Phase Diagram Structure of QCD under Critical Conditions

I. Aref'eva

Steklov Mathematical Institute, RAS
Physical Department, MGU

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QCD Phase Diagram

- QCD Phase Diagram
  - Confinement/deconfinement
  - First-order phase transitions

- HQCD Phase Diagram
  - Confinement/deconfinement under extreme conditions
    - high densities and magnetic fields
  - First-order phase transitions under extreme conditions
    - high densities and magnetic fields

- Predictions of HQCD and the extent to which they can be experimentally tested
  - energy losses, scattering cross-sections, in particular, direct photons

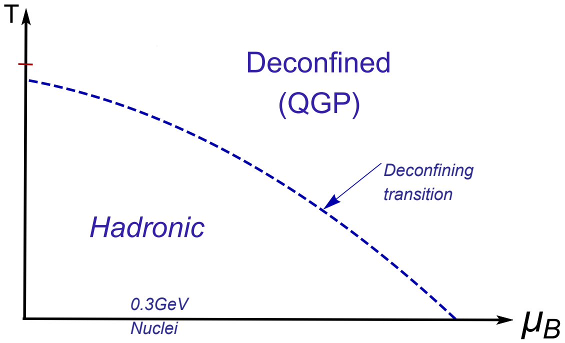
- How to configure HQCD calculations to assess phase structure agreement with experiment via machine learning.

Confinement in QCD

- Wilson criterion (Wilson '74): $<W(C)>_{vac}$ e $^{string\,A(C)}$
- D=4
  - according to pert. theory - not satisfied
  - on a lattice - satisfied (in strong coupling, Wilson '74); record calculations **96** **48^3**
  - D=2 QCD_2 - is satisfied ('t Hooft, Gross)
- On the lattice the main question $\neq 0$ - $_B = \frac{1}{3}[\ _u + \ _d + \ _s]$
- The problem of the sign [Fugacity expansion]

QCD Phase Diagram: Early Conjecture

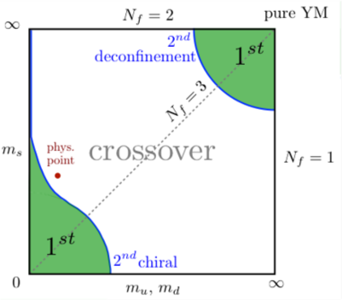
Cabibbo and Parisi, 1975



- a measure of the imbalance between quarks and antiquarks in the system

QCD Phase Diagram: Lattice

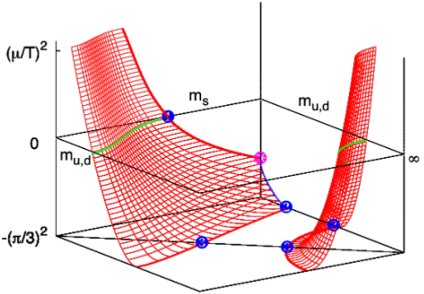
Dependence of phase diagram
on quark mass



Columbia plot

Brown et al., PRL (1990)

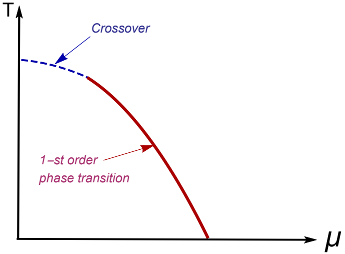
Main problem with $\neq 0$
Imaginary chemical potential method



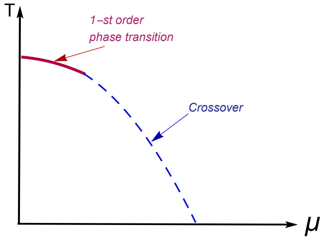
Philipsen, Pinke, PRD (2016)

“Heavy” and “light” quarks from Columbia plot

Light quarks



Heavy quarks



$\mathrm{T}_c = 155 \quad 160 MeV$

$_{CEP} \quad 400 MeV$

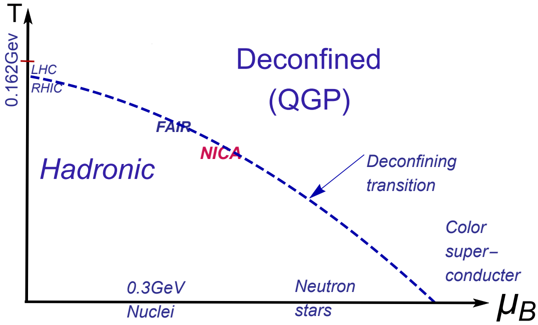
QCD Phase Diagram: Experiments

- LHC, RHIC (2005);
- FAIR (Facility for Antiproton and Ion Research),

  NICA (Nuclotron-based Ion Collider fAcility)

