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Excited states in spaces of constant curvature

Plan

1. Definitions and motivation
2. Local quench
3. Local quench in curved spaces:
    a. Anti-de Sitter (AdS)
    b. BTZ black hole
4. Holography and BDHM dictionary
5. de Sitter space

Nonequilibrium dynamics: quenches

Quantum quench - nonequilibrium process following (a smooth or instantaneous) change of parameters.

Global quench - это homogeneous change of parameters, for example, coupling constant or chemical potential. [Calabrese and Cardy, `06; Das, Galante, Myers, `15]

Local quench - local change of parameter or local interaction/perturbation. [Calabrese and Cardy, `07]

Motivation

- General consideration in nonequilibrium quantum field theory [Das, Galante, Myers, `15]
- Applications:
  Condensed matter physics
- High energy physics, information theory, early Universe physics,
- Duality in AdS/CFT and dS/CFT (entanglement entropy, thermalization)
  [Balasubramanian, `11; Nozaki, Numasawa, Takayanagi, `13;
  Hartman, Maldacena, `13;
  Ageev and Aref’eva, `16, `18, `19;
  Ageev, `19 и многие другие]

We study the dynamics of energy density evolution following a local quench in the theory of a massive scalar field in the background metric of AdS space, BTZ black hole and dS space.

Conformal field theory	AdS
Global quench	Black hole formation
Local quench	Falling particle

Operator local quench

[Nozaki, Numasawa, Takayanagi, `14]

A convenient and universal way to introduce and study local perturbations.

A local operator acting on the ground state creates a locally excited state

$$|\Psi(t)\rangle = \mathcal{N}_O \cdot e^{-iH(t-t_0)} \cdot e^{-\varepsilon H} O(t_0, x_0)|0\rangle$$

Observable

Quench operator

$$\langle \mathcal{O}(t,x)\rangle_\Psi = \frac{\langle\Psi|\mathcal{O}(t,x)|\Psi\rangle}{\langle\Psi|\Psi\rangle} \longrightarrow \langle \mathcal{O}(t,x)\rangle_\Psi = \frac{\langle 0|O(i\varepsilon,0)\mathcal{O}(x,t)O(-i\varepsilon,0)|0\rangle}{\langle 0|O(i\varepsilon,0)O(-i\varepsilon,0)|0\rangle}$$

Free massive scalar field in AdS background

$$S = -\frac{1}{2}\int d^3x\sqrt{g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right)$$

Metric in Poincare coordinates:

$$ds^2 = \frac{L^2}{z^2}\left[dz^2 + d\tau^2 + dx^2\right]$$

Energy density:

$$T_{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta S}{\delta g^{\mu\nu}} \; ; \; T_{\tau\tau} \equiv \mathcal{E} = \frac{1}{2}\left(-(\partial_\tau\phi)^2 + (\partial_x\phi)^2 + (\partial_z\phi)^2 + \frac{L^2m^2}{z^2}\phi^2\right)$$

Calculation of the energy density evolution dynamics following a local quench

1) Calculate euclidean two point correlation function;

2) Regularize composite operator, e.g. $\phi^2(x)$, by splitting the spacetime point into two distinct points

$$\phi^2(x) \to \lim_{y \to x} \phi(x)\phi(y);$$

3) Calculate 4-point function using Wick theorem;

4) Take the limit and extract the finite part;

5) Perform a Wick rotation to obtain real time result;

6) Взять оператор квенча в точках $(-i\varepsilon, 0)$ и $(i\varepsilon, 0)$.

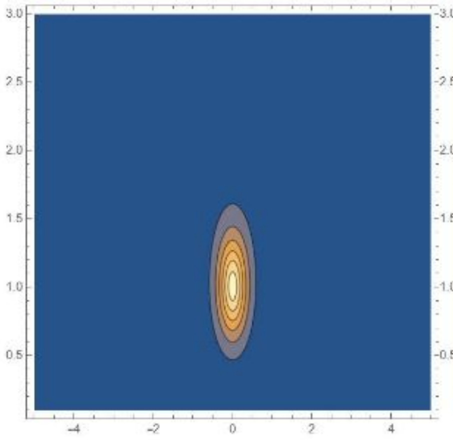
Free massive scalar field in AdS background: quench dynamics

Local quench operator $O = \phi$

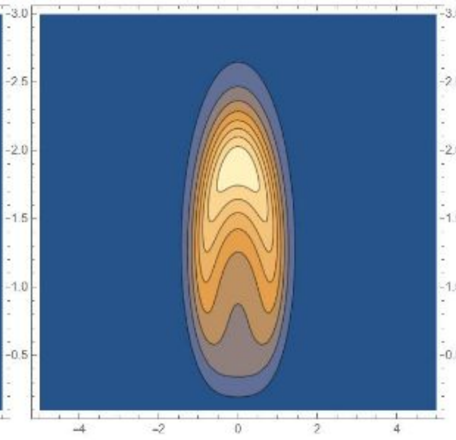
$$\langle \phi(z_1,\tau_1,x_1)\phi(z_2,\tau_2,x_2)\rangle_\phi = \frac{\langle 0|\phi(z_0,-\varepsilon,0)\phi(z_1,\tau_1,x_1)\phi(z_2,\tau_2,x_2)\phi(z_0,\varepsilon,0)|0\rangle}{\langle 0|\phi(z_0,-\varepsilon,0)\phi(z_0,\varepsilon,0)|0\rangle}$$

