On the interplay between the BFKL resummation and high-energy factorization in Mueller-Navelet dijet production[1][2]

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Contents

Motivation I

The DGLAP / RG equation:

$$\frac{\partial}{\partial \ln \mu_R^2} \mathscr{O} = \hat{K}_{\text{DGLAP}}\, \mathscr{O}$$

- $z = \mu^2/s$ is fixed: $z \sim 1$;
- N^{k}LO resum N^{k}LL $\sim \bar{\alpha}_s^{n+k}(\mu_R^2) \ln^n \mu_R^2$ in the limit $\mu_R^2 \to \infty$.

The **Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation:**

$$\boxed{\frac{\partial}{\partial Y} \mathscr{O} \;=\; \hat{K}_{\text{BFKL}}\, \mathscr{O}}$$

- Regge limit: $z \ll 1$, while $\bar{\alpha}_s(\mu_R^2) Y \sim 1$ with $Y = \ln(1/z)$.

State-of-the-art:

$$\hat{K}_{\text{BFKL}} \;=\; \underbrace{\bar{\alpha}_s(\mu_R^2)\,\hat{K}^{(0)}}_{\text{[BFKL '76-78]}} \;+\; \underbrace{\bar{\alpha}_s^2(\mu_R^2)\,\hat{K}^{(1)}}_{\text{[FL '98]}} \;+\; \underbrace{\mathscr{O}(\bar{\alpha}_s^3)}_{\text{[...]}}.$$

- LO resum LL $\sim \bar{\alpha}_s^n Y^n$;
- NLO resum NLL $\sim \bar{\alpha}_s^{n+1}(\mu_R^2) Y^n$, partially includes $\bar{\alpha}_s(\mu_R^2) \ln \mu_R^2$;
- NNLO is in progress[V. Fadin, L. Lipatov '18; V. Del Duca *et.al.* '21; and more. . .].

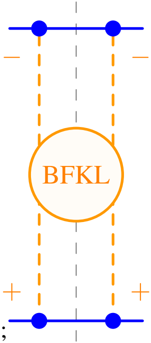
Motivation II

White paper on the BFKL physics[M. Hentschinski *et.al.* '23]: different jets topologies, Higgs, heavy quarkonia, diffraction, etc.

The **Mueller-Navelet (MN) dijets**[MN '86] as a tool to probe the BFKL:

$$Y \sim \ln\left(\frac{M_{12}^2}{|\mathbf{p}_{T1}||\mathbf{p}_{T2}|}\right) \gg 1 \quad \Longrightarrow \quad \bar{\alpha}_s(\mu_R^2)\, Y \sim 1,$$

where $|\mathbf{p}_{T1,2}| \gg \Lambda_{\text{QCD}}$. Selected results:



- Collinear improvement of the NLL Green's function[A. Sabio-Vera '05];
- NLO impact factors consists $\ln(\mathbf{l}_T^2/\mu_R^2)$[D. Colferai *et.al.* '10; F. Caporale *et.al.* '13];
- Instability of the NLO BFKL computations when $\Delta\phi \to 0$. A *special scale setting* is required[B. Duclou´ *et.al.* '14; F. Caporale *et.al.* '15];
- Sudakov resummation was found to be important[A. Mueller *et.al.* '15].

Resume:

- The correct treatment of the Green's function RG-invariance is crucial;
- Implementation of the Sudakov resummation should be done.