Main goals

- search for signs of the phase transition between hadronic matter and QGP;
- search for new phases of baryonic matter



Holographic QCD -phenomenological approach

- Perturbation methods are not applicable to describe QCD phase diagram
- Lattice methods do not work, because of problems with the chemical potential.
- Holographic QCD - phenomenological model(s)

- One of goals of Holographic QCD – describe QCD phase diagram

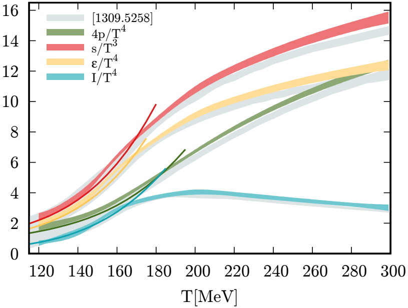
- Requirements:
  - reproduce the QCD results from perturbation theory at short distances
  - reproduce Lattice QCD results at large distances (1 fm) and small $_B$

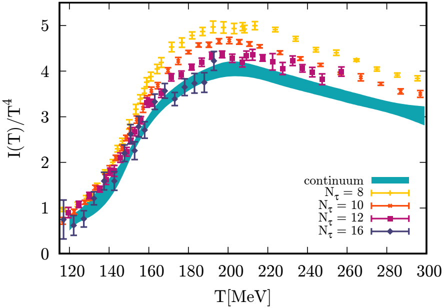
Holographic QCD vs exact AdS/CFT

Maldacena,1998

What is wrong with exact AdS/CFT applications to QCD:

- QCD is not conformal, conformal invariance is restored only in high energy





From:S. Borsanyi et al, arXiv:2502.10267

Left. Pressure (green), entropy (red), energy density (yellow) and trace anomaly (cyan) as functions of the temperature. EoS: $\frac{p(T_0 = 185)}{T_0^4}$ = 1:371. The gray bands from

S. Borsanyi et al, arXiv: 1309.5258

Right Trace anomaly

Holographic QCD vs exact AdS/CFT

Maldacena,1998

What is wrong with exact AdS/CFT applications to QCD:

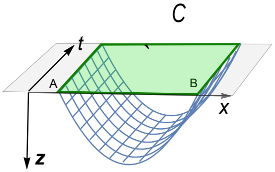
- QCD is not conformal, conformal invariance is restored only in high energy
- $T > T_{HP}$ for BHAdS (check for Poincare BH)
  ADD Plots

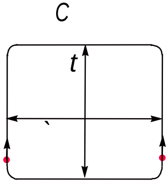
Holographic QCD vs exact AdS/CFT

Maldacena,1998

What is wrong with exact AdS/CFT applications to QCD:

- QCD is not conformal, conformal invariance is restored only in high energy
- $T > T_{HP}$ for BHAdS
- No confinement in $BHAdS_5$





$$W_R[C] \underset{t\to\infty}{\sim} e^{-V(\ell)t}$$

$$V(\ell) \underset{\ell\to\infty}{\sim} \frac{1}{\ell}$$

corresponds to conformal invariance

+ Remark about Sakai-Sugamoto model.

Holographic QCD

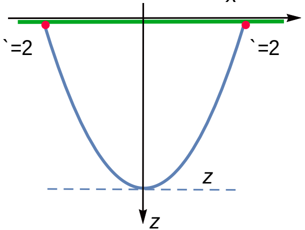
- Temperature in QCD ( ) black hole temperature in (deform.)AdS
- Thermalization in QCD ( ) formation of black hole in (deform.)AdS_5

Time-like Wilson loops in a deformed metric

O.Andreev, V.Zakharov, PRD'07, I.A., K.Rannu, JHEP'18 + P.Slepov, PLB'19.

