Rare Four-leptonic B^- Meson Decays

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Motivation

We study four-leptonic decays $B^- \to \ell^+ \ell^- \bar{\nu}'_\ell \ell'^-$.

- Rare decays allow a precise test of Standard Model. Deviations from theoretical predictions may indicate physics Beyond the SM.
- The previous theoretical predictions have certain limitations:
  - Massless lepton approximation (simplifies bremsstrahlung contribution and kinematics).
  - Only light mesons resonances contribution.
  - Form factors are taken in non-gauge invariant form.

  Our aim is to address these limitations to provide more accurate theoretical prediction.

Effective Leptonic Hamiltonian

The effective Hamiltonian for the decay $B^- \to \ell^+\ell^-\bar{\nu}_\ell\ell^-$ consists of:

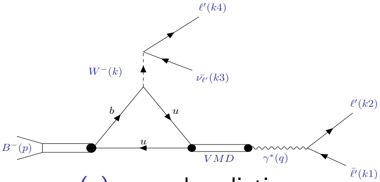
$$\mathcal{H}_{\text{eff}}(x) = \mathcal{H}_W(x) + \mathcal{H}_{\text{em}}(x).$$

The contribution from $b \to uW^- \to u\ell\bar{\nu}_\ell$ transition is

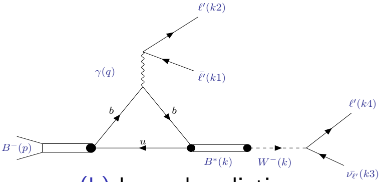
$$\mathcal{H}_{weak}(x) = -\frac{G_F}{\sqrt{2}} V_{ub} \left(\bar{u}(x)\gamma^\mu(1-\gamma^5)b(x)\right) \left(\bar{\ell}(x)\gamma_\mu(1-\gamma^5)\nu_\ell(x)\right) + \text{h.c.},$$

the electromagnetic:

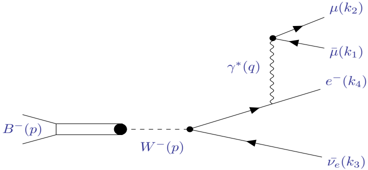
$$\mathcal{H}_{\text{em}}(x) = -e\sum_f Q_f \left(\bar{f}(x)\gamma^\mu f(x)\right) A_\mu(x) = -j^\mu_{\text{em}}(x)A_\mu(x)$$



(a) u-quark radiation



(b) b-quark radiation



(c) bremsstrahlung

The amplitude for the process:

$$
\begin{aligned}
\mathcal{M}_{fi}(q^2, k^2) &\sim \frac{1}{q^2} T^{\nu\mu}(q,k)\, j_\nu(k_2,k_1)\, J_\mu(k_4,k_3) \\
\mathrm{T}_{\nu\mu}(q,k) &= \mathrm{T}^{(u)}{}_{\nu\mu}(q,k) + \mathrm{T}^{(b)}{}_{\nu\mu}(q,k) + \mathrm{T}^{(brem)}{}_{\nu\mu}(q,k)
\end{aligned}
$$

Bremsstrahlung

The bremsstrahlung amplitude:

$$\mathcal{M}^{(brem)} = -\frac{\mathcal{A}}{q^2}\mathrm{i}\, f_{B_u}\, g_{\mu\nu}\, j^{\nu}(k_2, k_1)\, \tilde{J}^{\mu}(k_4, k_3),$$

$$\tilde{J}^{\mu}(k_4, k_3) = J^{\mu}(k_4, k_3) + \frac{m_{\ell'}}{(p-k_3)^2 - m^2{}_{\ell'}}\left(\bar{\ell}'(k_4)\gamma^{\mu}(\hat{p} + m_{\ell'})\,(1-\gamma_5)\,\nu_{\ell'}(-k_3)\right)$$