The high–energy factorization

HEF

Leading-twist **high–energy factorization (HEF):**

$$\sigma = \int \frac{dx_1}{x_1} \int_{\mathbf{q}_{T1}} \Phi_i(x_1, \mathbf{q}_{T1}^2, \mu^2) \int \frac{dx_2}{x_2} \int_{\mathbf{q}_{T2}} \Phi_j(x_2, \mathbf{q}_{T2}^2, \mu^2)$$

$$\times \quad H_{ij}\left(x_{1,2}, \mathbf{q}_{T1,2}, \bar{\alpha}_s(\mu_R^2)\right) + \mathcal{O}\left((\Lambda/\mu_F)^{\#}, \mu^2/s, \mathrm{N}^{\#}\mathrm{LL}\right),$$

where *unintegrated PDF (UPDF)*

$$\Phi_i(x, \mathbf{q}_T^2, \mu^2) = \int_x^1 \frac{dz}{z} \tilde{f}_{i'}\left(\frac{x}{z}, \mu_F^2\right) C_{ii'}(z, \mathbf{q}_T, \mu_F^2, \mu^2).$$

Advantages[J. Collins, C. Ellis '91; S. Catani, M. Ciafaloni, F. Hautman '91, 94; M. Nefedov, A. van Hameren '25]:

- Proven up to NLL and NLP;
- Coefficient function H_{ij} is gauge invariant;
- Evolution kernels $C_{ii'}$ are universal. General structure:

$$C_{ii'} \sim \sum_{n_k} \bar{\alpha}_s^{n_1}(\mu_R^2) \underbrace{z^{n_2}}_{\text{LP/NLP}} \underbrace{\ln^{n_3}(1/z)}_{\text{BFKL}} \underbrace{\ln^{n_4}(\mu^2/\mathbf{q}_T^2)}_{\text{LL/NLL Sudakov}},$$

also should include effects $z \to 1$.

PRA

Parton Reggeization approach (PRA)$_{\text{[M. Nefedov, V. Saleev, A. Shipilova '13; M. Nefedov, V. Saleev '20]}}$:

- Based on the Lipatov's EFT$_{\text{[L. Lipatov '95]}}$;
- Feynman rules of the EFT$_{\text{[E. Antonov, I. Cherednikov, E. Kuraev, L. Lipatov '05]}}$;
- Includes NLP (Reggeized quarks)$_{\text{[V. Fadin, V. Sherman '77; L. Lipatov, M. Vyazovsky '02]}}$;
- Modified KMR(W) UPDFs$_{\text{[M. Kimber, A. Martin, M. Ryskin '01; M. Nefedov, V. Saleev '20]}}$ contains only the Sudakov resummation up to NLL $\bar{\alpha}_s(\mu_R^2)\ln^2(\mu^2/\mathbf{q}_T^2)$.

Relevant results:

- Dijet$_{\text{[M. Nefedov, V. Saleev, A. Shipilova '13]}}$: all LO EFT diagrams have been computed;
- Pair $J/\psi$$_{\text{[Z.-G. He, B. Kniehl, M. Nefedov, V. Saleev '19]}}$: PRA + NRQCD + LL BFKL;
- New UPDFs with exact normalization$_{\text{[M. Nefedov, V. Saleev '20]}}$;
- Interplay between the BFKL and Sudakov resummation$_{\text{[M. Nefedov '21]}}$.

The BFKL equation

The BFKL equation

The BFKL equation for the *Green's function*:

$$\frac{\partial}{\partial Y} G(\mathbf{l}_{T1}, \mathbf{l}_{T2}, Y) = \int_{\mathbf{q}_T} K(\mathbf{l}_{T1}, \mathbf{q}_T)\, G(\mathbf{q}_T, \mathbf{l}_{T2}, Y).$$

The LO kernel is conformal invariant ($f^{(0)}_{n,\gamma}(\mathbf{q}_T) \sim \mathbf{q}_T^{2(\gamma-1)}$)[I. Balitsky, L. Lipatov '78]:

$$\int_{\mathbf{q}_T} K(\mathbf{l}_T, \mathbf{q}_T)\; f^{(0)}_{n,\gamma}(\mathbf{q}_T) = \bar{\alpha}_s(\mu_R^2)\, \chi^{(0)}(n,\gamma)\, f^{(0)}_{n,\gamma}(\mathbf{l}_T).$$

