Nonfactorizable charming loops in exclusive rare FCNC B decays

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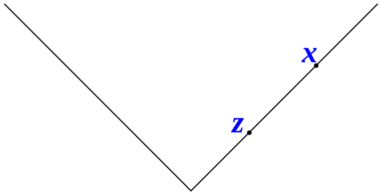
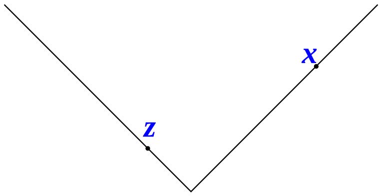
I compare nonfactorizable correction induced by charm-quark loops in exclusive FCNC B-decays with a specific correction to SL B-decay form factor; both are given via three-particle distribution amplitude (3DA) of the B-meson

$\langle 0|\bar{q}(x)G_{\mu\nu}(z)b(0)|B(p)\rangle$:

- **The SL form factor correction is dominated by the *collinear* LC configuration:** $x^2=0$, $z^2=0$, **and** $z_\mu = u x_\mu$, $0<u<1$.



- **In contrast, the FCNC amplitude is dominated by a different LC configuration with *non-collinear* arguments:** $x^2=0$, $z^2=0$, **but** $(x-z)^2\neq 0$.



- **The basic object for our analysis is**

$$A(p|q,q') = \int dx\, \exp(iqx)\langle 0|T\left\{j(x), J(0)\right\}|B(p)\rangle,$$

where j and J are bilinear quark currents.

SL B-decays:

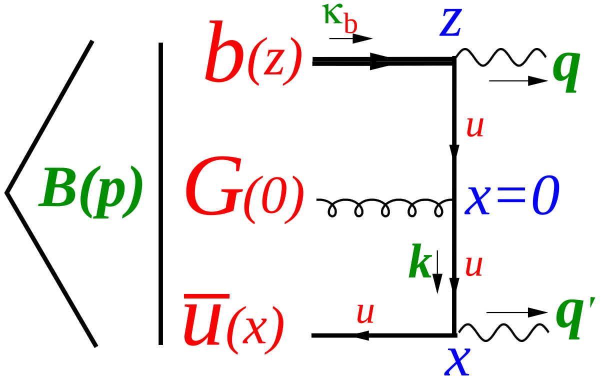
$$A_{\rm SL}(p|q,q') = \int dx\, \exp(iq'x)\langle 0|T\left\{\bar u(x)\gamma_\nu u(x), \bar u(0)\gamma_\mu b(0),\right\}|B_u(p)\rangle.$$

FCNC B decays:

$$A_{\rm FCNC}(p|q,q') = \int dy\, \exp(iqy)\langle 0|T\left\{\bar c(y)\gamma_\mu c(y), \bar s(0)\gamma_\nu s(0)\right\}|B_s(p)\rangle,$$

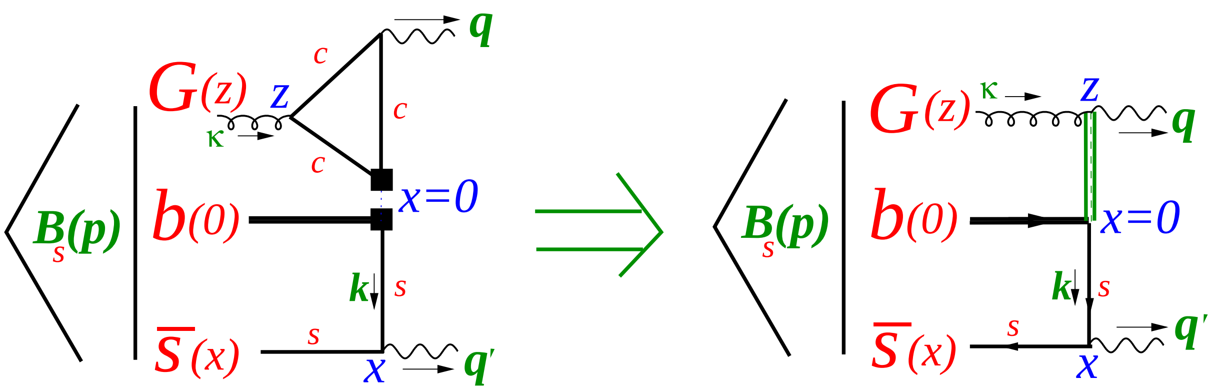
We expand the T-product to the necessary order in G_F and QCD couplings when the 3DA contributions emerge.