$$\langle \phi^2(z,t,x)\rangle_\phi = -\frac{\Delta-1}{4\pi L} + \frac{1}{\pi L}\left(\frac{|\xi_t|^2}{2\xi_0}\right)^{\Delta} \frac{\left|{}_2F_1(\frac{\Delta}{2},\frac{\Delta+1}{2};\Delta;\xi_t^2)\right|^2}{{}_2F_1\left(\frac{\Delta}{2},\frac{\Delta+1}{2};\Delta;\xi_0^2\right)}$$

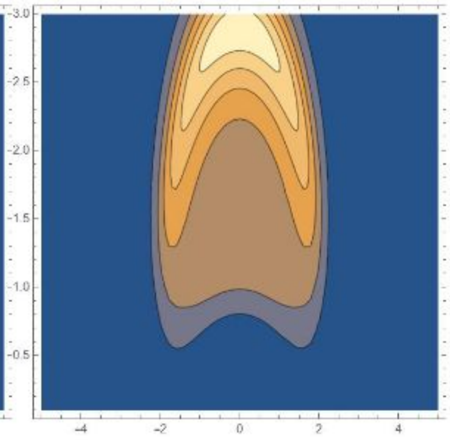
$$\xi_t = \frac{2zz_0}{z^2+z_0^2+(\varepsilon-it)^2+x^2}; \qquad \xi_0 = \frac{z_0^2}{z_0^2+2\varepsilon^2}$$



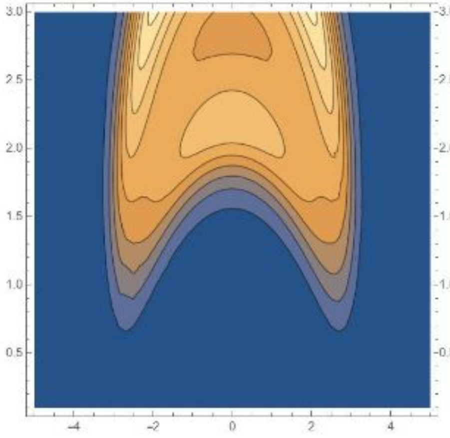
(a) $t = 0$



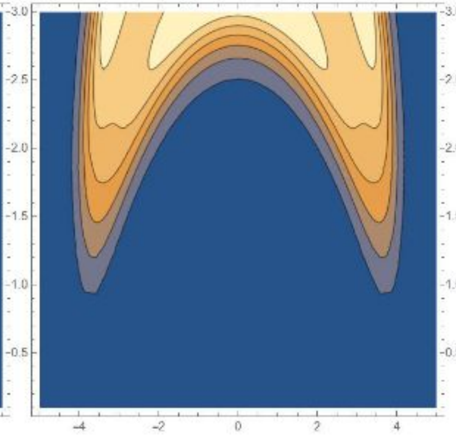
(b) $t = 1$



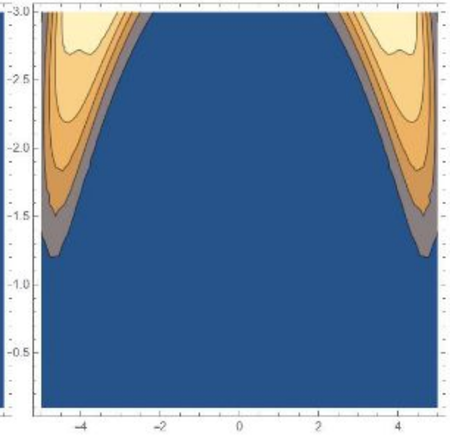
(c) $t = 2$



(d) $t = 3$



(e) $t = 4$



(f) $t = 5$

Free massive scalar field in BTZ black brane background

BTZ black brane metric in Poincare coordinates:

$$ds^2 = \frac{L^2}{z^2}\left[\frac{dz^2}{\left(1-\frac{z^2}{z_h^2}\right)} + \left(1-\frac{z^2}{z_h^2}\right)d\tau^2 + dx^2\right]$$

Energy density:

$$\mathcal{E} = \frac{1}{2}\left(-(\partial_\tau\phi)^2 + \left(1-\frac{z^2}{z_h^2}\right)(\partial_x\phi)^2 + \left(1-\frac{z^2}{z_h^2}\right)^2(\partial_z\phi)^2 + \frac{L^2m^2}{z^2}\left(1-\frac{z^2}{z_h^2}\right)\phi^2\right)$$