Expansion of the NLO kernel over the LO eigenfunctions[V. Fadin, L. Lipatov '98]:

$$\int_{\mathbf{q}_T} K(\mathbf{l}_T, \mathbf{q}_T)\; f^{(0)}_{n,\gamma}(\mathbf{q}_T) = \bar{\alpha}_s(\mathbf{l}_T^2) \left[\chi^{(0)}(n,\gamma) + \bar{\alpha}_s(\mathbf{l}_T^2)\, \frac{\delta(n,\gamma)}{4}\right] f^{(0)}_{n,\gamma}(\mathbf{l}_T).$$

The LO eigenfunctions are not the basis of the NLO kernel:

- Coupling running:

$$\bar{\alpha}_s(\mathbf{l}_T^2) = \bar{\alpha}_s(\mu_R^2)\left(1 - \bar{\alpha}_s(\mu_R^2)\,\frac{\beta_0}{4\pi}\ln\left(\frac{\mathbf{l}_T^2}{\mu_R^2}\right)\right) + \mathscr{O}(\bar{\alpha}_s^3);$$

- Structure of the $\delta(n,\gamma)$[A. Kotikov, L. Lipatov '00]:

$$\frac{\delta(n,\gamma)}{4} = \chi^{(1)}(n,\gamma) \underbrace{- \frac{1}{2}\,\frac{\beta_0}{4\pi}\,\frac{\partial \chi^{(0)}(n,\gamma)}{\partial \gamma}}_{\text{asym. } \gamma \leftrightarrow 1-\gamma}.$$

The BFKL equation

The BFKL equation for the *Green's function*:

$$\frac{\partial}{\partial Y} G\left(\mathbf{l}_{T1}, \mathbf{l}_{T2}, Y\right) = \int_{\mathbf{q}_T} K\left(\mathbf{l}_{T1}, \mathbf{q}_T\right) G\left(\mathbf{q}_T, \mathbf{l}_{T2}, Y\right).$$

- Perturbative expansion of the kernel:

$$K \;=\; \bar{\alpha}_s(\mu_R^2)\, K^{(0)} \;+\; \bar{\alpha}_s^2(\mu_R^2)\, K^{(1)} \;+\; \mathcal{O}(\bar{\alpha}_s^3);$$

- Characteristic equation (order-by-order in $\bar{\alpha}_s$):

$$\int_{\mathbf{q}_T} K\left(\mathbf{l}_T, \mathbf{q}_T \mid \bar{\alpha}_s(\mu_R^2)\right) H_{n,\gamma}(\mathbf{q}_T, \mu_R^2) \;=\; \bar{\alpha}_s(\mu_R^2)\, \chi(n,\gamma)\, H_{n,\gamma}(\mathbf{l}_T, \mu_R^2),$$

where

$$\chi(n,\gamma) \;=\; \chi^{(0)}(n,\gamma) \;+\; \bar{\alpha}_s(\mu_R^2)\, \chi^{(1)}(n,\gamma) \quad +\; \mathcal{O}(\bar{\alpha}_s^2),$$

$$H_{n,\gamma}(\mathbf{l}_T, \mu_R^2) \;=\; f_{n,\gamma}^{(0)}(\mathbf{l}_T) \quad +\; \bar{\alpha}_s(\mu_R^2)\, f_{n,\gamma}^{(1)}(\mathbf{l}_T, \mu_R^2) \;+\; \mathcal{O}(\bar{\alpha}_s^2);$$

- **The RG–invariant solution** (w.r.t. $\mathcal{O}(\bar{\alpha}_s^3)$)**:**

$$\boxed{G\left(\mathbf{l}_{T1}, \mathbf{l}_{T2}, Y\right) \;=\; \sum_n \int \frac{d\gamma}{2\pi i}\, \exp\left[\bar{\alpha}_s(\mu_R^2)\, \chi(n,\gamma)\, Y\right] H_{n,\gamma}(\mathbf{l}_{T1}, \mu_R^2)\, H^{\star}_{n,\gamma}(\mathbf{l}_{T2}, \mu_R^2).}$$

