The XXV International Workshop-School
High Energy Physics and
Quantum Field Theory
QFTHEP-270

On description of neutrino oscillation within QFT

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04.07.2025

Introduction

Neutrino oscillations: the standard QM description in terms of plane waves or wave packets
➔ inconsistent, since the neutrino flavor states cannot be understood as true quantum states.

QFT description in terms of wave packets
➔ cumbersome calculations.

The reason is:
The standard S-matrix formalism is not appropriate for describing the processes passing at finite distances and lasting finite time intervals,
$t, L \longrightarrow \pm\infty$.

We use a modified perturbative formalism that allows one to describe processes passing at finite distances during finite time intervals.

The approach is based on the Feynman diagram technique in the coordinate representation supplemented by modified rules of passing to the momentum representation. The latter reflect the geometry of neutrino oscillation experiments and lead to a modification of the Feynman propagators of the neutrino mass eigenstates in the momentum representation.

The idea behind the approach comes from the paper

- R.P. Feynman, “Space-Time Approach to Quantum Electrodynamics,”
  Phys. Rev. **76**, 769 (1949)

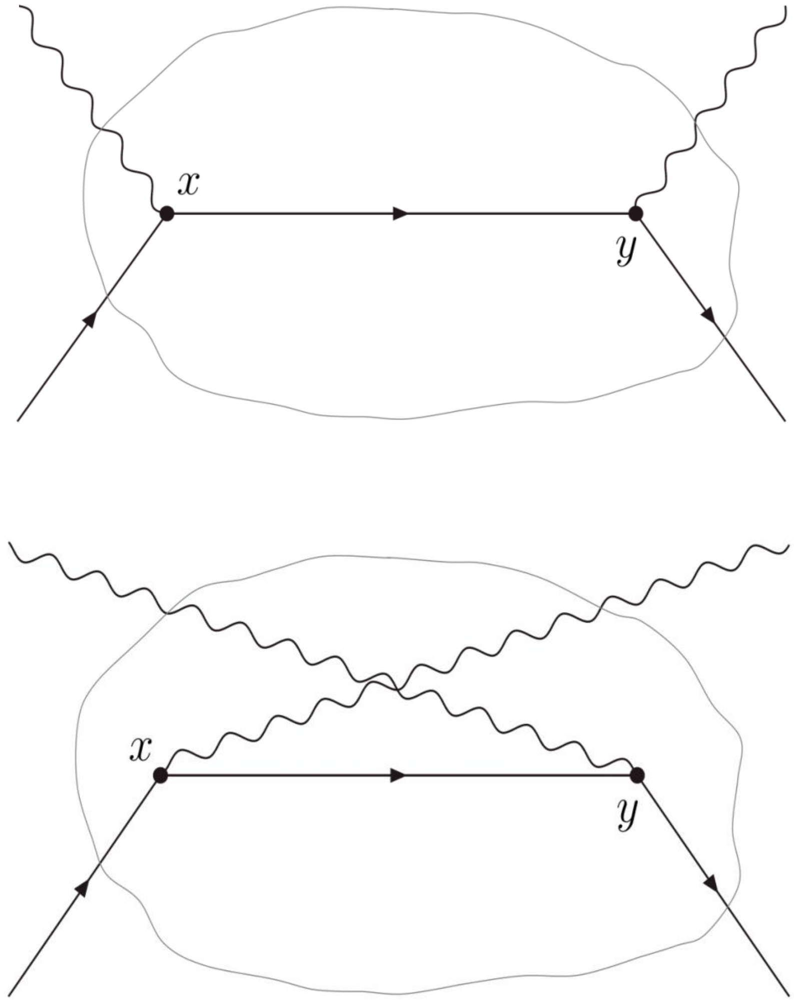
and was first presented in the paper

- I.P. Volobuev, “Quantum field-theoretical description of neutrino and neutral kaon oscillations,”
  Int. J. Mod. Phys. A **33**, 1850075 (2018).

