

# Formalism of Open Quantum Systems in Neutrino Scattering on Superfluid Helium

SATURNE Project

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# Presentation Outline

- 1 Neutrinos
  - Window to New Physics
  - Neutrino Electromagnetic Properties
  - Neutrino-Atom Scattering
- 2 Superfluid Helium
  - Dynamic Structure Factor
- 3 Open Quantum Systems
  - Formalism and Principles
- 4 SATURNE Project
  - Single Quasiparticle Generation
  - OQS Formalism for Neutrino-Hell Scattering
- 5 Conclusion

# Neutrinos - Window to New Physics

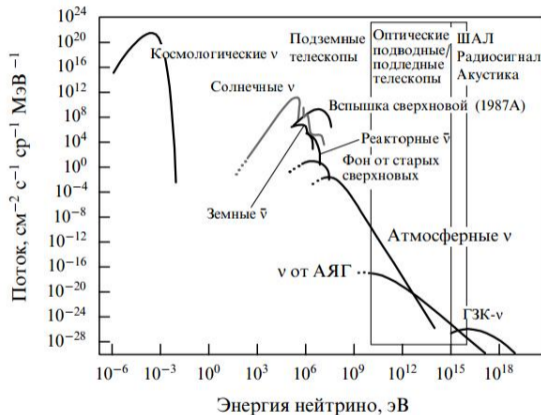
## Relevance

Neutrinos are a window to new physics.

C. Guinti, A. Studenikin, "Neutrino electromagnetic interactions: a window to new physics" Rev. Mod. Phys. 87 (2015) has over 450 citations.

## SATURNE Project

Study of coherent elastic neutrino-atom scattering and setting record constraints on neutrino magnetic moment.



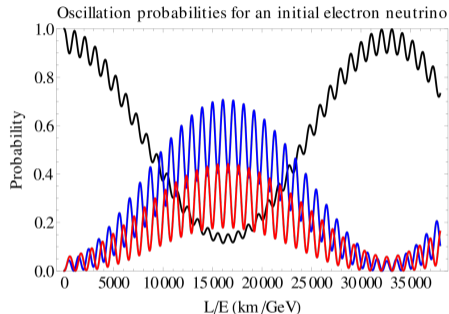
# Neutrino Oscillations: Essence and Significance

## Physical essence:

- Flavor transformation ( $\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$ )
- Requires massive neutrinos ( $m_\nu \neq 0$ )

## Historical milestones:

- 1957 - B. Pontecorvo predicts oscillations
- 1998 - [Super-Kamiokande](#) discovers atmospheric  $\nu_\mu$  oscillations
- 2001 - [SNO](#) confirms solar  $\nu_e$  oscillations
- 2015 - Nobel Prize for neutrino mass discovery



## Fundamental implications

- **New physics:** Standard Model requires modification

## Key fact

Neutrinos have **no electric charge** in SM but massive neutrinos imply non-zero magnetic moment.

## Magnetic moment

SM prediction

$$\mu_\nu^{\text{SM}} \approx 3.2 \times 10^{-19} \mu_B$$

Experimental limits

- Borexino (solar  $\nu$ ):  $\mu_\nu < 2.8 \times 10^{-11} \mu_B$
- GEMMA (reactor  $\bar{\nu}$ ):  $\mu_\nu < 2.9 \times 10^{-11} \mu_B$

C.Guinti, A.Studenikin, "Neutrino electromagnetic interactions: a window to new physics" Rev. Mod. Phys. 87 (2015)

# Coherent Neutrino-Nucleus Scattering (CE $\nu$ NS)

## Physics:

- Neutrino interacts with entire nucleus
- Coherent when  $qR < 1$  ( $q$  - momentum transfer)
- Cross section  $\propto N^2$  ( $N$  - neutron number)
- Sensitive to:
  - Weak angle  $\theta_W$
  - Magnetic moment  $\mu_\nu$
  - New interactions