The BFKL equation solution

The LO solution[I. Balitsky, L. Lipatov '78]:

$$\chi^{(0)}(n,\gamma) = 2\psi(1) - 2\,\mathrm{Re}\,\psi\left(\frac{n}{2}+\gamma\right),$$

$$f^{(0)}_{n,\gamma}(\mathbf{l}_T) = \frac{1}{\sqrt{\pi}}\mathbf{l}_T^{2(\gamma-1)}e^{in\phi_l}.$$

The NLO solution[G. Chirilli, Y. Kovchegov '13, 14]:

$$\chi^{(1)}(n,\gamma) = \frac{1}{2}\frac{\beta_0}{4\pi}\frac{\partial\chi^{(0)}(n,\gamma)}{\partial\gamma} + \frac{\delta(n,\gamma)}{4},$$

$$f^{(1)}_{n,\gamma}(\mathbf{l}_T,\mu_R^2) = \sum_m c_m(n,\gamma)\ln^m\left(\frac{\mathbf{l}_T^2}{\mu_R^2}\right)f^{(0)}_{n,\gamma}(\mathbf{l}_T),$$

see $\delta(n,\gamma)$ in[A. Kotikov, L. Lipatov '00], coefficients:

$$c_0 = 0,\quad c_1(n,\gamma) = \partial_\gamma c_2(n,\gamma),\quad c_2(n,\gamma) = \frac{1}{2}\frac{\beta_0}{4\pi}\frac{1}{\partial_\gamma \ln\chi^{(0)}(n,\gamma)},\quad c_{m>2} = 0.$$

NOTE: the NLO eigenfunctions should be treated as a distributions due to pole of the c_2 at $\gamma = 1/2$.

Resummation of collinear poles

The *collinear-improvement* (CI)[G. Salam '98]: $\chi(n,\gamma) + \Delta\chi(n,\gamma)$.

- A (anti-) collinear poles:

$$\chi^{(l)}(n,\gamma) = \sum_{k=1}^{2l+1} \kappa_{l,k}\, d_0^{-k}\, \bar{\alpha}_s^l + \mathscr{O}(\gamma) + \{\gamma \to 1-\gamma\},$$

where $\bar{\alpha}_s = \bar{\alpha}_s(\mu_R^2)$, $\kappa_{l,k} = \kappa_{l,k}(n)$, and $d_m = d_m(n,\gamma) = m + \gamma + n/2$;

- The ω-shift scheme[G. Salam '98]:

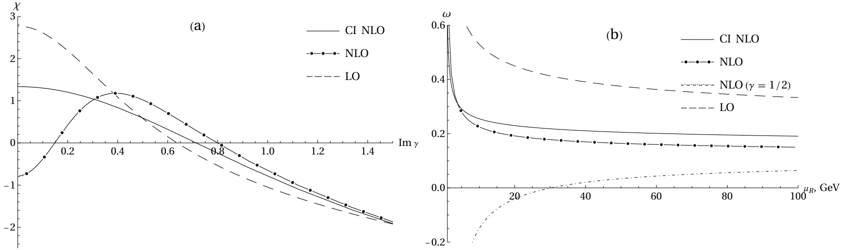
$$d_0^{-k}\left(n, \gamma + \frac{1}{2}\bar{\alpha}_s\,\chi(n,\gamma)\right)\bar{\alpha}_s^l \supset \sum_{l'=l}^{\infty} d_0^{-k-2(l-l')}\,\bar{\alpha}_s^{l'};$$

