Radiative transitions of charmonium states in covariant confined quark model

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Charmonium states below $D\bar{D}$-threshold

J^{PC}	${}^{2S+1}L_J$	quark current	M_{cc} (MeV)	Γ_{cc}^{tot} (MeV)
0^{-+}	${}^1S_0 = \eta_c$	$\bar{q}\, i\gamma^5\, q$	2984.1(4)	30.5(5)
1^{--}	${}^3S_1 = J/\psi$	$\bar{q}\, \gamma^\mu\, q$	3096.900(6)	0.0926(17)
0^{++}	${}^3P_0 = \chi_{c0}$	$\bar{q}\, q$	3414.71(30)	10.5(8)
1^{++}	${}^3P_1 = \chi_{c1}$	$\bar{q}\, \gamma^\mu\gamma^5\, q$	3510.67(5)	0.88(5)
1^{+-}	${}^1P_1 = h_c(1P)$	$\bar{q}\ \overset{\leftrightarrow}{\partial}{}^\mu\ \gamma^5\, q$	3525.37(14)	0.78(28)
2^{++}	${}^3P_2 = \chi_{c2}$	$\frac{i}{2}\bar{q}\left(\gamma^\mu\ \overset{\leftrightarrow}{\partial}{}^\nu + \gamma^\nu\ \overset{\leftrightarrow}{\partial}{}^\mu\right) q$	3556.17(7)	2.00(11)
1^{--}	${}^3S_1 = \psi(2S)$	$\bar{q}\, \gamma^\mu\, q$	3686.097(11)	0.293(9)

Charmonium in covariant confined quark model

The covariant confined quark model (CCQM) is based on a relativistic Lagrangian describing the interaction of a hadron with its constituent quarks. The charmonium is described by a field $\phi_{cc}(x)$ which couples with an interpolating quark current $J_{cc}(x)$. The interaction Lagrangian reads as

$$\mathcal{L}_{\rm int}(x) = g_{cc}\, \phi_{cc}(x) \cdot J_{cc}(x).$$

The quark current $J_{cc}(x)$ is the nonlocal generalization of the quark currents shown in the above Table.

$$\begin{aligned} J_{cc}(x) &= \iint dx_1 dx_2\, F_{\rm cc}(x, x_1, x_2) \cdot \bar{c}(x_1)\, \Gamma_{cc}\, c(x_2), \\ F_{cc}(x, x_1, x_2) &= \delta(x - w\, x_1 - w\, x_2)\, \Phi_{cc}\left((x_1 - x_2)^2\right), \qquad (w = 1/2). \end{aligned}$$

Charmonium in covariant confined quark model

We use a simple Gaussian form for the Fourier transform of Φ_{cc}:

$$\widetilde{\Phi}_{cc}\left(-p^2\right) = \exp\left(s_{cc} \cdot p^2\right), \qquad s_{cc} \equiv 1/\Lambda_{cc}^2 ,$$

where Λ_{cc} is an adjustable charmonium size-related parameter of the CCQM.

For radial excitation $\psi(2S)$ the shape of the vertex function looks differently as

$$\widetilde{\Phi}_{2S}\left(-p^2\right) = (1 + c_1\, s_{2S}\, p^2)\exp\left(s_{2S} \cdot p^2\right), \qquad s_{2S} \equiv 1/\Lambda_{2S}^2 ,$$

Gauging of nonlocal quark current

Gauge invariance of the nonlocal strong interaction Lagrangian is provided by multiplying each quark field $q(x_i)$ by a gauge field exponentional according to

$$q(x_i) \to Q(x_i) = e^{-ie_q I(x_i,x,P)}\, q(x_i), \qquad I(x_i,x,P) = \int\limits_x^{x_i} dz_\mu A^\mu(z),$$

where P is the path taken from x to x_i.

It is readily seen that the neutral nonlocal quark current defined by

$$J^{\,\mathrm{em}}(x) = \iint dx_1 dx_2 \delta(x - wx_1 - wx_2)\Phi\left((x_1 - x_2)^2\right) \bar{Q}(x_1)\Gamma Q(x_2), \ \ (w = 1/2)$$

is invariant under the local gauge transformations

$$\begin{aligned} q(x_i) &\to e^{ie_q f(x_i)} q(x_i), \qquad \bar{q}(x_i) \to e^{-ie_q f(x_i)} \bar{q}(x_i), \\ A^\mu(z) &\to A^\mu(z) + \partial^\mu f(z), \Rightarrow I(x_i,x,P) \to I(x_i,x,P) + f(x_i) - f(x), \end{aligned}$$

if the matix Γ has no derivative.