Metrics in the "Einstein Frame":

$$ds^2 = \frac{b(z)}{z^2} \quad g(z)dt^2 + d\vec{x}^2 + \frac{1}{g(z)}dz^2 \qquad \text{in "string frame"} \qquad b_s = e^{\sqrt{\frac{2}{3}}} \; b \tag{1}$$



String action "on the barn":

$$S = t \int_{`=2}^{`=2} d \quad M(z(\ )) \sqrt{F(z(\ )) + (z^{0}(\ ))^2}; \tag{2}$$

$$M(z) = \frac{b_s(z)}{z^2}; \qquad F(z) = g(z): \tag{3}$$

Lemma: If there is a solution to the equation defining the dynamic wall $(z = z_{DW})$

$$\left. \frac{M^{0}(z)}{M(z)} + \frac{1}{2}\frac{F^{0}(z)}{F(z)} \right|_{z=z_{DW}} = 0; \tag{4}$$

then when` $! \; 1$ $\quad S \quad _{DW}$ `, где $\quad _{DW} = \frac{b(z_{DW})}{(z_{DW})^2} \sqrt{g(z_{DW})}$

this is confinement.

Holographic model of an anisotropic plasma in a magnetic field at a nonzero chemical potential

I.A, K. Rannu, P.Slepov, JHEP, 2021

$$S = \int d^5x \sqrt{g}\left[R \quad \frac{f_1(\ )}{4} F^2_{(1)} \quad \frac{f_B(\ )}{4} F^2_{(B)} \quad \frac{1}{2}\partial_M \ \partial^M \quad V(\ )\right]$$

$$ds^2 = \frac{L^2}{z^2}\, b(z) \left[\ g(z)\, dt^2 + dx^2 + dy_1^2 + e^{c_B z^2} dy_2^2 + \frac{dz^2}{g(z)}\right]$$

$A_{(1);m} = A_t(z)\ ^0_m;\ A_t(0) = \ ;\qquad F_{(B)} = dx \wedge dy^1$

Giataganas'13; IA, Golubtsova'14; Gürsoy, Järvinen '19; Dudal et al.'19

$\mathfrak{b}(z) = e^{2A(z)}$, quarks mass "Bottom-up approach"

Heavy quarks (b, t):

$A(z) = \ cz^2/4$ Andreev, Zakharov'06

$A(z) = \ cz^2/4 + pz^4$ IA, Hajilou, Rannu, Slepov, EPJ C (2023)83

Light quarks (d, u)

$A(z) = \ a \ln(bz^2 + 1)$ Li, Yang, Yuan'17

φ - dilaton, $(z) = e^{\varphi(z)}$ - running coupling in HQCD

Holographic Equation of Motions

$$\phi'' + \phi'\left(\frac{g'}{g} + \frac{3b'}{2b} \quad \frac{3}{z} + c_B z\right) + \left(\frac{z}{L}\right)^2 \frac{\partial f_1}{\partial \phi}\frac{(A_t')^2}{2bg} \quad \left(\frac{z}{L}\right)^2 \frac{\partial f_B}{\partial \phi}\frac{q_B^2}{2bg} = 0;$$

$$A_t'' + A_t'\left(\frac{b'}{2b} + \frac{f_1'}{f_1} \quad \frac{1}{z} + c_B z\right) = 0;$$

$$g'' + g'\left(\frac{3b'}{2b} \quad \frac{3}{z} + c_B z\right) \quad \left(\frac{z}{L}\right)^2 \frac{f_1 (A_t')^2}{b} \quad \left(\frac{z}{L}\right)^2 \frac{q_B^2 \; f_B}{b} = 0;$$

$$b'' \quad \frac{3(b')^2}{2b} + \frac{2b'}{z} \quad \frac{4b}{3z^2} \quad \left(\frac{c_B z^2}{2} \quad \frac{c_B^2 z^4}{2}\right) + \frac{b(\phi')^2}{3} = 0;$$

$$c_B z^2 \quad 2g' + 3g\left(\frac{b'}{b} \quad \frac{4}{3z} + \frac{2c_B z}{3}\right) \quad \left(\frac{z}{L}\right)^3 \frac{L q_B^2 f_B}{b} = 0;$$

$$\frac{b''}{b} + \frac{(b')^2}{2b^2} + \frac{3b'}{b} \quad \frac{g'}{2g}\left(\frac{2}{z} + \frac{2c_B z}{3}\right) \quad \frac{g'}{3zg} \; 9 \quad 3c_B z^2 \quad \frac{2c_B}{3} \; 5 \quad c_B z^2$$

$$+ \frac{8}{z^2} + \frac{g''}{3g} + \frac{2}{3}\left(\frac{L}{z}\right)^2 \frac{bV}{g} = 0;$$