This approach was further developed in the papers by I.P. Volobuev and V.O. Egorov:

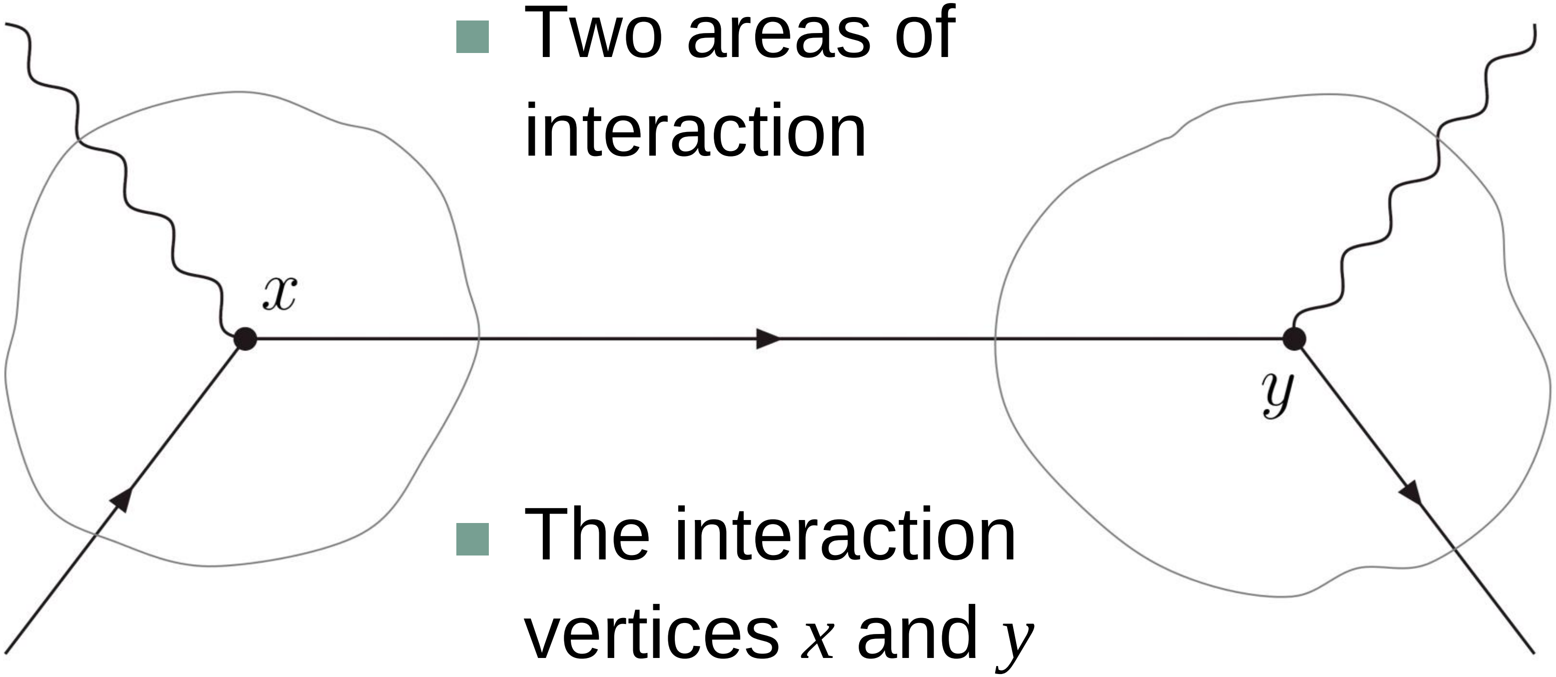
- Phys. Rev. D **97**, 093002 (2018),
- Theor. and Math. Phys. **199**, 562 (2019) / ТМФ **199**, 104 (2019),
- JETP **128**, 713 (2019) / ЖЭТФ **155**, 839 (2019),
- Phys. Rev. D **100**, 033004 (2019),
- JETP **135**, 197 (2022) / ЖЭТФ **162**, 226 (2022).

Standard scattering process



- One single common area of interaction
- No restrictions on the position of the interaction vertices x and y

