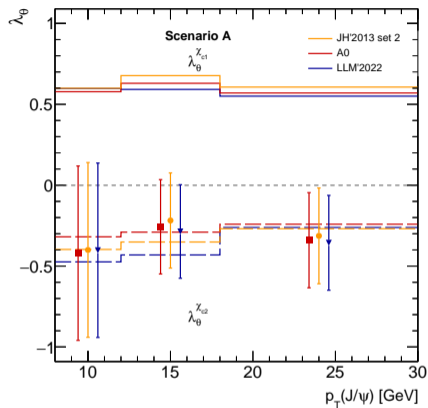
$$\lambda_\theta^{\chi_{c2}} = (-0.76 + 0.80\lambda_\theta^{\chi_{c1}}) \pm (0.26 + 0.05\lambda_\theta^{\chi_{c1}}),$$

$$12 < p_T^{J/\psi} < 18 \text{ GeV},$$

$$\lambda_\theta^{\chi_{c2}} = (-0.78 + 0.77\lambda_\theta^{\chi_{c1}}) \pm (0.26 + 0.06\lambda_\theta^{\chi_{c1}}),$$

$$18 < p_T^{J/\psi} < 30 \text{ GeV}.$$

χ_{cj} : polarization predictions (see [Eur.Phys.J.C 84, 348 (2024)])



- ▶ $\lambda_\theta^{X_{c2}}$ points (expected values) are obtained from the CMS relations.
- ▶ The $2 \rightarrow 2$ contribution provide lower polarization of the $^3P_J^{[1]}$ mesons as compared to the $2 \rightarrow 1$ contribution (NLO* $^3P_J^{[1]}$ more important in scenario B).