Gauging of nonlocal quark current

Superficially the results appear to depend on the path P when one expands the gauge exponential in powers of $I(x_i, x, P)$. However, one needs to know only derivatives of the path integrals when doing the perturbative expansion. One can make use of the formalism developed in

S. Mandelstam, Annals Phys. 19, 1-24 (1962), J. Terning, Phys. Rev. D 44, no.3, 887-897 (1991)

and based on the path-independent definition of derivative of $I(x, y, P)$:

$$\frac{\partial}{\partial x^\mu} I(x, y, P) = A_\mu(x)$$

which states that the derivative of the path integral $I(x, y, P)$ does not depend on the path P originally used in the definition.

It is easy to check that such procedure of gauging the free quark lagrangian leads to the standard form of $e_q\, \bar{q}((x)\, \hat{A}(x) q(x)$.

Gauging of nonlocal quark current

The evaluation of the Feynman diagrams involving the strong quark vertex with emitting photon leads to the typical integral:

$$R(x;k_1,k_2) = \iint dx_1 dx_2\, \delta(x - w\,x_1 - w\,x_2)\Phi\left((x_1-x_2)^2\right) I(x_1,x_2)\, e^{ik_1x_1-ik_2x_2}$$

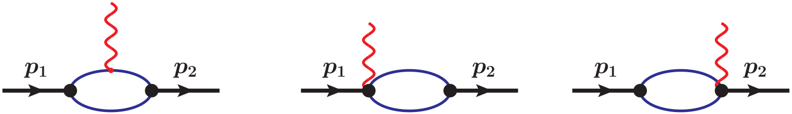
By using the definition of derivative of $I(x,y,P)$ and choosing the free electromagnetic field as $A_\alpha(x) = \epsilon^*_\alpha e^{iqx}$ one has

$$\begin{aligned} R(x;k_1,k_2) &= i\,\epsilon^*_\alpha\, e^{i(k_1-k_2+q)x} \\ &\times \int\limits_0^1 d\tau \left\{ \widetilde{\Phi}'\left(-z_\tau^+\right)(k+w^2q)^\alpha + \widetilde{\Phi}'\left(-z_\tau^-\right)(k-w^2q)^\alpha \right\} \end{aligned}$$

where $k = \frac{1}{2}(k_1+k_2)$ **and** $z_\tau^\pm = (k \pm w\,q)^2\,\tau + k^2\,1-\tau)$.

Radiative decays $(\bar{c}c)_1 \to (\bar{c}c)_2 + \gamma$: Feynman diagrams

Diagrams describing the $(\bar{c}c_1) \to (\bar{c}c)_2 + \gamma$ transition are shown below.



(a) (b) (c)

We start with the group of decays $\chi_{cJ}(p_1) \to J/\psi(p_2) + \gamma(q)$ $(J = 0, 1, 2)$. The invariant matrix element describing these decays are written as

$$M_{\chi_{cJ} \to J/\psi + \gamma} = 6\, g_{\chi_{cJ}}\, g_{J/\psi}\, e_q\, \epsilon^*_{2\,\beta}(p_2)\, \epsilon^*_{\gamma\,\alpha}(q) \left(M^{\beta\alpha}_{\triangle\,a} + M^{\beta\alpha}_{\bigcirc\,b} + M^{\beta\alpha}_{\bigcirc\,c} \right)$$

The amplitudes $M^{\beta\alpha}$ is written via the loop integrals corresponding to the Feynman diagrams.