- The "all-poles" approximation[A. Sabio-Vera '05; A. Sabio-Vera, F. Schwennsen '07]:

$$\begin{aligned}\bar{\alpha}_s\,\Delta\chi(n,\gamma) \;=\; & \sum_{m=0}^{\infty}\Bigg[\kappa_{1,2}\,\bar{\alpha}_s - d_m + \sqrt{2\bar{\alpha}_s\left(\kappa_{0,1} + \kappa_{1,1}\,\bar{\alpha}_s\right) + \left(\kappa_{1,2}\,\bar{\alpha}_s - d_m\right)^2} \\ & - \sum_{l=0}^{1}\sum_{k=1}^{2l+1} \kappa_{l,k}\, d_m^{-k}\,\bar{\alpha}_s^{l+1}\Bigg] + \{\gamma \to 1-\gamma\};\end{aligned}$$

- Consistent with characteristic equation: $\Delta\chi \sim \mathscr{O}(\bar{\alpha}_s^3)$.

The characteristic function



The CI NLO:

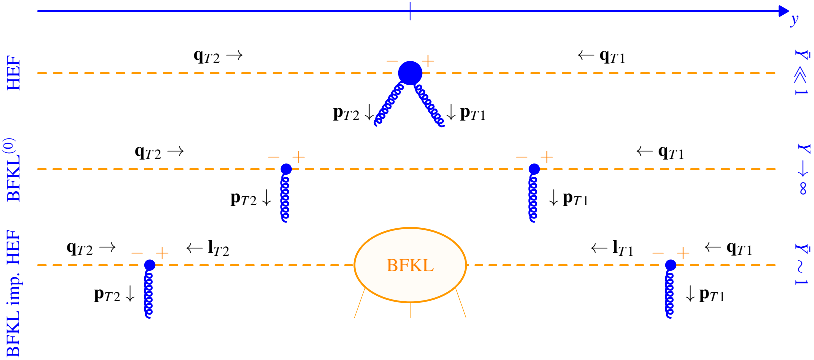
- Only one saddle point located at $\gamma = 1/2$;
- The LO definition of the intercept is preserved at NLO:

$$\omega = \bar{\alpha}_s(\mu_R^2)\,\chi(0, 1/2)$$

- The ω is approximately scale-independent at relatively large μ_R.

Mueller–Navelet dijet production

Matching



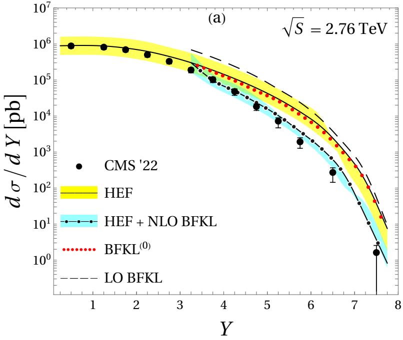
The matching scheme:

$$H_{ij}^{(\mathrm{HEF+BFKL})} = H_{ij}^{(\mathrm{HEF})} + H_{ij}^{(\mathrm{BFKL})} - H_{ij}^{(\mathrm{BFKL},0)},$$

where

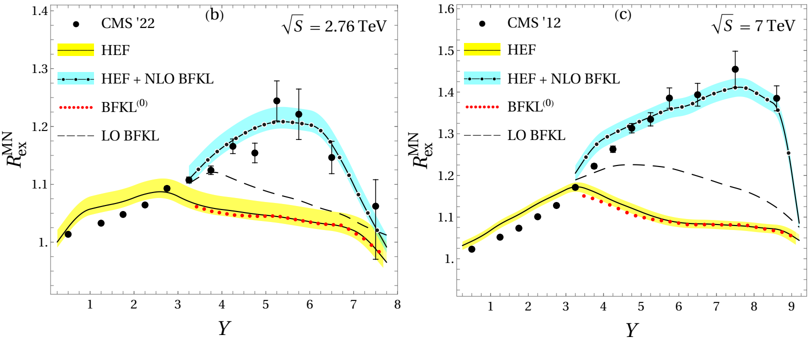
- $H_{ij}^{(\mathrm{HEF})} - H_{ij}^{(\mathrm{BFKL},0)} = 0$ (w.r.t. NLP) for $Y \to \infty$;
- $H_{ij}^{(\mathrm{BFKL})} - H_{ij}^{(\mathrm{BFKL},0)} = 0$ for $\bar{Y} \ll 1$.