## Differential cross section:

$$\frac{d\sigma^{\text{CE}\nu\text{NS}}}{dE_R} = \frac{G_F^2}{\pi} C_V^2 m_N \left( 1 - \frac{m_N E_R}{2E_\nu^2} \right),$$

$$C_V = \frac{1}{2} [(1 - 4 \sin^2 \theta_W) Z F_Z - N F_N]$$

## Modern experiments:

- **COHERENT** (USA):
  - First CE $\nu$ NS observation (2017)
  - CsI[Na], Ar, Ge detectors
- **CONUS** (Germany):
  - Ge detectors at reactor
- **MINER** (USA):
  - Cryogenic detectors
- **RED-100** (Russia):
  - Liquid xenon

## Coherence condition:

$$q \cdot R_{\text{atom}} \ll 1$$

where:  $q$  – momentum transfer,  $R_{\text{atom}}$  – atomic radius

## Differential cross section (nucleus):

$$\frac{d\sigma^{\text{CE}\nu\text{NS}}}{dE_R} = \frac{G_F^2}{\pi} C_V^2 m_N \left( 1 - \frac{m_N E_R}{2E_\nu^2} \right),$$

$$C_V = \frac{1}{2} [(1 - 4 \sin^2 \theta_W) Z F_Z - N F_N]$$

## Modification for atom:

$$C_V^{\text{Atom}} = C_V + \frac{1}{2} (\pm 1 + 4 \sin^2 \theta_W) Z F_e(q^2)$$

- + for  $\nu_e, \bar{\nu}_e$ , – for other neutrinos
- $F_e(q^2)$  – electron form factor

## Key effects:

- Electrons screen the weak charge of the nucleus
- Sharp dip in cross section at  $T_R \sim 9$  meV
- Complete screening when  $C_V^{\text{Atom}} = 0$

## Why helium specifically?

- **Smallest atomic radius** ( $R_{\text{atom}} \approx 0.5$  Å)
- Light atom  $\Rightarrow$  large recoil energy
- Detection possibility via quantum evaporation

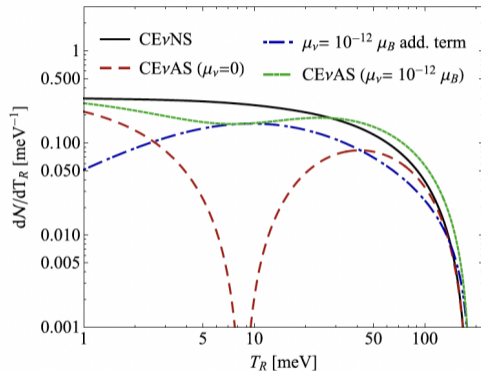
## Cross section modification:

$$\left. \frac{d\sigma^{\text{CE}\nu\text{AS}}}{dT_R} \right|_{\mu_\nu \neq 0} \simeq \frac{d\sigma^{\text{CE}\nu\text{AS}}}{dT_R} + \frac{\pi\alpha^2 Z^2}{m_e^2} \left( \frac{\mu_\nu}{\mu_B} \right)^2 \cdot \left( \frac{1}{T_R} - \frac{1}{E_\nu} \right) (1 - F_e(T_R))^2,$$

## Features:

- Atomic form factor  $(1 - F_e(T_R))^2$
- Enhancement at low  $T_R$
- For helium ( $Z = 2$ ):

$$\propto \left( \frac{\mu_\nu}{\mu_B} \right)^2 \frac{1}{T_R}$$



M. Cadeddu et al., Phys. Rev. D 100 (2019)

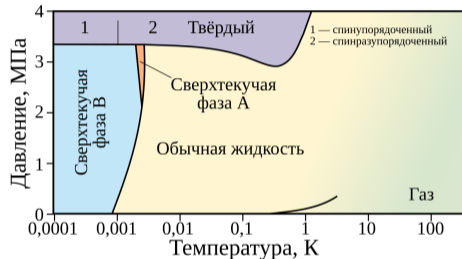
# Superfluid Helium (He-II)

## Unique properties:

- Transition temperature  $T_\lambda = 2.17$  K
- Density  $\rho = 145$  kg/m<sup>3</sup>
- Sound velocity  $c_1 \approx 238$  m/s
- Second sound velocity  $c_2 \approx 20$  m/s

## Discovery history:

- **1908** - H. Kamerlingh Onnes: **Liquefaction**
- **1938** - P. Kapitsa: **Superfluidity**
- **1941** - L. Landau: **Two-fluid model**
- **1995** - E. Cornell, C. Wieman: **BEC in gases** (Nobel Prize 2001)
  - Normal component ( $\rho_n$ )
  - Superfluid component ( $\rho_s$ )