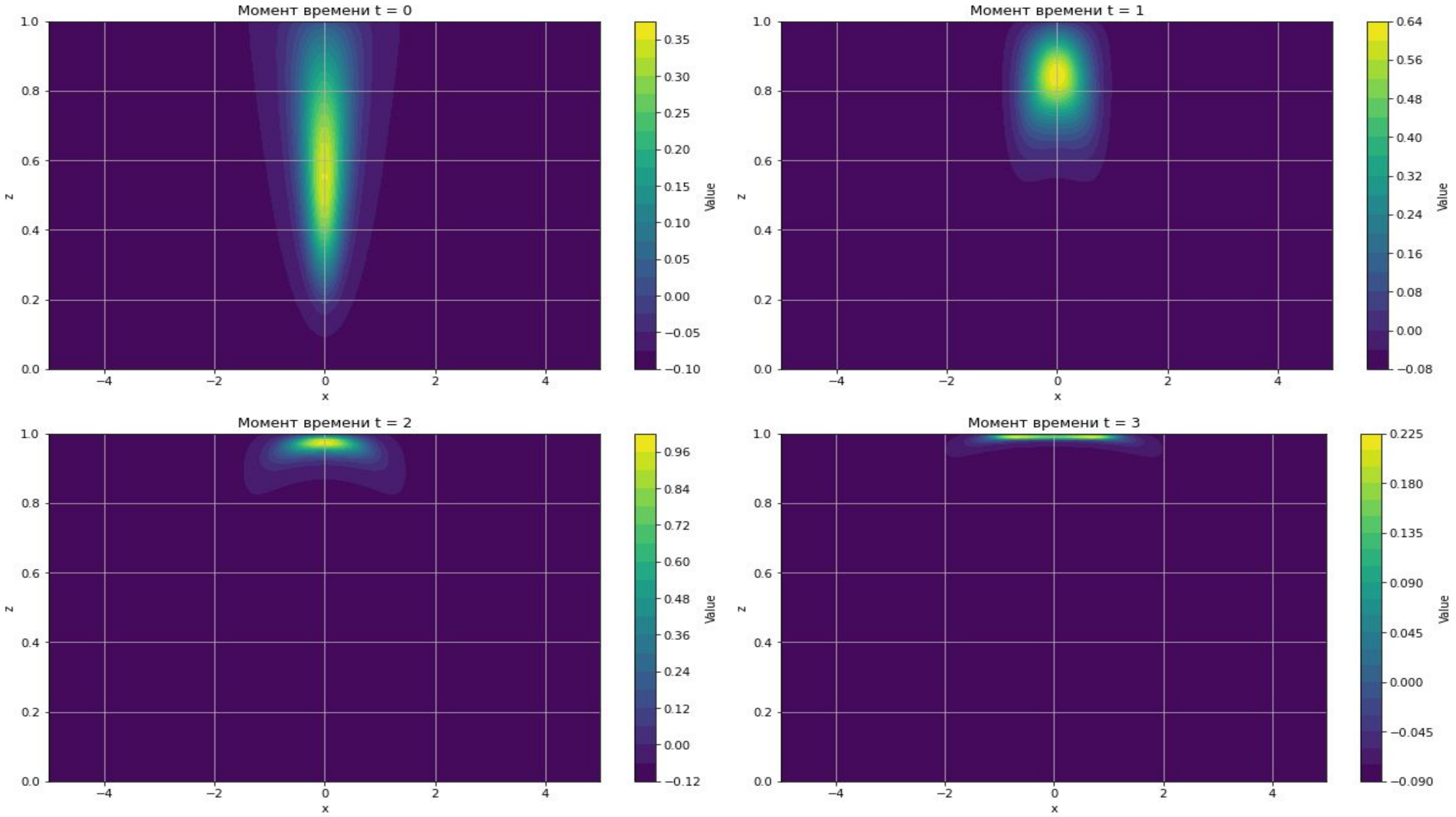
Massive scalar field in BTZ black brane background: quench dynamics

Local quench operator $O = \phi$

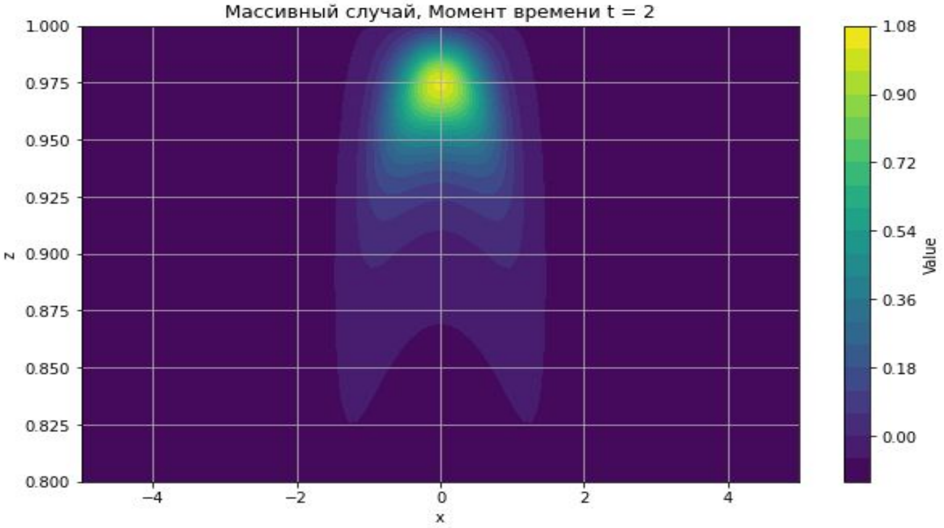
$$\langle \phi(z_1,\tau_1,x_1)\phi(z_2,\tau_2,x_2)\rangle_\phi = \frac{\langle 0|\phi(z_0,-\varepsilon,0)\phi(z_1,\tau_1,x_1)\phi(z_2,\tau_2,x_2)\phi(z_0,\varepsilon,0)|0\rangle}{\langle 0|\phi(z_0,-\varepsilon,0)\phi(z_0,\varepsilon,0)|0\rangle}$$