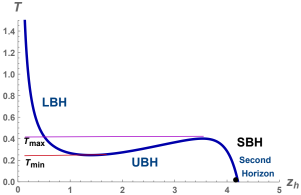
We choose (LQ): $f_1 = e^{\;cz^2 \quad A(z)} = (1 + bz^2)^a e^{\;cz^2}$,

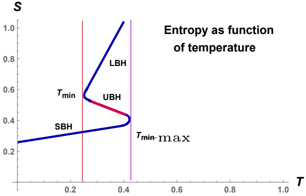
B.C.: $A_t(0) = \;$; $A_t(z_h) = 0$; $g(0) = 1$; $g(z_h) = 0$; $\phi(z_0) = 0$ We have to fix z_0.

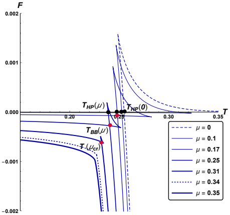
Origin of 1-st order phase transition in HQCD

- $g(z)$ blackenning function. The form of $g(z)$ depends on $A(z)$.
- Due non-monotonic dependence of $T = T(z_h) = g^0(z)=4 \Big|_{z=z_h}$ on z_h, the entropy $s = s(T)$ is not monotonic
- As a consequence the free energy $F = \text{R}\ sdT$ undergoes the phase transition

1-st order phase transition describes transition
from small black holes ! large black holes



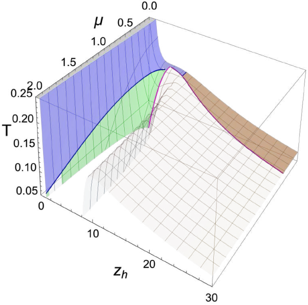




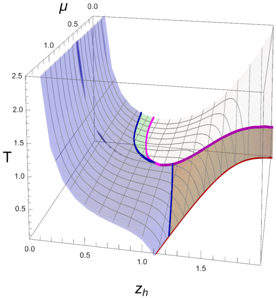
The swallow-tailed shape

- Physical quantities that probe backgrounds are smooth relative to z_h
) their dependence on T should be taken from stable region
- Non-monotonic dependence of $T = T(z_h)$ gives the 1-st PT for corresponding characteristic of QCD

1-st order phase transition in HQCD. 1/3



LQ

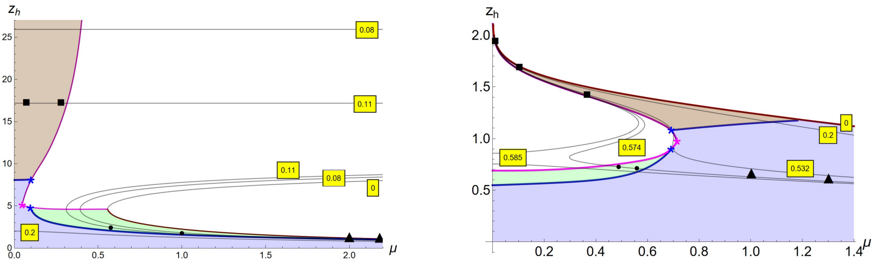


HQ

3D plot $T = T(\;; z_h)$. The brown part of the surface corresponds to the hadronic phases, the blue one corresponds to the quark-gluon plasma and the green one to the quarkyonic phase. 3D plot $T = T(\;; z_h)$

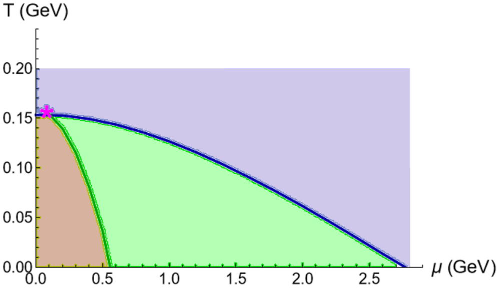
I.A., A.Hajilou, P.Slepov, M.Usova, PRD (2024) **110**, 126009; [arXiv:2402.14512]