Results



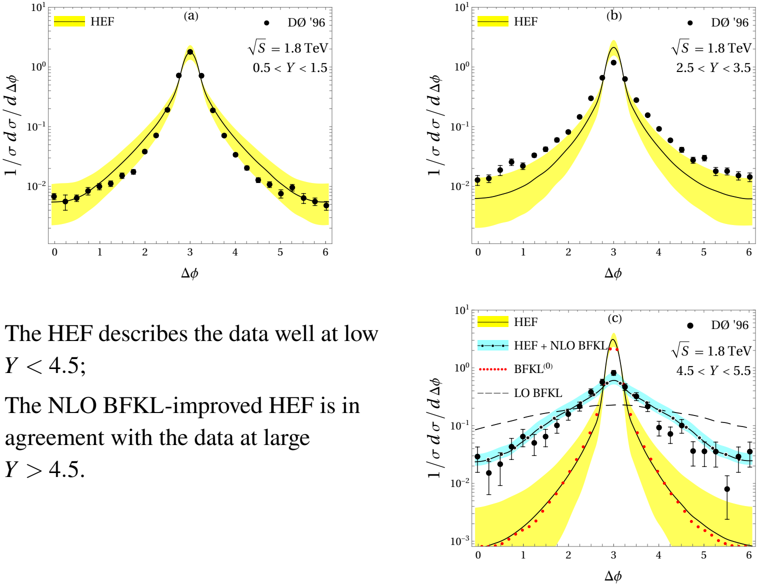
- The HEF describes the data well at low $Y < 2 - 3$;
- The HEF gets subtracted by the BFKL$^{(0)}$ starting from $Y > 3 - 3.5$;
- The NLO BFKL improved HEF is in agreement with the data at large $Y > 3.5$.

Results



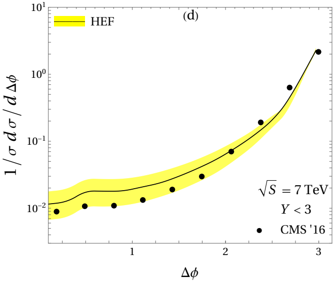
- The HEF describes the data well at low $Y < 2-3$;
- The HEF gets subtracted by the $\mathrm{BFKL}^{(0)}$ starting from $Y > 3-3.5$;
- The NLO BFKL-improved HEF is in agreement with the data at large $Y > 3.5$.

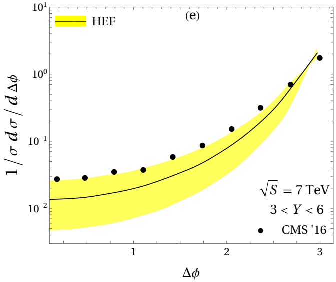
Results



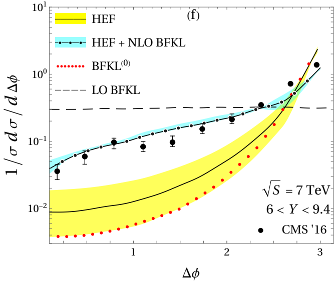
- The HEF describes the data well at low $Y < 4.5$;
- The NLO BFKL-improved HEF is in agreement with the data at large $Y > 4.5$.

Results





- The HEF describes the data well at low $Y < 6$;
- The NLO BFKL-improved HEF is in agreement with the data at large $Y > 6$.