Kapitsa, P. L. Viscosity of Liquid Helium below the  $\lambda$ -Point. *Nature*, 1938

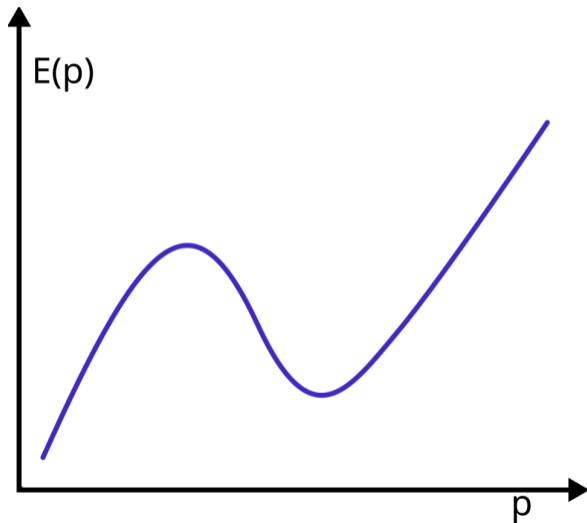
# Superfluid Helium: Quasiparticles

## Fundamental properties:

- Zero viscosity at  $T < T_\lambda$
- Anomalous thermal conductivity ( $10^6$  times > copper)
- Quantized vortices with circulation  $\kappa = h/m$
- Existence of second sound

## Main types of quasiparticles:

- Phonons:
  - Linear dispersion law:  $\epsilon(p) = c_1 p$
  - Dominate at  $T < 0.5$  K
- Rotons :
  - Minimum at  $p_0 \approx 1.92 \text{ \AA}^{-1}$
  - Energy:  $\Delta \approx 8.6$  K
  - Mass:  $m^* \approx 0.16 m_{\text{He}}$



# Dynamic Structure Factor in Superfluid Helium

The dynamic structure factor  $S(\mathbf{q}, \omega)$  characterizes:

- System response to perturbation with wave vector  $\mathbf{q}$  and frequency  $\omega$

## Key formulas:

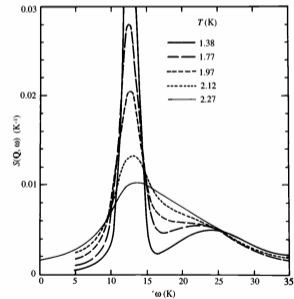
- Definition:

$$S(\mathbf{q}, \omega) = \frac{1}{2\pi N} \int dt e^{i\omega t} \langle \rho_{\mathbf{q}}(t) \rho_{-\mathbf{q}}(0) \rangle$$

- Peaks in  $S(\mathbf{q}, \omega)$  correspond to **elementary excitations**
- Peak width characterizes **lifetime** of excitations
- 

$$\frac{d^2\sigma}{d\Omega d\omega} = b_c^2 \frac{k'}{k} S(Q, \omega)$$

Alan Griffin, *Excitations in a Bose-condensed Liquid*, Cambridge University Press, 1993



$$S(q, \omega) = S_s + S_m + S_{\text{inc}}$$

# Open Quantum Systems: Formalism and Applications

## Definition:

- Systems interacting with environment (reservoir)
- Irreversible processes: dissipation, decoherence
- Described by **reduced density matrix**  
 $\rho_s = \text{Tr}_E(\rho_{\text{total}})$

## Key equations:

- **Master equation (Lindblad equation):**

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \mathcal{D}[\rho]$$

$$\mathcal{D}[\rho] = \sum_k \gamma_k \left( L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right)$$

## Historical development:

- **1960s:** Works by Haake, Zubarev
- **1976:** G. Lindblad - **Lindblad equation**
- **1980s:** Quantum trajectories (Dalibard, Harrow)
- **1990-2000:** Quantum information (Kraus, Nielsen)