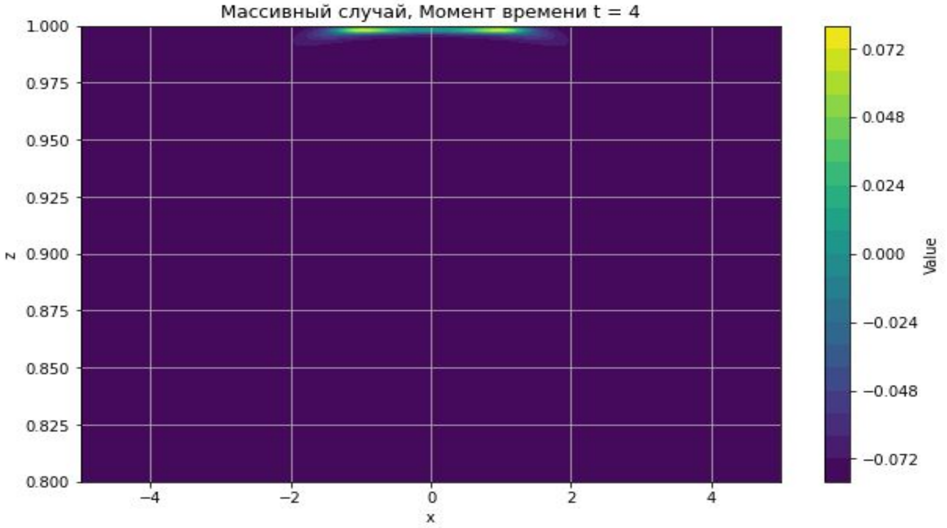
$$\langle \phi^2(z,t,x)\rangle_\phi = -\frac{\Delta-1}{4\pi L} + \frac{1}{\pi L}\left(\frac{|\xi_t|^2}{2\xi_0}\right)^\Delta \frac{\left|{}_2F_1(\frac{\Delta}{2},\frac{\Delta+1}{2};\Delta;\xi_t^2)\right|^2}{{}_2F_1\left(\frac{\Delta}{2},\frac{\Delta+1}{2};\Delta;\xi_0^2\right)}$$

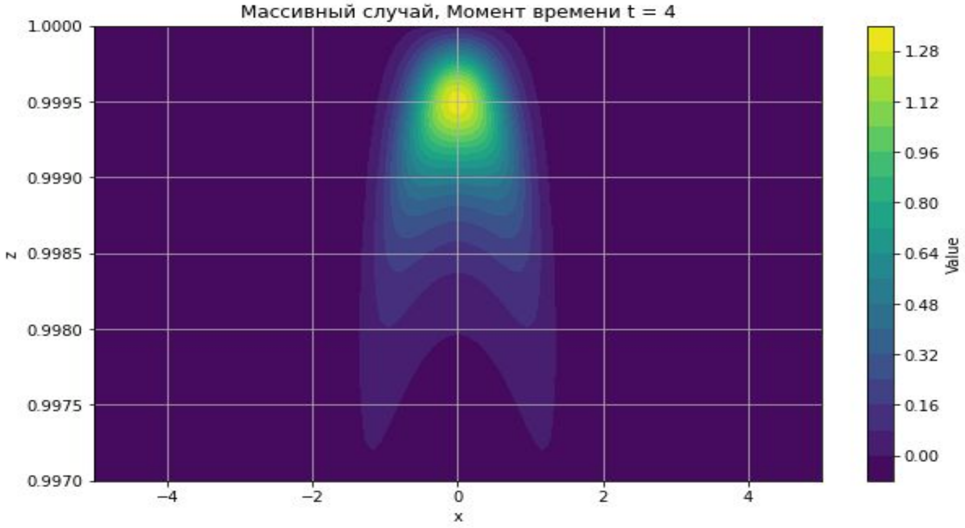
$$\xi = \frac{z_1 z_2}{z_h^2\cosh\left(\frac{x_2-x_1}{z_h}\right) - \sqrt{(z_h^2-z_1^2)(z_h^2-z_2^2)}\cos\left(\frac{\tau_2-\tau_1}{z_h}\right)}$$



Момент времени t = 0
Момент времени t = 1
Момент времени t = 2
Момент времени t = 3
x
z
Value
1.0
0.8
0.6
0.4
0.2
0.0
−4
−2
0
2
4
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
0.64
0.56
0.48
0.40
0.32
0.24
0.16
0.08
0.00
−0.08
0.96
0.84
0.72
0.60
0.48
0.36
0.24
0.12
0.00
−0.12
0.225
0.180
0.135
0.090
0.045
0.000
−0.045
−0.090