Radiative decays $(\bar{c}c)_1 \to (\bar{c}c)_2 + \gamma$: loop integrals

$$
\begin{aligned}
M^{\beta\alpha}_{\triangle\, a} &= -\int \frac{d^4\ell}{(2\pi)^4 i}\,\widetilde{\Phi}_{cJ}\big(-(\ell+wp_1)^2\big)\,\widetilde{\Phi}_{J/\psi}\big(-(\ell+wp_2)^2\big)\\
&\quad\times \mathrm{tr}\big[\gamma^\beta S(\ell+p_2)\gamma^\alpha S(\ell+p_1)\tilde{\Gamma}_{cJ} S(\ell)\big]\\
M^{\beta\alpha}_{\bigcirc\, b} &= +\int \frac{d^4\ell}{(2\pi)^4 i}\int_0^1 d\tau\,\widetilde{\Phi}'_{cJ}(-z_\tau^+)\,\widetilde{\Phi}_{J/\psi}\big(-\ell^2\big)\,\ell^\alpha\\
&\quad\times \mathrm{tr}\big[\gamma^\beta S(\ell+wp_2)\tilde{\Gamma}_{cJ} S(\ell-wp_2)\big]\\
M^{\beta\alpha}_{\bigcirc\, c} &= +\int \frac{d^4\ell}{(2\pi)^4 i}\,\widetilde{\Phi}_{cJ}\big(-\ell^2\big)\int_0^1 d\tau\,\widetilde{\Phi}'_{J/\psi}(-z_\tau^+)\,\ell^\alpha\\
&\quad\times \mathrm{tr}\big[\gamma^\beta S(\ell+wp_1)\tilde{\Gamma}_{cJ} S(\ell-wp_1)\big]
\end{aligned}
$$

Here $\tilde{\Gamma}_{c0} = I,\qquad \tilde{\Gamma}_{c1} = \epsilon_\mu(p_1)\gamma^\mu\gamma_5,\quad \tilde{\Gamma}_{c2} = 2\,\epsilon_{\mu\nu}(p_1)\ell^\mu\gamma^\nu$

and $\quad z_\tau^+ = (\ell + w\,q)^2\,\tau + \ell^2\,(1-\tau)$

Radiative decays $(\bar{c}c)_1 \to (\bar{c}c)_2 + \gamma$: loop integrals

There is an additional diagram in the case of spin 2 due to the derivative in the Lagrangian. One has

$$M^{\beta\alpha}_{\bigcirc\, d} = -\int \frac{d^4\ell}{(2\pi)^4 i} \widetilde{\Phi}_{cJ}\big(-(\ell+wq)^2\big)\widetilde{\Phi}_{J/\psi}(-\ell^2)\,\epsilon_{\mu\nu}(p_1)\,g^{\nu\alpha} \times \mathrm{tr}\big[\gamma^\beta S(\ell+wp_2)\gamma^\mu S(\ell-wp_2)\big]$$

We have used the property of transversality and symmetry of the polarization vectors: $\epsilon^*_\alpha q^\alpha = 0$, $\epsilon_{\mu\nu} = \epsilon_{\nu\mu}$ and $\epsilon_{\mu\nu} p_1^\mu = 0$.

Check the gauge invariance

The first step is to check the gauge invariance before the loop integration, i.e.

$$M^{\beta\alpha}\, q_\alpha = 0.$$

It maybe done by using two identities:

$$\begin{aligned} S(\ell + p_2)\, \not{q}\, S(\ell + p_1) &= S(\ell + p_1) - S(\ell + p_2)\,, \\ \int_0^1 d\tau \widetilde{\Phi}'(-z_\tau)(\ell + w^2 q)^\alpha q_\alpha &= \widetilde{\Phi}(-\ell^2) - \widetilde{\Phi}(-(\ell + wq)^2) \end{aligned}$$

The second step is to reduce the loop integrals to the three-fold integrals which are evaluated numerically.

The calculation of the matrix elements of the decays $\psi(2S) \to \chi_{cJ} + \gamma$ and $J/\psi \to \eta_c + \gamma$, $h_c \to \eta_c + \gamma$ is performed in a similar manner.

Decay widths: $\chi_{c0} \to J/\psi + \gamma$ transition

$$\begin{aligned} M_{\chi_{c0}\to J/\psi+\gamma} &= e\, \epsilon^{*}_{2\,\beta}(p_2)\, \epsilon^{*}_{\gamma\,\alpha}(q)\, M^{\beta\alpha}_{\chi_{c0}} , \\ M^{\beta\alpha}_{\chi_{c0}} &= (m_1 A_1)\Big(g^{\beta\alpha} - \frac{q^{\beta} p_2^{\alpha}}{p_2 q}\Big), \qquad p_2 q = \frac{m_1^2 - m_2^2}{2} , \end{aligned}$$

$$\Gamma(\chi_{c0} \to J/\psi + \gamma) = \alpha\, |\mathrm{q}|\, A_1^2, \qquad |\mathrm{q}| = \frac{m_1^2 - m_2^2}{2m_1}.$$