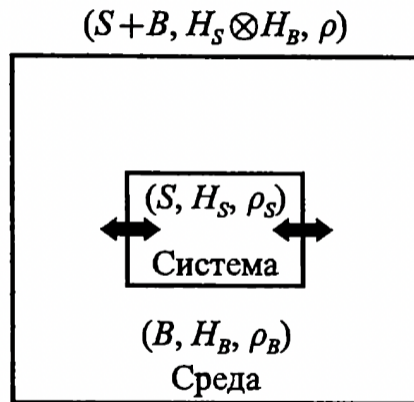
## Application areas:

- **Quantum optics:** lasers, resonators
- **Quantum information:** qubit decoherence
- **Chemical physics:** quantum molecular dynamics
- **Biophysics:** photosynthesis
- **Quantum technologies:** sensors

## Main components:

- **System**: Studied system
- **Environment**: Surrounding medium
- Interaction:  $H_{int}$

H.-P. Breuer, F. Petruccione, "The Theory of Open Quantum Systems"



# Derivation of the Lindblad Equation: Main Steps

## Step 1: Total system

- System + Environment = Isolated system
- Hamiltonian:

$$H_{\text{total}} = H_S \otimes I_E + I_S \otimes H_E + H_{\text{int}}$$

- Interaction:  $H_{\text{int}} = \sum_k A_k \otimes B_k$

## Step 2: von Neumann equation

$$\dot{\rho}_{\text{total}} = -\frac{i}{\hbar} [H_{\text{total}}, \rho_{\text{total}}]$$

## Step 3: Approximations

- **Born approximation:** Weak interaction
- **Markov approximation:** No environment memory
- Initial state:  $\rho(0) = \rho_S(0) \otimes \rho_E$

## Step 4: Partial trace

$$\dot{\rho}_S = \text{Tr}_E \left( -\frac{i}{\hbar} [H_{\text{total}}, \rho_{\text{total}}] \right)$$

## Step 5: Lindblad equation

$$\dot{\rho}_S = -\frac{i}{\hbar} [H_S, \rho_S] + \mathcal{D}[\rho_S]$$

where dissipator:

$$\mathcal{D}[\rho_S] = \sum_k \gamma_k \left( L_k \rho_S L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho_S\} \right)$$

## Lindblad operators $L_k$ :

- Describe dissipation channels
- $\gamma_k$  - relaxation rate

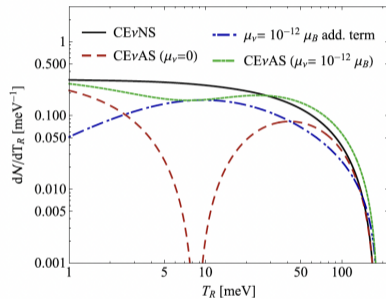
SATURNE aims to study coherent elastic neutrino-atom scattering and search for neutrino magnetic moment.

## Project goals:

- First detection of neutrino-atom scattering
- Search for neutrino magnetic moment

## He-II based detector:

- Sensitivity to recoil energies  $\sim 2$  meV
- Sensitivity to neutrino-atom scattering



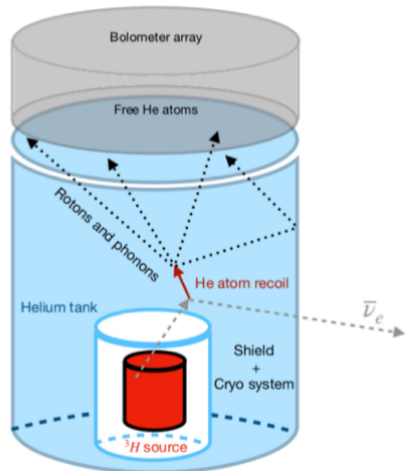
M. Cadeddu et al., Phys. Rev. D 100 (2019)

## Experimental stages:

- 1 Neutrino scattering
- 2 Recoil atom production
- 3 Quasiparticle generation
- 4 Detection

## Aim:

- Obtain quasiparticle spectrum from tritium source neutrino scattering



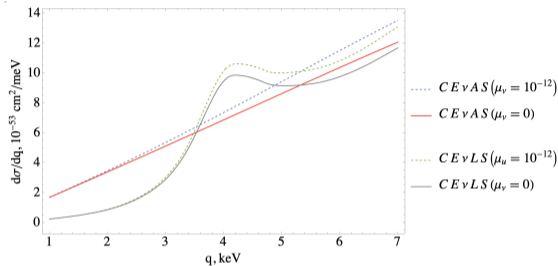
# Neutrino Scattering with Recoil Energy $< 2$ meV