Массивный случай, Момент времени t = 2
z
x
Value
1.000
0.975
0.950
0.925
0.900
0.875
0.850
0.825
0.800
−4
−2
0
2
4
1.08
0.90
0.72
0.54
0.36
0.18
0.00


Массивный случай, Момент времени t = 4
z
x
Value
1.000
0.975
0.950
0.925
0.900
0.875
0.850
0.825
0.800
−4
−2
0
2
4
0.072
0.048
0.024
0.000
−0.024
−0.048
−0.072


Массивный случай, Момент времени t = 4
z
x
Value
1.0000
0.9995
0.9990
0.9985
0.9980
0.9975
0.9970
−4
−2
0
2
4
1.28
1.12
0.96
0.80
0.64
0.48
0.32
0.16
0.00

Periodic BTZ black hole

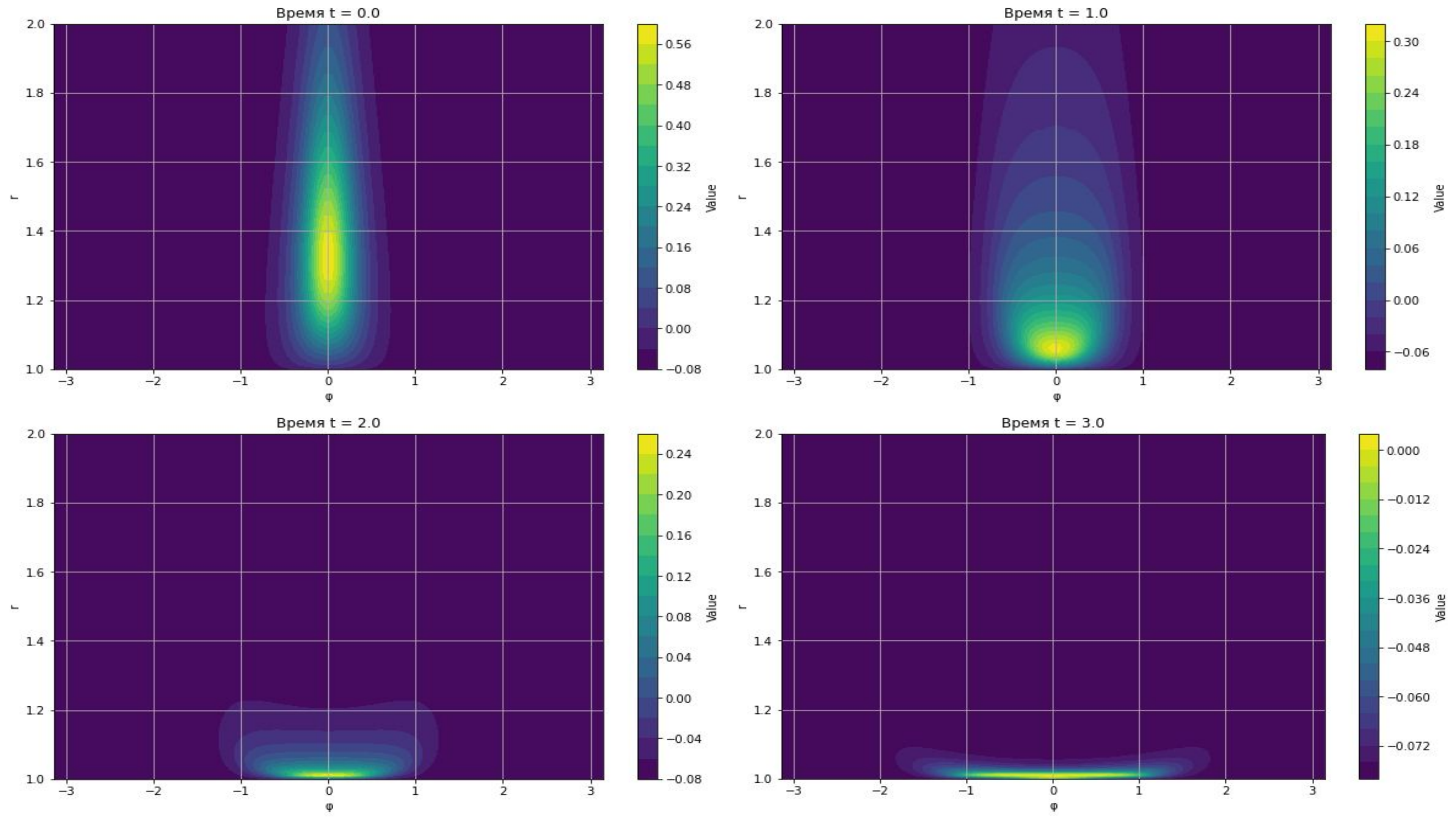
Metric

$$\mathrm{d}s^2 = \left(\frac{r^2}{L^2} - \frac{h^2}{L^2}\right)\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{\left(\frac{r^2}{L^2} - \frac{h^2}{L^2}\right)} + r^2\mathrm{d}\phi^2$$