Decay widths: $\chi_{c1} \to J/\psi + \gamma$ transition

There are two independent helicity amplitudes $H_{\lambda_1;\lambda_2\lambda}$ which we denote by H_i $(i = L, T)$ according to the helicity of the final meson state J/ψ, where $\lambda_2 = 0$ and $\lambda_2 = \pm 1$ stand for the longitudinal and transverse helicities of the J/ψ. From parity one has $H_{+;0-} = -H_{-;0+} = H_L$ and $H_{0;++} = -H_{0;--} = H_T$. The invariant matrix element is written as

$$\begin{aligned} M_{\chi_{c1}\to J/\psi+\gamma} &= e\,\epsilon_{1\,\mu}(p_1)\epsilon^*_{2\,\beta}(p_2)\,\epsilon^*_{\gamma\,\alpha}(q)\,M^{\mu\beta\alpha}_{\chi_{c1}} \\ H_L &= H_{+;0,-} = -H_{-;0,+} = \varepsilon_{1\,\mu}(+)\varepsilon^\dagger_{2\,\beta}(0)\bar{\varepsilon}^\dagger_\alpha(-)M^{\mu\beta\alpha}_{\chi_{c1}} \\ H_T &= H_{0;+,+} = -H_{0;-,-} = \varepsilon_{1\,\mu}(0)\varepsilon^\dagger_{2\,\beta}(+)\bar{\varepsilon}^\dagger_\alpha(+)M^{\mu\beta\alpha}_{\chi_{c1}} \end{aligned}$$

$$\Gamma(\chi_{c1} \to J/\psi + \gamma) = \frac{\alpha}{3}\,|\mathrm{q}|\left(|H_L|^2 + |H_T|^2\right).$$

Decay widths: $\chi_{c2} \to J/\psi + \gamma$ transition

There are three independent helicity amplitudes $H_{\lambda_1;\lambda_2\lambda}$ characterizing the decay $\chi_{c2} \to J/\psi + \gamma$.

$$\begin{aligned}
H_{+2;+1-1} &= H_{-2;-1+1} = \epsilon_{1\,\mu\nu}(+2)\epsilon^*_{2\,\beta}(+)\,\bar{\epsilon}^*_{\alpha}(-)\,M^{\mu\nu\beta\alpha}_{\chi_{c2}}\,, \\
H_{+1;0-1} &= H_{-1;0+1} = \epsilon_{1\,\mu\nu}(+1)\epsilon^*_{2\,\beta}(0)\,\bar{\epsilon}^*_{\alpha}(-)\,M^{\mu\nu\beta\alpha}_{\chi_{c2}}\,, \\
H_{0;+1+1} &= H_{0;-1-1} = \epsilon_{1\,\mu\nu}(0)\epsilon^*_{2\,\beta}(+)\,\bar{\epsilon}^*_{\alpha}(+)\,M^{\mu\nu\beta\alpha}_{\chi_{c2}}\,.
\end{aligned}$$

The invariant matrix element $M^{\mu\nu\beta\alpha}_{\chi_{c2}}$ is represented in terms of the five form factors. By using the gauge invariance the number of the form factors is reduced to three.

$$\begin{aligned}
M_{\chi_{c2}\to J/\psi+\gamma} &= e\,\epsilon_{1\,\mu\nu}(p_1)\epsilon^*_{2\,\beta}(p_2)\,\epsilon^*_{\gamma\,\alpha}(q)\,M^{\mu\nu\beta\alpha}_{\chi_{c2}} \\
M^{\mu\nu\beta\alpha}_{\chi_{c2}} &= F_1\,(p_2^{\alpha}q^{\beta} - p_2q\,g^{\alpha\beta})q^{\mu}q^{\nu} + F_2\,(p_2^{\alpha}q^{\nu} - p_2q\,g^{\nu\alpha})g^{\mu\beta} \\
&\quad + F_3(g^{\mu\alpha}q^{\nu}q^{\beta} - g^{\alpha\beta}q^{\mu}q^{\nu})
\end{aligned}$$

Decay widths: $\chi_{c2} \to J/\psi + \gamma$ transition

The decay width via helicity amplitudes:

$$\Gamma(\chi_{c2} \to J/\psi + \gamma) \quad = \quad \frac{\alpha}{5}\frac{|\mathrm{q}|}{m_1^2}\left(|H_{+2;+1-1}|^2 + |H_{+1;0-1}|^2 + |H_{0;+1+1}|^2\right),$$