Conclusions & outlook

- The HEF with NLL BFKL resummation is pushed to the MN dijets production;
- Both, the HEF and NLL BFKL, are crucial for the uniform description of the data across all values of the rapidity difference;
- The matching scheme interpolates predictions between the two descriptions;
- The basis of the NLO eigenfunctions together with the collinear improvement provides the RG-invariant NLL BFKL Green's function without any pathologies;
- A preprint[A. Polizzi, M. Fucilla, A. Papa '25] that also uses the NLO basis have recently appeared. The stability of such calculations has been independently confirmed.

Outlook:

- NLO HEF[M. Nefedov, A. Hameren '25];
- Sub-eikonal corrections[I. Balitsky, A. Tarasov '15; G. Chirilli '21; M. Nefedov '21].

Thank you for your attention!

End Matter materials

The NLO characteristic function:

$$\chi^{(1)}(n,\gamma) = \mathscr{S}\chi^{(0)}(n,\gamma) + \frac{3}{2}\zeta(3) - \frac{\beta_0}{8N_c}\left(\chi^{(0)}(n,\gamma)\right)^2$$
$$-\frac{\pi^2\cos(\pi\gamma)}{4(1-2\gamma)\sin^2(\pi\gamma)}\left[\left(3+\left(1+\frac{N_f}{N_c^3}\right)\frac{2+3\gamma(1-\gamma)}{(1+2\gamma)(3-2\gamma)}\right)\delta_n^0 - \left(1+\frac{N_f}{N_c^3}\right)\frac{\gamma(1-\gamma)}{2(1+2\gamma)(3-2\gamma)}\delta_n^2\right]$$
$$+\frac{1}{4}\left[\psi''\left(\gamma+\frac{n}{2}\right)+\psi''\left(1-\gamma+\frac{n}{2}\right)-2\left(\phi(n,\gamma)+\phi(n,1-\gamma)\right)\right],$$

where $\mathscr{S} = (4-\pi^2+5\beta_0/N_c)/12$ and $\beta_0 = (11N_c - 2N_f)/3$. The function ϕ is:

$$\begin{aligned}\phi(n,\gamma) &= \sum_{m=0}^{\infty}\frac{(-1)^{m+1}}{d_m}\Big(\psi'(m+n+1)-\psi'(m+1)\\ &+ (-1)^{m+1}\left(\beta'(m+n+1)-\beta'(m+1)\right)+\frac{\psi(m+1)-\psi(m+n+1)}{d_m}\Big),\end{aligned}$$

where

$$\beta(z) = \frac{1}{2}\left(\psi\left(\frac{z+1}{2}\right)-\psi\left(\frac{z}{2}\right)\right).$$

End Matter materials

The coefficients $\kappa_{l,k}$:

$$\begin{aligned}
\kappa_{0,1} &= 1, \\
\kappa_{1,1} &= \mathscr{S} - \frac{\pi^2}{24} + \frac{\beta_0}{4N_c}\,\mathrm{H}_n + \frac{1}{8}\left(\psi'\left(\frac{n+1}{2}\right) - \psi'\left(\frac{n+2}{2}\right)\right) \\
&+ \frac{1}{2}\,\psi'(n+1) - \frac{1}{36}\left(67 + 13\frac{N_\mathrm{f}}{N_c^3}\right)\delta_n^0 - \frac{47}{1800}\left(1 + \frac{N_\mathrm{f}}{N_c^3}\right)\delta_n^2, \\
-\kappa_{1,2} &= \frac{\beta_0}{8N_c} + \frac{1}{2}\,\mathrm{H}_n + \frac{1}{12}\left(11 + 2\frac{N_\mathrm{f}}{N_c^3}\right)\delta_n^0 + \frac{1}{60}\left(1 + \frac{N_\mathrm{f}}{N_c^3}\right)\delta_n^2, \\
\kappa_{1,3} &= \frac{1}{24}\left(1 + \frac{N_\mathrm{f}}{N_c^3}\right),
\end{aligned}$$

where $\mathrm{H}_n = \psi(n+1) - \psi(1)$ is harmonic number.