It is periodic in the angular coordinate. The Green's function is obtained using the method of images

$$G = \sum_n \frac{1}{4\pi L}\frac{1}{\sqrt{1-\xi^2}}\xi^\Delta\left(1+\sqrt{1-\xi^2}\right)^{1-\Delta}$$

$$\xi = \frac{1}{-\sqrt{-1+\frac{r_1^2}{h^2}}\sqrt{-1+\frac{r_2^2}{h^2}}\cos\left(\frac{h(\tau_1-\tau_2)}{L^2}\right) + \frac{r_1 r_2}{h^2}\cosh\left(\frac{h(\phi_1-\phi_2+2\pi n)}{L}\right)}$$

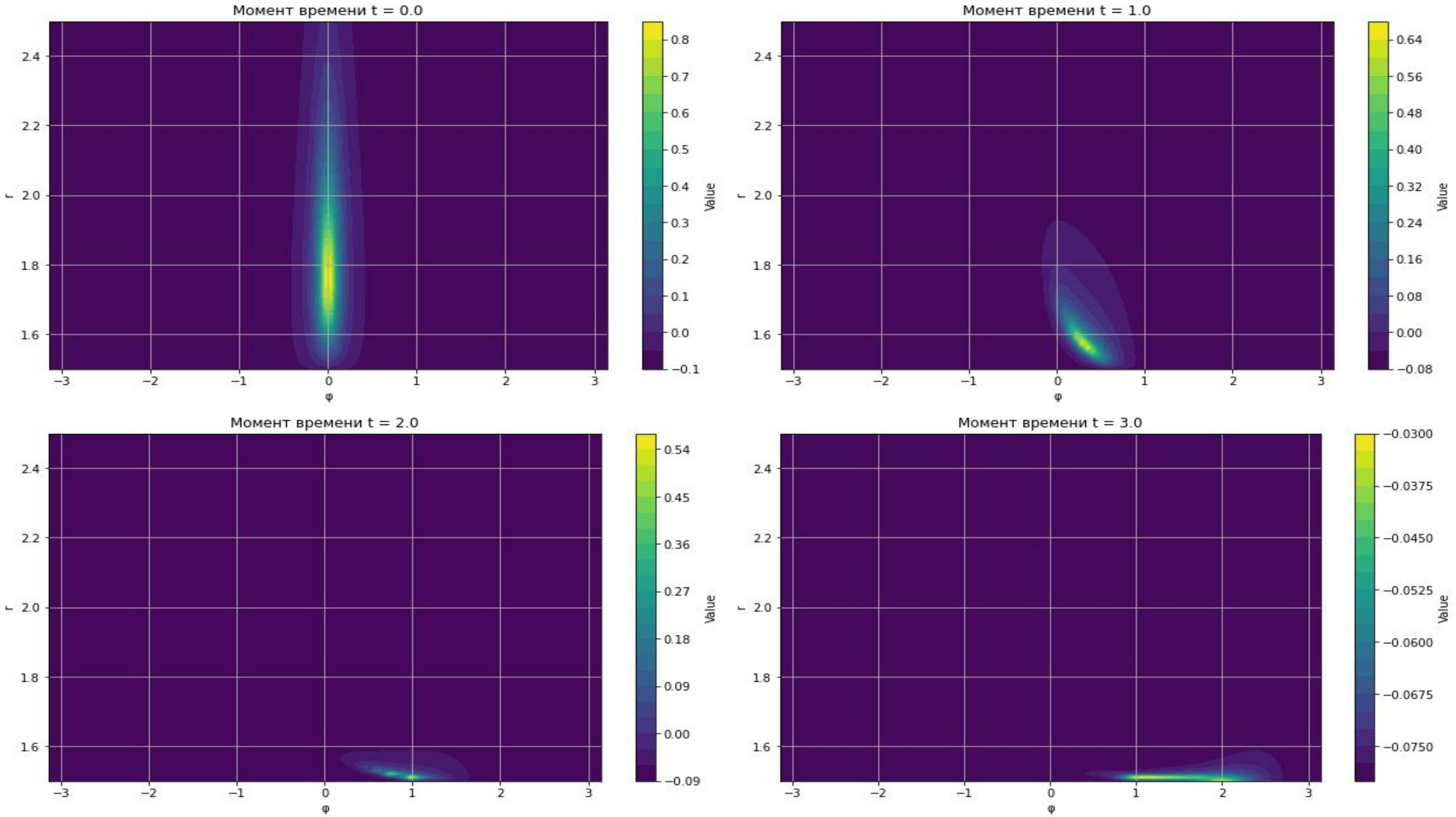

Время t = 0.0
Время t = 1.0
Время t = 2.0
Время t = 3.0
r
φ
Value
2.0
1.8
1.6
1.4
1.2
1.0
−3
−2
−1
0
1
2
3
0.56
0.48
0.40
0.32
0.24
0.16
0.08
0.00
−0.08
0.30
0.24
0.18
0.12
0.06
0.00
−0.06
0.24
0.20
0.16
0.12
0.08
0.04
0.00
−0.04
−0.08
0.000
−0.012
−0.024
−0.036
−0.048
−0.060
−0.072