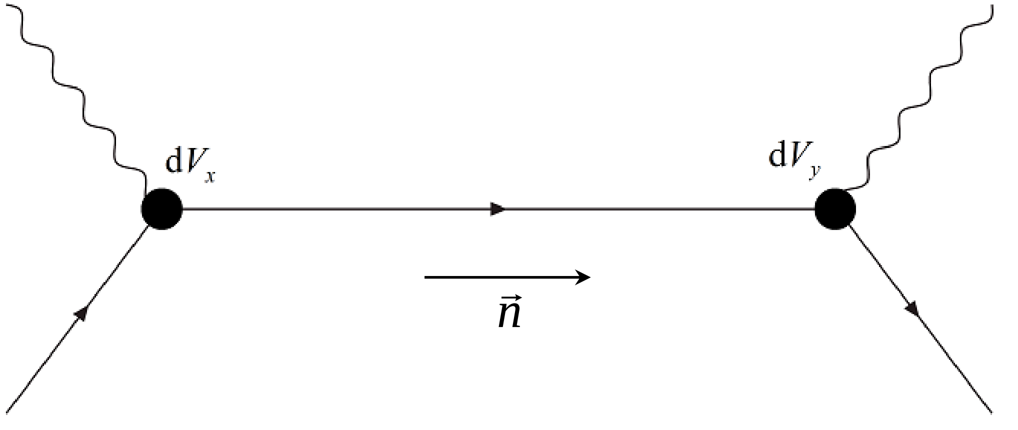
Scattering process in neutrino oscillation experiments



- Two areas of interaction
- The interaction vertices x and y are ordered in space and time

Distance-dependent propagators

To take into account the geometry of the experiment we have to integrate with respect to the coordinates x and y in such a way that the distance between these points along the unit vector $\vec{n}$ directed from the source to the detector is fixed and equal to L.



This can be achieved by introducing the delta function $\delta(\vec{n}(\vec{y}-\vec{x})-L)$ into the integrand. Formally, this is equivalent to replacing the standard Feynman fermion propagator $S^{c}(y-x)$ by $S^{c}(y-x)\delta(\vec{n}(\vec{y}-\vec{x})-L)$.

The Fourier transform of this product is called *the distance-dependent fermion propagator in the momentum representation*:

$$S^{c}\left(p,\vec{L}\right) \equiv \int d^{4}z\ e^{ipz}\, S^{c}\left(z\right)\,\delta\left(\vec{n}\vec{z}-L\right).$$

This integral can be evaluated exactly. In the approximation of the proximity to the mass shell:

$$S^{c}\left(p,\vec{L}\right) = i\,\frac{\hat{p}+m}{2\,\vec{p}\vec{n}}\, e^{-i\frac{m^{2}-p^{2}}{2\,\vec{p}\vec{n}}L}.$$

Neutrino oscillations in vacuum (charged current)

We work in the minimal extension of the SM by right neutrino singlets,

Neutrino mixing matrix (PMNS matrix)

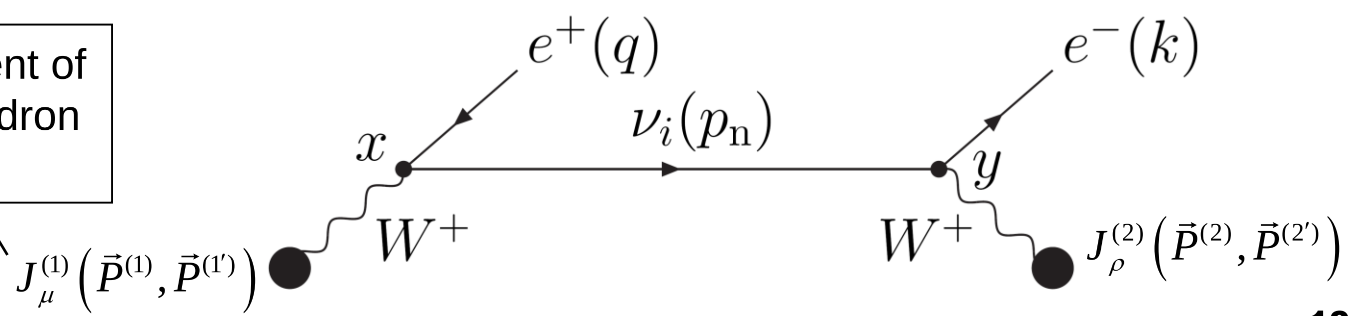
$$L_{\mathrm{cc}} = -\frac{g}{2\sqrt{2}}\left(\sum_{\alpha=e,\mu,\tau}\sum_{i=1}^{3}\overline{l_\alpha}\,\gamma^\mu\left(1-\gamma^5\right)U_{\alpha i}\,\nu_i\,W_\mu^- + \mathrm{H.c.}\right).$$