the 2nd term is taken into account

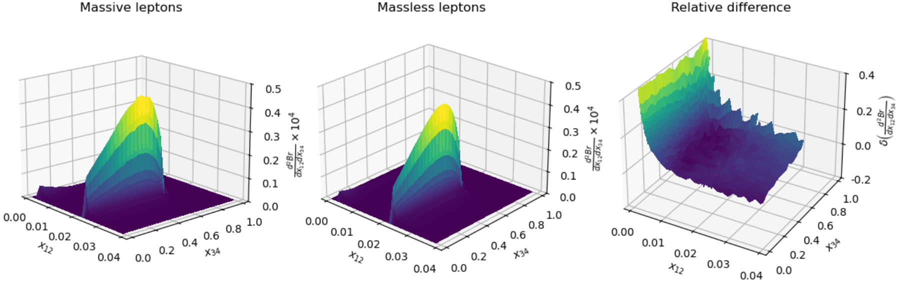


Figure: Double differential distribution $\frac{d^2Br(B^- \to \mu^+\mu^-\bar{\nu}_e e^-)}{dx_{12}\,d_{x34}}$, where x_{12}, x_{34} - lepton pair invariant masses squared

Heavy Vector Meson Resonances

All resonances are described using the Breit-Wigner:

$$\mathcal{M}^{(u)} = -\mathrm{i}\frac{\mathcal{A}}{q^2}\left[\sum_{V=\rho\,,\omega}\frac{I_V\, M_V\, f_V}{q^2 - M_V^2 + \mathrm{i}\,\Gamma_V M_V}\,\mathcal{F}^{(V)}_{\mu\nu}(k^2)\right]\times j^{\nu}(k_2,k_1)\,J^{\mu}(k_4,k_3)$$

$$T_{\nu\mu}(q,k) = \varepsilon_{\nu\mu qk}\frac{e\,a(q^2,k^2)}{M_1} - \mathrm{i}\left(g_{\nu\mu} - \frac{q_\nu q_\mu}{q^2}\right) eM_1\, b(q^2,k^2)$$
$$-\mathrm{i}e\left(k_\nu - \frac{(q\cdot k)}{q^2}q_\nu\right)\left(k_\mu\frac{2d(q^2,k^2)}{M_1} - q_\mu\frac{2c(q^2,k^2)}{M_1}\right) \quad - \mathrm{i}Q_{B_u} e f_{B_u}\frac{q_\nu k_\mu}{q^2},$$

At $q^2 = 0$ gauge invariance constraints form factors $a(q^2,k^2)$,...,$d(q^2,k^2)$.
All form factors are modified:

$$\tilde{a}(q^2,k^2) = a(0,k^2) + \frac{q^2}{{M_V}^2}a(q^2,k^2)$$
$$a(0,k^2) = 0, \quad b(0,k^2) = 0$$
$$d(0,k^2) = \frac{Q_B f_B}{M_1}\frac{1}{1-x_{34}}, \quad c(0,k^2) = -\frac{Q_B f_B}{M_1}\frac{1}{1-x_{34}} + \frac{b(0,k^2)}{1-x_{34}}$$

Heavy Meson Contribution

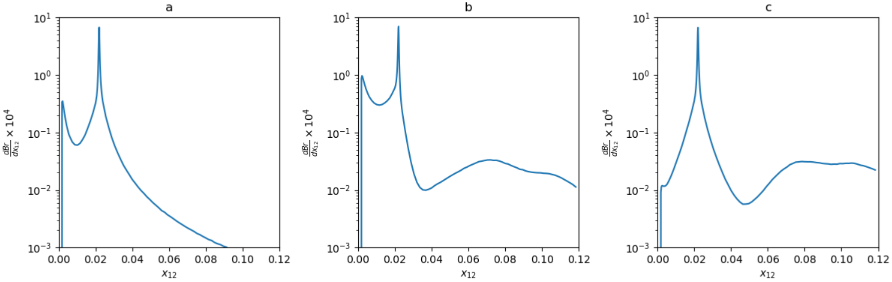


Figure: Differential distributions for $dBr(B^- \to \mu^+\mu^-\bar{\nu}_e e^-)/dx_{12}$

a) $\rho(770)$ and $\omega(787)$ only,

b) $\rho(1450)$, $\omega(1420)$, $\rho(1700)$ and $\omega(1650)$ added,

c) form factors are taken in gauge invariant form.