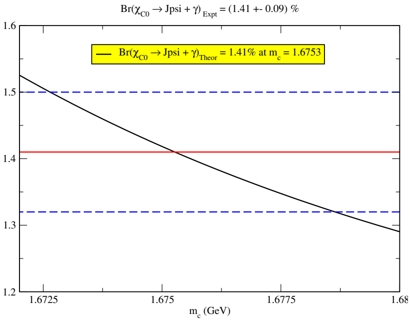
$$\begin{aligned} H_{+2;+1-1} &= -m_1|\mathrm{q}|\,F_2\,, \\ H_{+1;0-1} &= -\frac{1}{\sqrt{2}}\frac{m_1}{m_2}|\mathrm{q}|\left(E_2\,F_2 + |\mathrm{q}|\,F_3\right), \\ H_{0;+1+1} &= -\sqrt{\frac{2}{3}}m_1\,|\mathrm{q}|\left(|\mathrm{q}|^2 F_1 + \frac{1}{2}\,F_2 + \frac{|\mathrm{q}|}{m_1}\,F_3\right). \end{aligned}$$

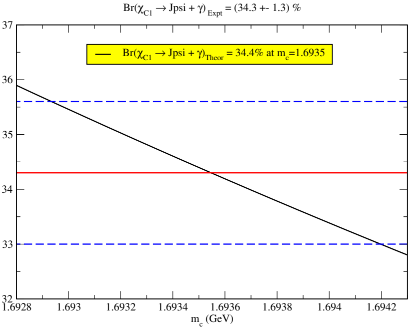
Numerical results

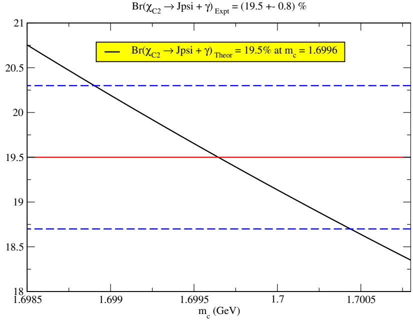
We observed that the charmonium radiative decay widths depend rather slowly on their size parameters Λ_{cc} in the interval [2,4] GeV. But the dependence on the charm quark mass m_c running in the loop is strong.

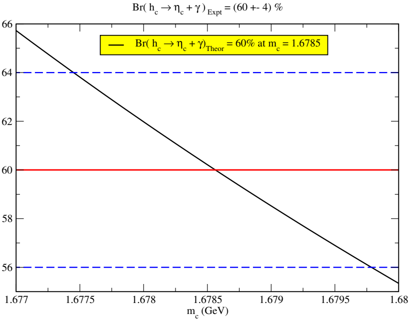
Therefore, we assume that the values of the size parameters Λ_{cc} are equal to the charmonium masses, i.e. $\Lambda_{cc} = M_{cc}$.

We then determine the values of the charm quark mass running in the loop in such a way as to fit the calculated branching fraction for the given decay mode to its experimental value within the experimental error bars.

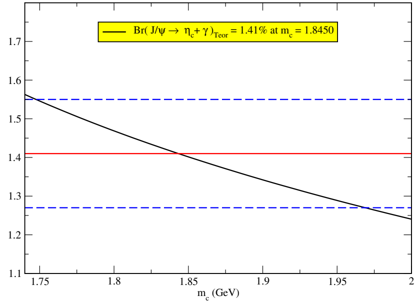

$\mathrm{Br}(\chi_{C0} \rightarrow \mathrm{Jpsi} + \gamma)_{\mathrm{Expt}} = (1.41 \text{ +- } 0.09)$ %


$\mathrm{Br}(\chi_{C1} \rightarrow \mathrm{Jpsi} + \gamma)_{\mathrm{Expt}} = (34.3 \text{ +- } 1.3)$ %

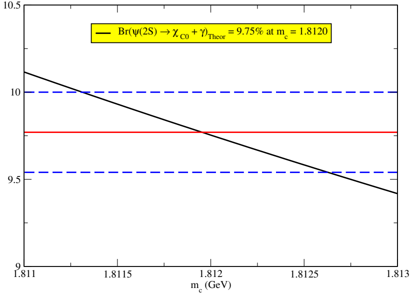

$\mathrm{Br}(\chi_{C2} \rightarrow \mathrm{Jpsi} + \gamma)_{\mathrm{Expt}} = (19.5 \text{ +- } 0.8)$ %