1-st order phase transition in HQCD. 2/3

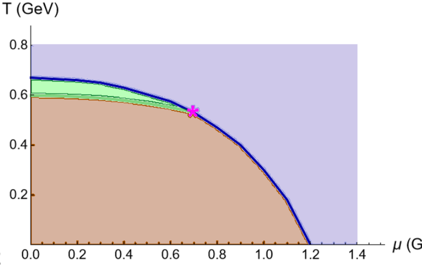


2D plots in (;z_h)-plane for light quarks (A) and heavy quarks (B). Hadronic, quarkyonic and QGP phases are denoted by brown, green and blue, respectively. Solid gray lines show the temperature indicated in rectangles. The intersection of the confinement/deconfinement and 1-st order phase transition lines is denoted by the blue stars. The magenta star indicates CEP

1-st order phase transition in HQCD. 3/3

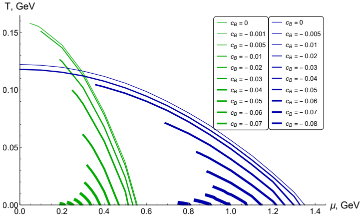


LQ



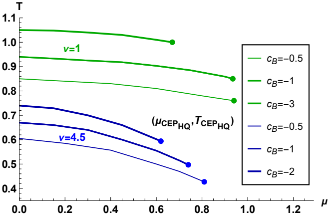
1-st order phase transition in HQCD, $B \neq 0$

Light quarks



I.A, Ermakov, Rannu, Slepov,EPJC'23

Heavy quarks



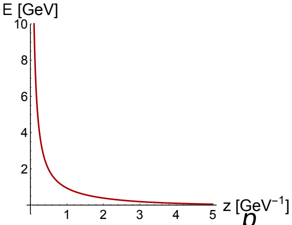
I.A, A. Hajilou, K.R., P.S.EPJC'23

QCD Phase Diagram: Experiments

- Main experimental method to find 1-st order phase transition: Beam Energy Scan Method
- We propose a new method related with study of scattering amplitudes of particles created in colliding heavy ions beams.
- It is related with special behaviour of scattering amplitudes near 1-st order phase transition

Running coupling in QCD vs running coupling in HQCD

- $\alpha(z) = e^{\varphi(z)}$ running coupling in HQCD, here ′ - dilaton,
  E. Kiritsis et al, 1401.0888, 1805.01769
- $\alpha = \alpha(E)$ - running coupling in QCD
- The energy scale as a function of the bulk coordinate z, corresponding to the warp factor for light quark model



Energy scale $E(z) = \frac{\sqrt{b(z)}}{z}$

Holographic Running coupling

$(z) = e^{'(z)}$ I.A. A.Hajilou, P.Slepov, M.Usova, 2402.14512

Light Quark Model

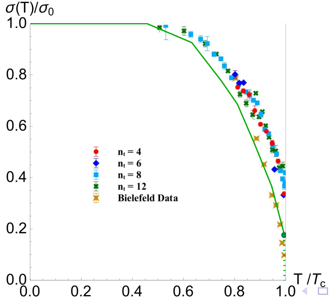
$'(z)$ - dilaton field $'(z)$ is defined up to a constant: $'(z)\Big|_{z=z_0} = 0.$

There are 3 choices: a) $z_0 = 0$ b) $z_0 = f(z_h)$ c) $z_0 = z_h$

$$z_0 = 10\exp[\ z_h/4] + 0.1$$

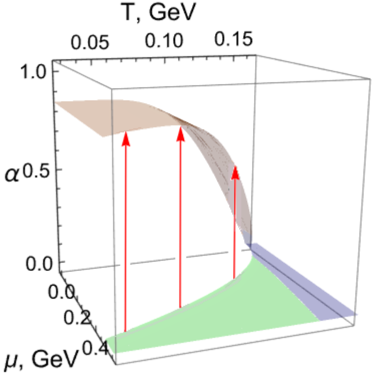
IA, K.Rannu, P.Slepov, JHEP'21

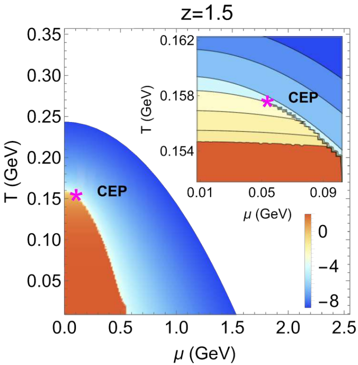
With this boundary condition the temperature dependence of $_s$ fits the known lattice data Cordaso, Bicudo 1111.1317