Numerical Results

Comparison of theoretical predictions obtained by different authors

Refs.	Kinematic range, GeV^2	Process
	$B^- \to \mu^+\mu^-\bar{\nu}_e e^-$	
[1]	full phase space	6×10^{-8}
[2]	$[4m_\mu^2, 1]$	6.02×10^{-8}
[3]	$[4m_\mu^2, 6]$	1.78×10^{-8}
[4]	full phase space	3.01×10^{-8}
[5]	full phase space	3.19×10^{-8}
	full phase space	3.7×10^{-8}
	$B^- \to e^+e^-\bar{\nu}_\mu \mu^-$	
[1]	$0.1 < q^2$	8×10^{-8}
[2]	$[0.0025, 1]$	7.59×10^{-8}
[3]	$[0.0025, 0.96]$	2.48×10^{-8}
[4]	full phase space	6.38×10^{-8}
[5]	full phase space	3.78×10^{-8}
	full phase space	3.9×10^{-8}

Summary

1. Lepton masses are fully taken into account, leading to reduced approximations in the bremsstrahlung contribution.
2. Contributions from heavy vector meson resonances $\rho\,(1450)$, $\omega\,(1420)$, $\rho\,(1700)$, $\omega\,(1650)$ are included.
3. Hadronic form factors have been modified to restore gauge invariance.

$$
\begin{aligned}
\mathcal{B}r\,(B^- \to e^+ e^- \nu_\mu \mu^-) &= 3.9 \times 10^{-8} \\
\mathcal{B}r\,(B^- \to \mu^+ \mu^- \nu_e e^-) &= 3.7 \times 10^{-8} \\
\mathcal{B}r\,(B^- \to \mu^+ \mu^- \nu_e e^-) &< 1.6 \times 10^{-8} \text{ (LHCb upper limit)}
\end{aligned}
$$

In progress:

- theoretical predictions for decays involving interference between identical leptons

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Backup

$$T_{\nu\mu}(q,k) = \varepsilon_{\nu\mu qk}\frac{e\,a(q^2,k^2)}{M_1} - \mathrm{i}\left(g_{\nu\mu} - \frac{q_\nu q_\mu}{q^2}\right) eM_1\, b(q^2,k^2)$$

$$- \quad \mathrm{i}e\left(k_\nu - \frac{(q\cdot k)}{q^2}q_\nu\right)\left(k_\mu\frac{2d(q^2,k^2)}{M_1} - q_\mu\frac{2c(q^2,k^2)}{M_1}\right) - \mathrm{i}Q_{B_u}ef_{B_u}\frac{q_\nu k_\mu}{q^2},$$

singularity at $q^2 \to 0$.

$$Q_{B_u}f_{B_u} - \frac{2(qk)}{M_1}d(q^2,k^2)|_{q^2\to 0} = 0 \quad \Rightarrow \quad d(0,k^2) = \frac{Q_{B_u}f_{B_u}}{M_1}\frac{1}{1-x_{34}}$$

$$d(q^2,k^2) = d(0,k^2) + \Delta(q^2,k^2)$$

VMD: $\Delta(q^2,k^2) = \frac{f(k^2)}{q^2-\delta}, \quad \delta = {M_V}^2 - i\Gamma_V M_V \approx {M_V}^2$

$\Delta(0,k^2) = -f(k^2)/\delta \neq 0 \Rightarrow$ subtraction procedure :

$$\tilde{\Delta}(q^2,k^2) = \Delta(q^2,k^2) - \Delta(0,k^2) \approx \frac{q^2}{{m_V}^2}\Delta(q^2,k^2)$$

$$\tilde{\Delta}(q^2,k^2) = 0$$

$$d(q^2,k^2) = \frac{Q_{B_u}f_{B_u}}{M_1}\frac{1}{1-x_{34}} + \frac{q^2}{{M_V}^2}\Delta(q^2,k^2)$$