$\mathrm{Br}(h_c \rightarrow \eta_c + \gamma)_{\mathrm{Expt}} = (60 \text{ +- } 4)$ %

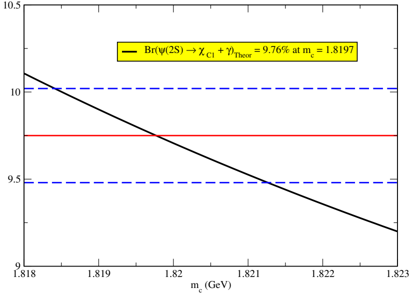
$Br(J/\psi \to \eta_c + \gamma)_{Expt} = (1.41 \pm 0.14)$ %



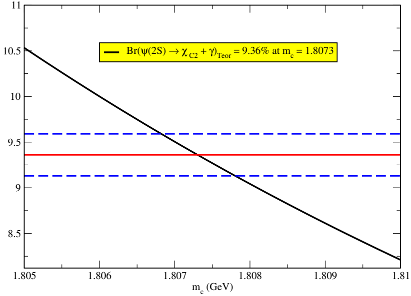
$Br(\psi(2S) \to \chi_{C0} + \gamma)_{Expt} = (9.77 \pm 0.23)$ %



$Br(\psi(2S) \to \chi_{C1} + \gamma)_{Expt} = (9.75 \pm 0.27)$ %



$Br(\psi(2S) \to \chi_{C2} + \gamma)_{Expt} = (9.36 \pm 0.23)$ %



Mode	m_c (GeV)	CCQM	Expt. (PDG'24)
$\chi_{c0} \to J/\psi + \gamma$	1.6753(27)	1.41(9)	1.41(9)
$\chi_{c1} \to J/\psi + \gamma$	1.6935(6)	34.4(1.4)	34.3(1.3)
$\chi_{c2} \to J/\psi + \gamma$	1.6996(7)	19.4(7)	19.5(8)
$h_c \to \eta_c + \gamma$	1.6785(11)	60(4)	60(4)
$J/\psi \to \eta_c + \gamma$	1.845(97)	1.41(14)	1.41(14)
$\psi(2S) \to \chi_{c0} + \gamma$	1.8120(6)	9.75(21)	9.77(23)
$\psi(2S) \to \chi_{c1} + \gamma$	1.8197(13)	9.76(26)	9.75(27)
$\psi(2S) \to \chi_{c2} + \gamma$	1.8073(5)	9.36(23)	9.36(23)

Fitting procedure

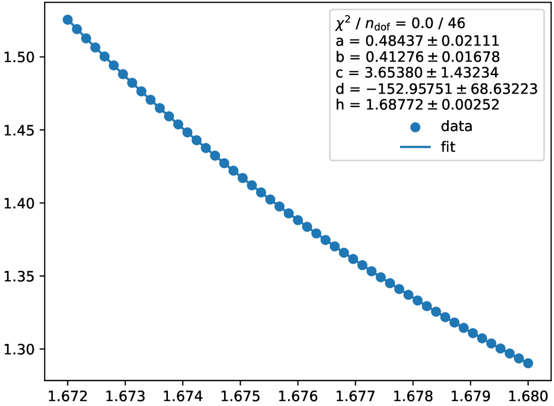
To determine uncertainties we have used iminuit which is a Python frontend to the Minuit2 library in C++, an integrated software that combines a local minimizer (called MIGRAD) and two error calculators (called HESSE and MINOS).

We were aiming to identify precision of our calculation with respect of the charm-quark mass. For proper describing of the data, we used three different fitting function

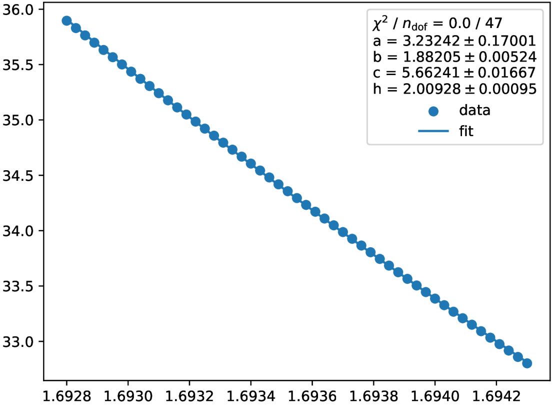
$$f(x) = \begin{cases} a/(b + c(x-h) + d(x-h)^2) \\ a/(b + c(x-h)) \\ a + \exp(b(x-h)) \end{cases} .$$