With collective effects:

$$\frac{d\sigma}{dT} = \int \frac{d\sigma^{\text{CE}\nu\text{AS}}}{dq} S(q, T) dq$$

Dynamic structure factor:

$$S(q, \omega) = S_s + S_m + S_{\text{inc}}$$



## Key findings

- Collective effects **significantly affect** cross section at  $T \sim 1$  meV
- Roton production shows **resonant behavior** at  $E \approx \Delta_{\text{rot}}$

K. Kouzakov et al., Phys. Atom. Nucl (2025) (accepted)

## Before:

- Neutrino + Atom (He)

## After:

- Neutrino + recoiling atom (He)
- Recoil energy: 2-100 meV

## Open quantum system:

- **System:** Recoil atom
- **Environment:** Superfluid helium
- Processes:
  - Quasiparticle excitation
  - Energy dissipation

## Theoretical description:

$$H = H_{\text{atom}} + H_{\text{Hell}} + H_{\text{int}}$$

## Post-scattering processes

- Cascade of energetic He atoms
- Phonon/roton excitations

# OQS Formalism for Neutrino Scattering on Hell

- Evolution equation:

$$\frac{d}{dt}\rho(t) = -i [H_I(t), \rho(t)]$$

- Density matrix decomposition:

$$\rho(t) \approx \rho_S(t) \otimes \rho_B$$

- Interaction:

$$H_I = \sum_{\alpha} A_{\alpha} \otimes B_{\alpha} \sim j_{\mu} J^{\mu}$$

K.Matchev, et al. Superfluid effective field theory for dark matter direct detection, JHEP, 2022.

- Equation for reduced density matrix:

$$\begin{aligned} \frac{d}{dt}\rho_S(t) = \int_0^{\infty} ds \operatorname{tr}_B \{ & H_I(t-s)\rho_S(t)\rho_B H_I(t) \\ & - H_I(t)H_I(t-s)\rho_S(t)\rho_B \} + \text{h.c.} \end{aligned}$$

$$\begin{aligned} \frac{d}{dt}\rho_S(t) \sim \sum_{\omega, \omega'} \sum_{\alpha, \beta} e^{i(\omega' - \omega)t} \Gamma_{\alpha\beta}(\omega) \\ \times [j_{\beta}(\omega)\rho_S(t)j_{\alpha}^{\dagger}(\omega') \\ - j_{\alpha}^{\dagger}(\omega')j_{\beta}(\omega)\rho_S(t)] + \text{h.c.} \end{aligned}$$

- Correlation function:

$$\Gamma_{\alpha\beta} \equiv \int_0^{\infty} ds e^{i\omega s} \langle J_{\alpha}^{\dagger}(t) J_{\beta}(t-s) \rangle$$

# Key Results

K. Stankevich, A. Studenikin, M. Vyalkov, "Generalized Lindblad master equation for neutrino evolution", Phys. Rev. D 111, **2025**

For superfluid helium

$$\Gamma_{\alpha\beta}(Q, \omega) \approx \Gamma_{00}(Q, \omega) \sim S(Q, \omega)$$

We obtain the Lindblad equation

$$\frac{d}{dt}\rho_S(t) = -i[H_{LS}, \rho_S(t)] + \mathcal{D}(\rho_S(t))$$

$$\mathcal{D}(\rho_S) = \sum_{\omega} \sum_{\alpha, \beta} S_{00}(\omega) \left( j_0(\omega) \rho_S j_0^\dagger(\omega) - \frac{1}{2} \left\{ j_0^\dagger(\omega) j_0(\omega), \rho_S \right\} \right)$$

## Results

- Evolution equation for the helium atom after neutrino scattering has been obtained for recoil energies above 2 meV
- Open quantum systems formalism has been developed to describe quasiparticle generation in superfluid helium for the SATURNE experiment

## Prospects:

- Development of the formalism to account for various scattering channels of helium atoms
- Obtaining estimates for the quasiparticle spectrum from neutrino scattering off a tritium source