Holographic Running coupling for $T \neq 0; \quad \neq 0$

I.A. Hajilou, Slepov, Usova, PRD, 2024



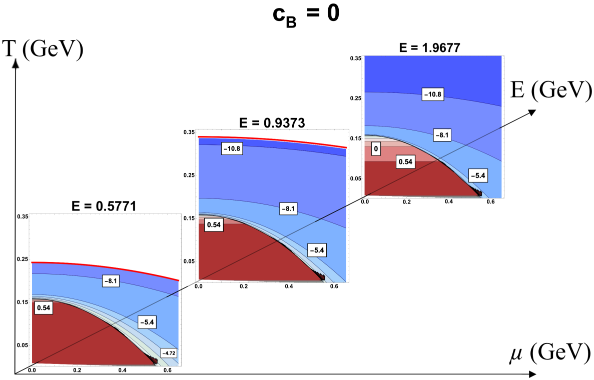


3D plot of $(z; \ ; T)$

Density plot of $\log \ (z; \ ; T)$

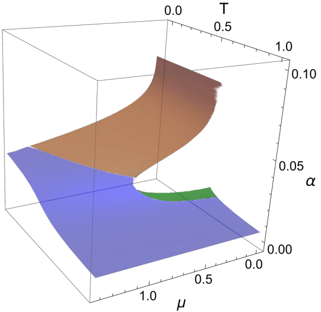
Light quark model, $z = 0$

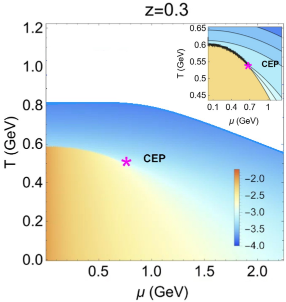
Running Coupling for Light Quark Model



Density plots of log $(E; \;\; ; T)$ at different energy scales.
All values of E on the top of each panel show fixed value of energy E-coordinate.

Running coupling for Heavy Quark Model



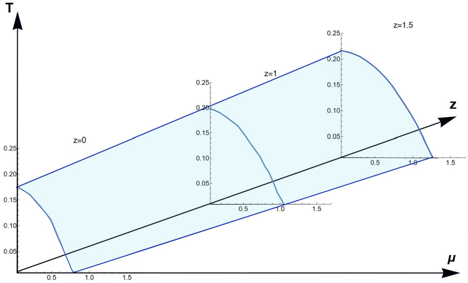


The 3D-plot for coupling constant $= \ _{HQ}(z; \ ; T)$ for heavy quarks at $z = 0$.