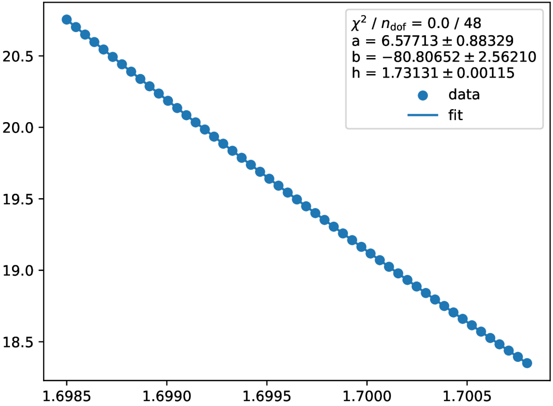
It was used the assumption that our analytical calculation has uncertainties 3 %.

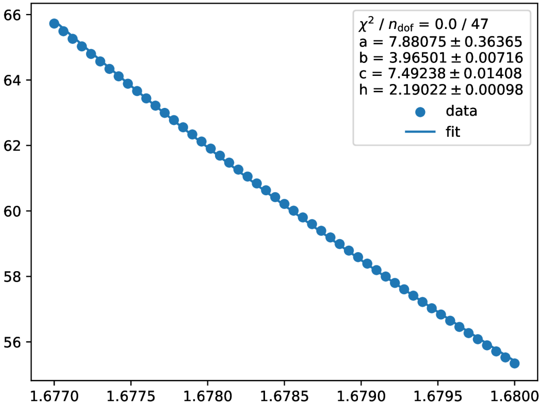
Fitting parameters

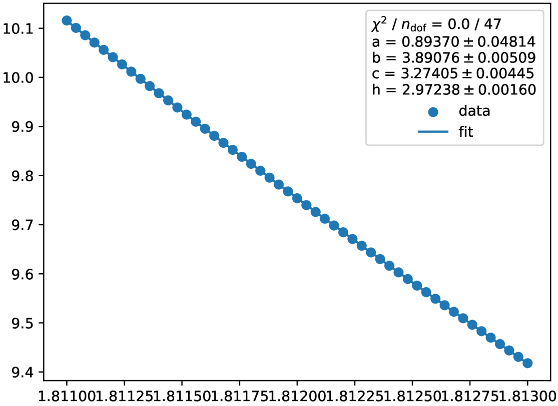


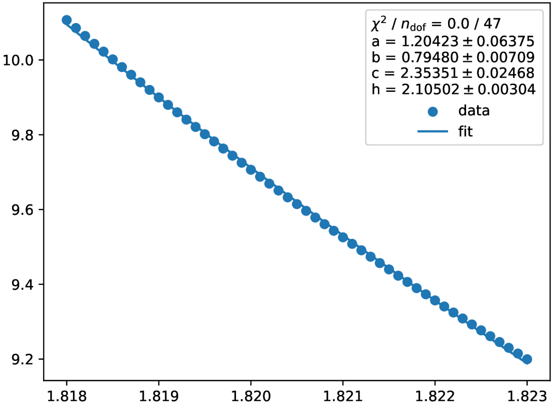
Fitting parameters

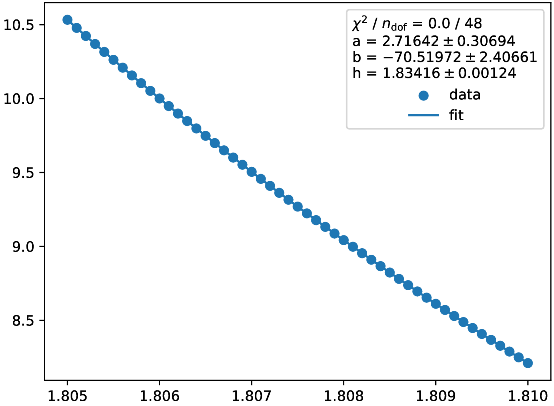


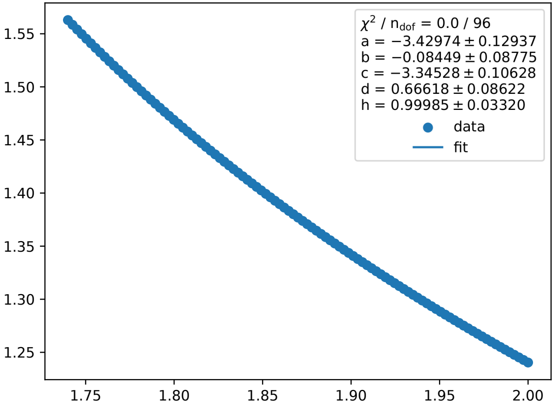
$\chi^2 / n_{\mathrm{dof}} = 0.0 / 48$
$a = 6.57713 \pm 0.88329$
$b = -80.80652 \pm 2.56210$
$h = 1.73131 \pm 0.00115$
data
fit
20.5
20.0
19.5
19.0
18.5
1.6985
1.6990
1.6995
1.7000
1.7005


χ^2 / n_{dof} = 0.0 / 47
a = 7.88075 ± 0.36365
b = 3.96501 ± 0.00716
c = 7.49238 ± 0.01408
h = 2.19022 ± 0.00098
data
fit
66
64
62
60
58
56
1.6770
1.6775
1.6780
1.6785
1.6790
1.6795
1.6800


χ^2 / n_{dof} = 0.0 / 47
a = 0.89370 ± 0.04814
b = 3.89076 ± 0.00509
c = 3.27405 ± 0.00445
h = 2.97238 ± 0.00160
data
fit
10.1
10.0
9.9
9.8
9.7
9.6
9.5
9.4
1.81100
1.81125
1.81150
1.81175
1.81200
1.81225
1.81250
1.81275
1.81300


χ^2 / n_{dof} = 0.0 / 47
a = 1.20423 ± 0.06375
b = 0.79480 ± 0.00709
c = 2.35351 ± 0.02468
h = 2.10502 ± 0.00304
data
fit
10.0
9.8
9.6
9.4
9.2
1.818
1.819
1.820
1.821
1.822
1.823


χ^2 / n_{dof} = 0.0 / 48
a = 2.71642 ± 0.30694
b = −70.51972 ± 2.40661
h = 1.83416 ± 0.00124
data
fit
10.5
10.0
9.5
9.0
8.5
1.805
1.806
1.807
1.808
1.809
1.810


χ^2 / n_{dof} = 0.0 / 96
a = −3.42974 ± 0.12937
b = −0.08449 ± 0.08775
c = −3.34528 ± 0.10628
d = 0.66618 ± 0.08622
h = 0.99985 ± 0.03320