Field of the charged lepton α

Neutrino field with a definite mass m_i

Let us consider the simplest process

Matrix element of the weak hadron current



The amplitude in the momentum representation:

$$M = -\mathrm{i}\,\frac{G_{\mathrm{F}}^2}{4\vec{p}_{\mathrm{n}}\vec{n}}\left(\sum_{i=1}^{3}\left|U_{ei}\right|^2 \mathrm{e}^{-\mathrm{i}\frac{m_i^2-p_{\mathrm{n}}^2}{2\vec{p}_{\mathrm{n}}\vec{n}}L}\right) J_{\rho}^{(2)}\left(\vec{P}^{(2)},\vec{P}^{(2')}\right)\times$$

$$\times\bar{u}\left(\vec{k}\right)\gamma^{\rho}\left(1-\gamma^{5}\right)\hat{p}_{\mathrm{n}}\,\gamma^{\mu}\left(1-\gamma^{5}\right)\upsilon\left(\vec{q}\right)\,J_{\mu}^{(1)}\left(\vec{P}^{(1)},\vec{P}^{(1')}\right).$$

$|M|^2$ factorizes. To find the probability we need:

- to multiply it by the delta function of 4-momentum conservation (as usually),
- also multiply by $4\pi\,\delta(\vec{p}_{\mathrm{n}}-\vec{p})\,\theta(p^0)\,\delta(p^2)$, where $\vec{p}=|\vec{p}|\,\vec{n}$,
- and then integrate with respect to the momenta of the final particles, nuclei, p^0 and $|\vec{p}|$.

As a result, we obtain the probability of registering an electron in the process under consideration:

$$\frac{\mathrm{d}W}{\mathrm{d}\Omega} = \int_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{\mathrm{d}^3 W_{\mathrm{P}}}{\mathrm{d}^3 p} W_{\mathrm{D}} P_{ee}\left(|\vec{p}|, L\right) |\vec{p}| \mathrm{d}|\vec{p}|.$$

Probability of the production of $\nu(\vec{p})$

Detection probability

Standard oscillating factor (survival probability)

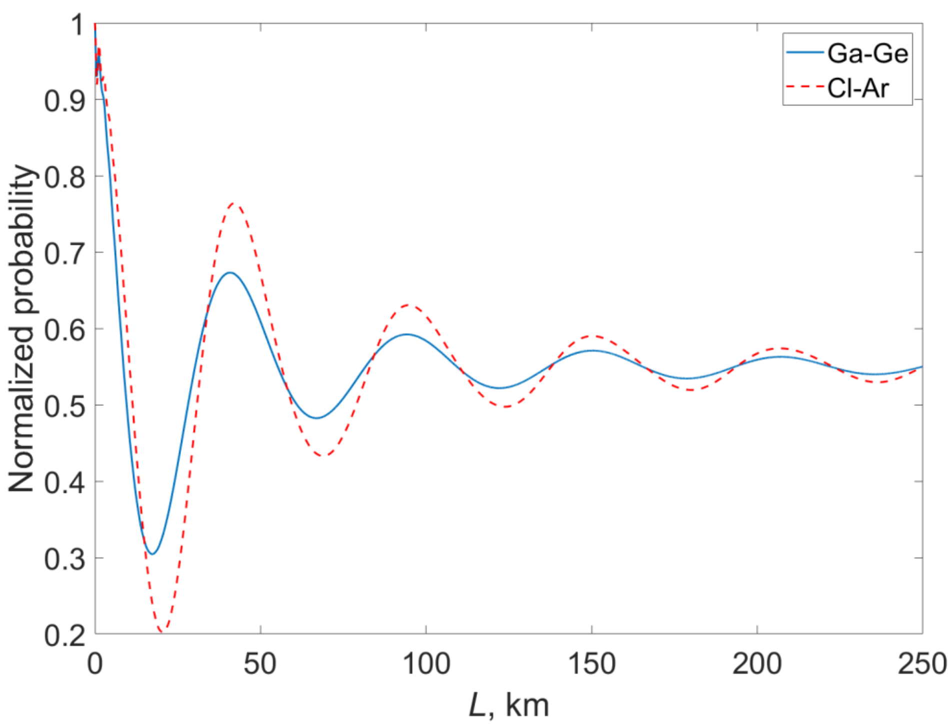
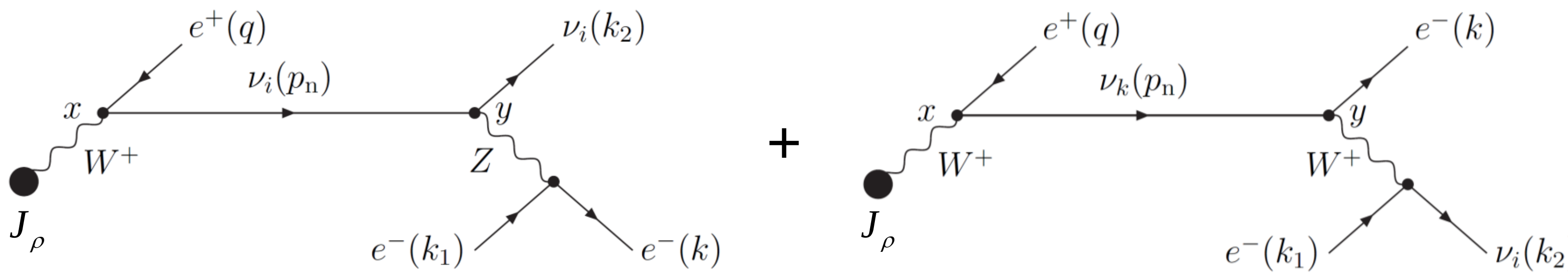