Density plots of $\log \ (z; \ ; T)$ for

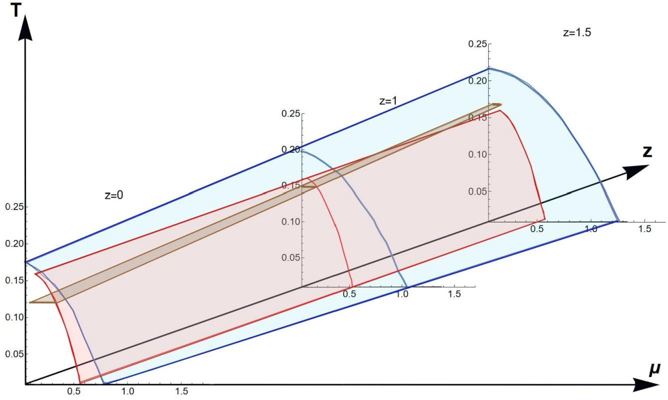
Heavy quarks, $z = 0$

Automodel behaviour of Running coupling



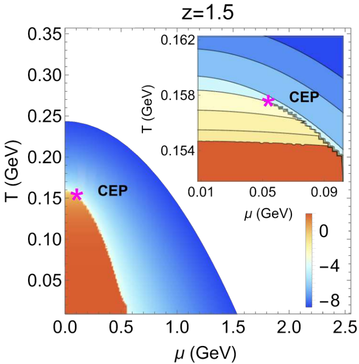
$$(T;\ \ ) = f_{up}(T^2 + c\ \ ^2)$$

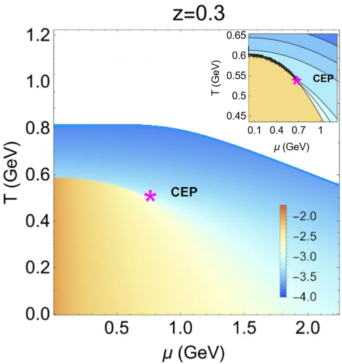
Automodel Behaviour of Running Coupling



$$(T;\ \ ) = f_{below}(T^2 + c\ \ ^2)$$

Light vs Heavy Quarks Phase Diagrams from Scattering Amplitudes. B=0





Density plots of log $(z; \;\; ;T)$

,

This is predicted by Holographic QCD

Dependence on magnetic field I.A., A.Hajilow, A.Nikolaev, P.Slepov

Jet Quenching as function of T and for LQ

1st order - magenta
2nd order - blue

Jet Quenching as function of T and for LQ

We see that junction in JQ is exactly on the line of the rst order transition (as should be)

2-nd order phase transition

1st order - magenta
2nd order - blue

IA, A.Hajilow, A.Nikolaev, P.Slepov

Jet Quenching as function of T and for heavy quarks

Heavy quarks

Jet Quenching as function of T and for heavy quarks

Heavy quarks

Jet Quenching as function of T and for heavy quarks

Heavy quarks

- Junction in JQ is exactly on rst order trans. (as should be).

- After conf./deconf. line - change of the slop

Conclusion

Phase structure in (T; ; B;)-space;
B - characterizes the magnetic eld,
-anisotropy

Dependence of the phase structure in (T; ; B;) - space on quark mass

Jumps in physical quantities (jet quenching, energy loss, etc.) at the rst-order phase transition and their dependence on B and anisotropy parameter .

Smooth variation of physical quantities (running couplings, jet quenching parameter, etc.) across the con nement-decon nement transition.

Main refs 1/2

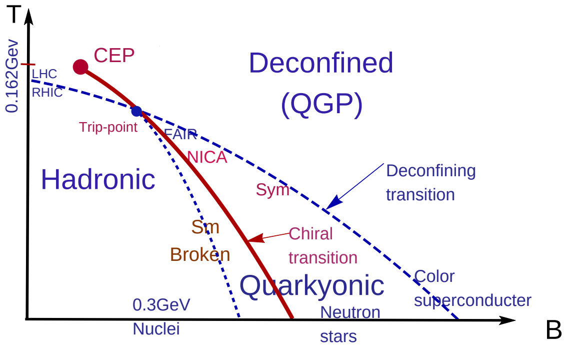
Main refs 2/2

Backup. The expected more detailed QCD phase diagram

Parameter of the chiral symmetry breaking $< \quad >$

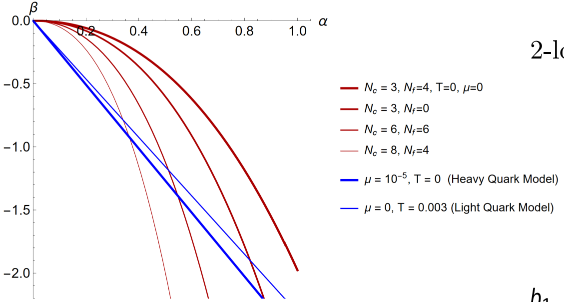
- $< \quad > = 0 \; ()$ -symmetry
- $< \quad > \neq 0 \; ()$ broken -symmetry

Backup. The expected more detailed QCD phase diagram



- Quarkyonic phase: baryon free $\rangle$ dense baryons *McLerran, Pisarski 0706.2191*
  - Baryon density jumps

Backup. function



Beta-function ( ) for QCD at 2-loop level for $T = 0$, $= 0$ at di erent N_c and N_f in red lines, and holographic -function for light quarks at $= 0$, $T = 0{:}003$ (light blue) and heavy quarks $= 10\ ^5$, $T = 0$ (dark blue); [ ] = [T] = GeV.

2-loops QCD -function:

$$(\ ) = \quad b_0 \quad ^2 \quad b_1 \quad ^3;$$

$$b_0 = \frac{1}{2} \quad \frac{11}{3} N_c \quad \frac{2}{3} N_f$$

$$b_1 = \frac{1}{8\ ^2} \quad \frac{34}{3} N_c^2 \quad \frac{13}{3} N_c \quad \frac{1}{N_f}$$

N_c # of colors, N_f #of flavors.

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