data
fit
1.55
1.50
1.45
1.40
1.35
1.30
1.25
1.75
1.80
1.85
1.90
1.95
2.00

Fitting results for branching ratios (in %) with uncertainties.

Decay mode	CCQM	Exp.
$\chi_{c0} \to J/\psi\gamma$	1.41 ± 0.15	1.41 ± 0.09
$\chi_{c1} \to J/\psi\gamma$	34.4 ± 3.8	34.3 ± 1.3
$\chi_{c2} \to J/\psi\gamma$	19.5 ± 1.8	19.5 ± 0.8
$J/\psi \to \eta_c\gamma$	1.41 ± 0.11	1.41 ± 0.14
$\Psi(2S) \to \chi_{c0}\gamma$	9.75 ± 1.09	9.77 ± 0.23
$\Psi(2S) \to \chi_{c1}\gamma$	9.77 ± 1.10	9.75 ± 0.27
$\Psi(2S) \to \chi_{c2}\gamma$	9.36 ± 0.79	9.36 ± 0.23
$h_c \to \eta_c(1S)\gamma$	60.2 ± 6.4	60 ± 4

Summary

We have calculated the amplitudes and branching ratios of radiative decays of charmonium states: $\chi_{c0,c1,c2} \to J/\psi\gamma$, $\psi(2S) \to \chi_{c0,c1,c2}\gamma$, $h_c \to \eta_c\gamma$ and $J/\psi \to \eta_c\gamma$ in the framework of covariant confined quark model (CCQM).

We have applied the method of electromagnetic gauging of the nonlocal Lagrangian by using a gauge field exponential and the path-independent definition of its derivative.

We observed that the charmonium radiative decay widths depend rather slowly on their size parameters.

Therefore, we assumed that the values of the size parameters are equal to the charmonium masses.

The values of the charm quark masses running in the loop were determined by fitting the calculated branching fractions for the given decay mode to its experimental value within the experimental error bars.