Illustration: normalized probabilities of neutrino oscillation processes with neutrino production in ^{15}O decay and registration by Cl-Ar and Ga-Ge detectors.

Neutral currents

In the same manner one can consider the processes



The differential probability has the form

$$\frac{\mathrm{d}W}{\mathrm{d}\Omega} = \int_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{\mathrm{d}^3 W_{\mathrm{P}}}{\mathrm{d}^3 p} W_{\mathrm{D}}(L)\, |\vec{p}|\,\mathrm{d}|\vec{p}|,$$

Probabilities of the corresponding interactions calculated using the Standard Model

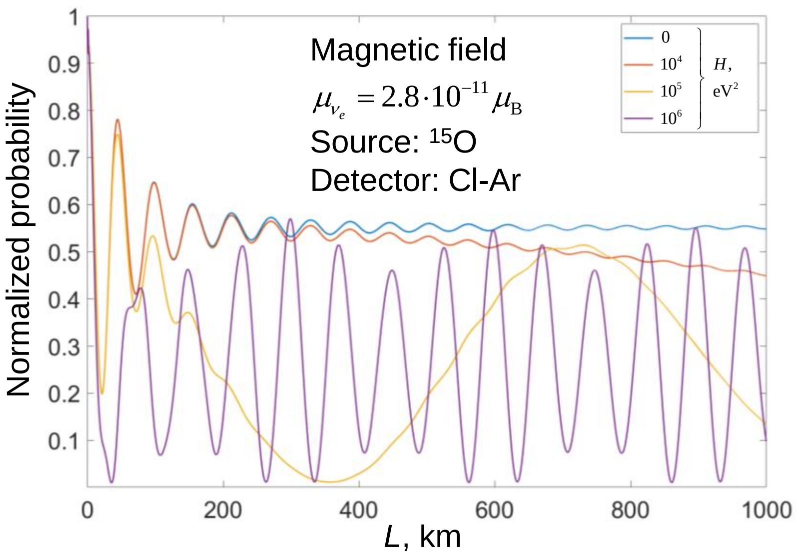
$$W_{\mathrm{D}}(L) = P_{ee}\left(|\vec{p}|, L\right) W_{\nu_e e} + \left[1 - P_{ee}\left(|\vec{p}|, L\right)\right] W_{\nu_\mu e}.$$