Rotating BTZ black hole

Metric

$$\mathrm{d}s^2 = \frac{(r^2 - a^2)(r^2 + b^2)}{r^2}\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{\frac{(r^2 - a^2)(r^2 + b^2)}{r^2}} + r^2\left(\mathrm{d}\phi + \frac{ab}{r^2}\mathrm{d}\tau\right)^2$$

where a, b - outer and inner horizons respectively


Момент времени t = 0.0
Момент времени t = 1.0
Момент времени t = 2.0
Момент времени t = 3.0
r
φ
Value
2.4
2.2
2.0
1.8
1.6
−3
−2
−1
0
1
2
3
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
0.64
0.56
0.48
0.40
0.32
0.24
0.16
0.08
0.00
−0.08
0.54
0.45
0.36
0.27
0.18
0.09
0.00
−0.09
−0.0300
−0.0375
−0.0450
−0.0525
−0.0600
−0.0675
−0.0750

BDHM (Banks, Douglas, Horowitz, Martinec) dictionary

The BDHM dictionary establishes a correspondence between Green's functions of massive scalar field in the bulk of AdS space and correlation functions of primary operators in the conformal field theory on the boundary

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_{\mathrm{CFT}} = \lim_{z\to 0} z^{-2\Delta}\langle \phi(x_1)\phi(x_2)\rangle_{\mathrm{bulk}}$$

BHDM dictionary: AdS space quench dynamics

Massive scalar field theory in three dimensional AdS space is dual to two dimensional conformal field theory

$$\langle \mathcal{O}(\tau_1, x_1)\mathcal{O}(\tau_2, x_2)\rangle_{\mathrm{ECFT}} = \frac{1}{2\pi L[(\tau_1-\tau_2)^2+(x_1-x_2)^2]^{\Delta}} + \frac{(\eta_1^+\eta_2^-)^{\Delta}+(\eta_1^-\eta_2^+)^{\Delta}}{2\pi L(2\xi_0)^{\Delta}\cdot {}_2F_1\left(\frac{\Delta}{2},\frac{\Delta+1}{2};\Delta;\xi_0^2\right)}$$

$$\eta_{1,2}^{\pm} = \frac{2z_0}{z_0^2+(\tau_{1,2}\pm\varepsilon)^2+x_{1,2}^2} \qquad \xi_0 = \frac{z_0^2}{z_0^2+2\varepsilon^2}$$

BHDM dictionary: BTZ black brane quench dynamics

Massive scalar field in BTZ black brane background is dual to two dimensional flat conformal field theory at finite temperature $T = \frac{1}{2\pi z_h}$

$$\langle \mathcal{O}(\tau_1, x_1)\mathcal{O}(\tau_2, x_2)\rangle_{\text{ECFT}} = \frac{1}{2\pi L (2z_h^2)^\Delta \left(\cosh\left(\frac{x_2 - x_1}{z_h}\right) - \cos\left(\frac{\tau_2 - \tau_1}{z_h}\right)\right)^\Delta} + \frac{(\eta_1^+ \eta_2^-)^\Delta + (\eta_1^- \eta_2^+)^\Delta}{2\pi L (2\xi_0)^\Delta \cdot {}_2F_1\left(\frac{\Delta}{2}, \frac{\Delta+1}{2}; \Delta; \xi_0^2\right)}$$

$$\eta_{1,2}^\pm = \frac{z_0}{z_h^2 \cosh\left(\frac{x_{1,2}}{z_h}\right) - z_h\sqrt{z_h^2 - z_0^2}\cos\left(\frac{\tau_{1,2} \pm \epsilon}{z_h}\right)} \quad ; \quad \xi_0 = \frac{z_0^2}{z_h^2 - (z_h^2 - z_0^2)\cos\left(\frac{2\epsilon}{z_h}\right)}$$

de Sitter space

Metric in Poincare coordinates

$$ds^2 = \frac{L^2}{\eta^2}(-d\eta^2 + dx^i dx^i)$$

Wightman function

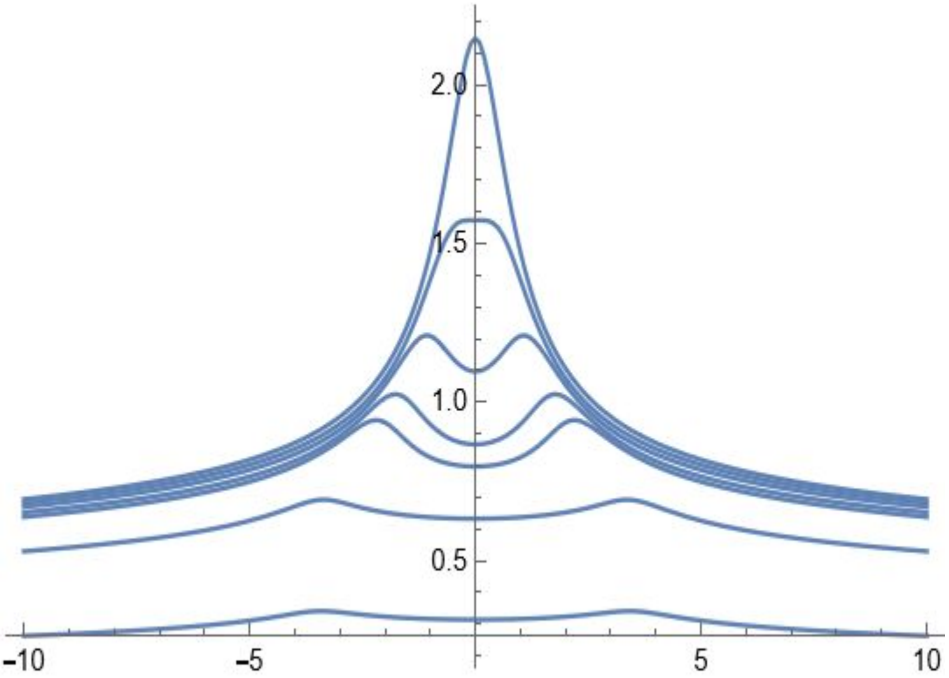
$$G(X_1, X_2) = \frac{\Gamma\left(\frac{d}{2} + i\nu\right)\Gamma\left(\frac{d}{2} - i\nu\right)}{(4\pi)^{(d+1)/2}\Gamma\left(\frac{d+1}{2}\right)} {}_2F_1\left(\begin{matrix}\frac{d}{2} + i\nu, \frac{d}{2} - i\nu \\ \frac{d+1}{2}\end{matrix}; \sigma_{\mathrm{dS}}\right)$$

$$\sigma_{\mathrm{dS}}^{\pm} = 1 + \frac{(\eta_1 - \eta_2 \pm \frac{i\epsilon}{2})^2 - |\vec{x}_1 - \vec{x}_2|^2}{4\eta_1\eta_2}$$

$$G_{+-}(X_1, X_2) = \langle 0|\hat{\phi}(X_2)\hat{\phi}(X_1)|0\rangle = G(\sigma_{\mathrm{dS}}^{+})$$

$$G_{-+}(X_1, X_2) = \langle 0|\hat{\phi}(X_1)\hat{\phi}(X_2)|0\rangle = G(\sigma_{\mathrm{dS}}^{-})$$

Mass-dependent behaviour



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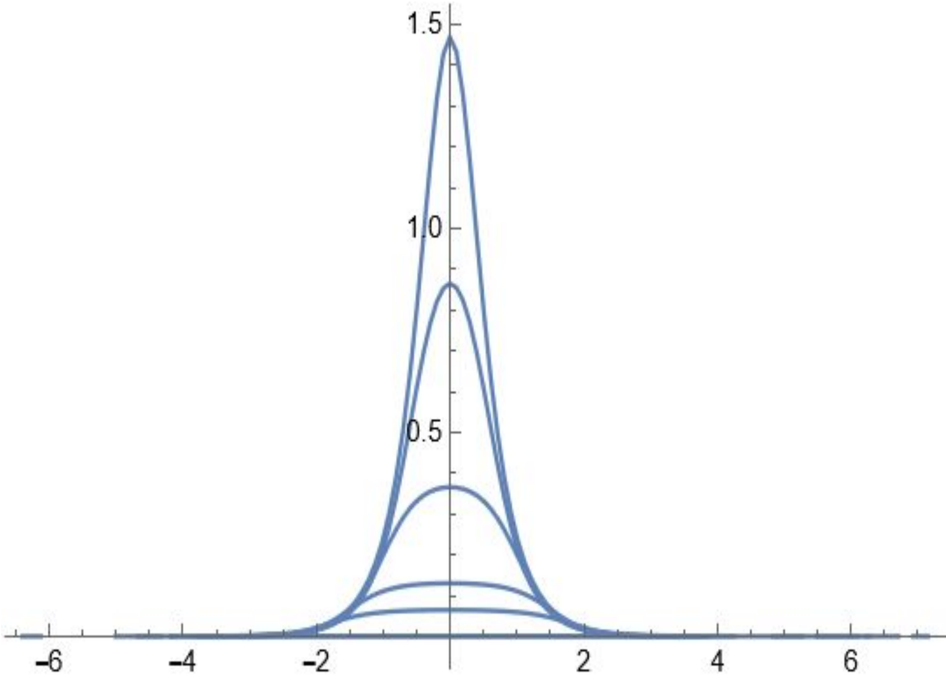

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Results and future studies

In this work, we studied local quenches in curved spacetimes: anti-de Sitter space, BTZ black hole and de Sitter space. We investigated the energy density dynamics of a massive scalar field following a local quench. Using holographic duality we derived correlation functions in CFT, which correspond to the bulk local quench, providing an example of nontrivial nonequilibrium dynamics in the CFT.

Future research will explore dS/CFT holography and dual description of quenches in de Sitter space. Additionally, the nature of the excited state in the CFT dual to the bulk local quench in AdS remains unclear.

Thank you for your attention!