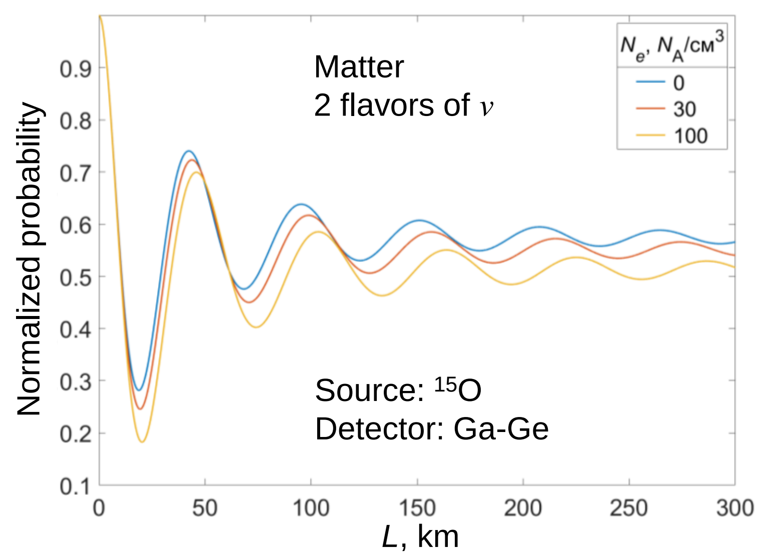
Oscillations in field and matter

Substituting the Green's function of neutrino in field or matter into the definition of the distance-dependent propagator

$$G_{\alpha\beta}\left(p,\vec{L}\right) \equiv \int \mathrm{d}^4 z \; \mathrm{e}^{ipz} \, G_{\alpha\beta}\left(z\right) \delta\left(\vec{n}\vec{z} - L\right)$$

and following the same procedure, one can describe these phenomena as well.





Conclusion

- A new QFT approach is applied to describe neutrino oscillations.
- The approach allows one to consistently describe oscillations in vacuum, field or matter, and to consider processes with charged and neutral weak currents.
- The advantages of the approach are technical simplicity and physical transparency. Particles are described by plane waves.

Thank you for your attention!

Backup slides

M. Agostini *et al.* [BOREXINO], «Comprehensive measurement of pp-chain solar neutrinos», Nature **562** (2018) no. 7728, 505–510, doi:10.1038/s41586-018-0624-y

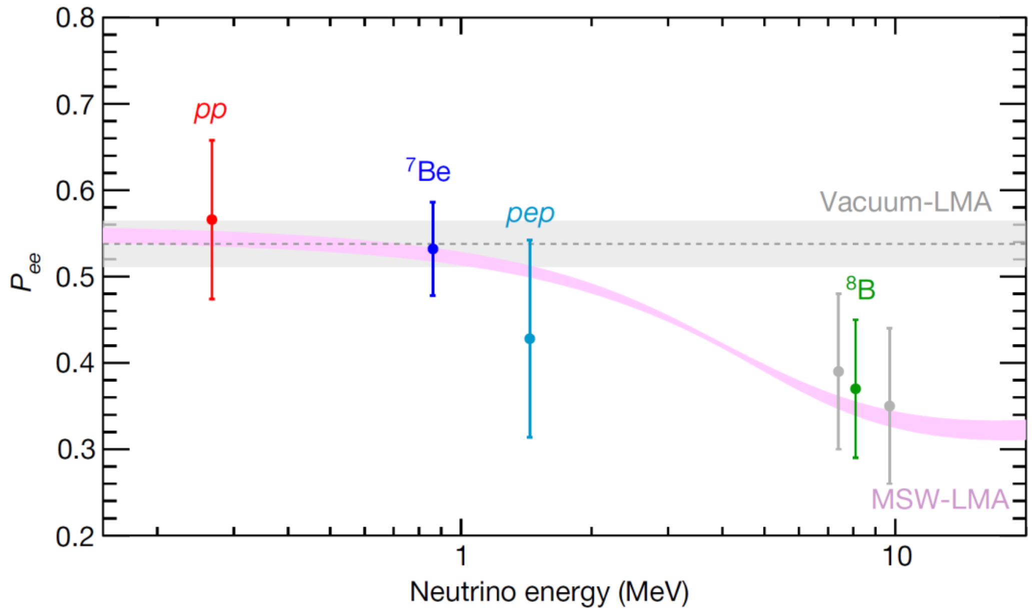


Fig. 3 | Electron neutrino survival probability P_{ee} as a function of neutrino energy. The pink band is the $\pm 1\sigma$ prediction of MSW-LMA with oscillation parameters determined from ref. [19]. The grey band is the vacuum-LMA case with oscillation parameters determined from refs [38,39]. Data points represent the Borexino results for *pp* (red), ^{7}Be (blue), *pep* (cyan) and ^{8}B (green for the HER range, and grey for the separate HER-I and HER-II sub-ranges), assuming HZ-SSM. ^{8}B and *pp* data points are set at the mean energy of neutrinos that produce scattered electrons above the detection threshold. The error bars include experimental and theoretical uncertainties.