- **SL Amplitude**



$$A_{\rm SL}(p|q,q') = \int dx dz d\kappa_b\, e^{i\kappa_b z}\frac{1}{m_u^2-(q-\kappa_b)^2}\frac{dk\, e^{-ikx+iq'x}}{m_u^2-k^2}\,\langle 0|\bar u(x)G(0)b(z)|B_s(p)\rangle.$$

- **FCNC Amplitude**



$$A_{\rm FCNC}(p|q,q') \; = \; \int dy \exp(iqy)\langle 0|T\left\{\bar c(y)\gamma_\mu c(y), \bar s(0)\gamma_\nu s(0)\right\}|B_s(p)\rangle$$

$$= \; \int dx dz d\kappa \, e^{i\kappa z}\Gamma_{cc}(\kappa,q)\frac{dk\, e^{-ikx+iq'x}}{m_s^2-k^2}\langle 0|\bar s(x)G(z)b(0)|B_s(p)\rangle$$

with Γ_{cc} quadratic function in momentum variables

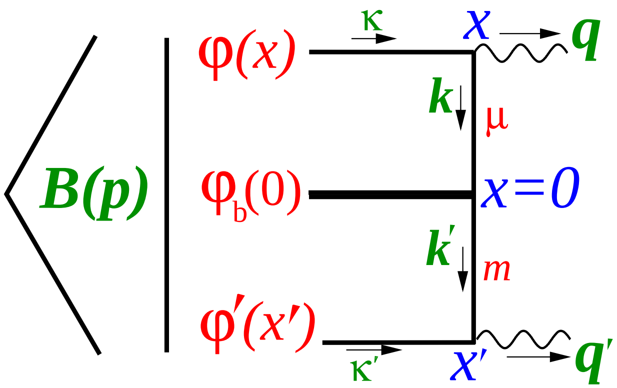
$$\Gamma_{cc}(\kappa,q) = \frac{1}{8\pi^2}\int\limits_0^1 du \int\limits_0^1 dv \frac{\theta(u+v<1)}{m_c^2-uv(\kappa-q)^2-u(1-u-v)\kappa^2-v(1-v-u)q^2}.$$

So, FCNC is similar to a SL amplitude, with the light-quark propagator replaced by an "effective" propagator $\Gamma_{cc}(\kappa,q)$. [We make use of $\Lambda_{\rm QCD}m_b/m_c^2\simeq 1$.]

The main difference between SL and FCNC ampltides is that in SL decay the b-quark hits the end-point of the line along which fast light degrees of freedom propagate, while in the FCNC the b-quark hits the middle point of this line.

Let us study the consequences of this difference.

• 3-particle BS contribution to amplitudes of B-decays of FCNC topology



$\phi_b(0)$ is heavy, and ϕ and ϕ' are light.

$$A(p|q,q') = \int \frac{dx dx' dk dk'}{(\mu^2-k^2)(m^2-k'^2)} e^{iqx+ikx+iq'x'-ik'x'} \langle 0|\phi(x)\phi_b(0)\phi'(x')|B(p)\rangle,$$

We shall derive the leading-order behaviour of this amplitude for large m_b.
Consider $q^2 = q'^2 = 0$, work in the rest frame of the B-meson, take q along $(+)$-axis, and q' along the $(-)$-axis, $q_+ \propto M_B$, $q'_- \propto M_B$.

The x-vertex

Introduce $\kappa = k + q$, the momentum carried by the constituent field $\phi(x)$:

$$\int \frac{dx d\kappa}{\mu^2-(\kappa-q)^2} e^{i\kappa x}\langle 0|\phi(x)...|B(p)\rangle.$$

Due to properties of the B-meson 3BS, the vector κ is soft, $\kappa_\mu \sim O(\Lambda_{\rm QCD})$.

The component $q_+ \sim M_B$ is large, and the propagator is highly virtual,
$\mu^2 - 2\kappa_-(\kappa_+ - q_+) + \kappa_\perp^2 \sim \Lambda_{\rm QCD} M_B$.

Expand the field operator $\phi(x)$ near $x = 0$. The expansion in powers of x_- and $x_\perp$ leads to a well behaved Taylor series for the amplitide because

$$x_- e^{i\kappa_+ x_-}\frac{1}{\mu^2 - 2\kappa_-(\kappa_+ - q_+) + \kappa_\perp^2} \to e^{i\kappa_+ x_-}\frac{\kappa_-}{(\mu^2 - 2\kappa_-(\kappa_+ - q_+) + \kappa_\perp^2)^2}.$$

Since $\kappa_- = O(\Lambda_{\rm QCD})$ and the virtuality of the propagator is $O(\Lambda_{\rm QCD} M_B)$, any term $(x_-)^n$ is suppressed by a factor $(1/M_B)^n$ compared to the term $(x_-)^0$. The same property holds for $(x_\perp)^n$.

However, for powers of the variable x_+ the situation is different:

$$x_+ e^{i\kappa_- x_+}\frac{1}{\mu^2 - 2\kappa_-(\kappa_+ - q_+) + \kappa_\perp^2} \to e^{i\kappa_- x_+}\frac{q_+}{(\mu^2 - 2\kappa_-(\kappa_+ - q_+) + \kappa_\perp^2)^2}.$$

$q_+ \sim M_B$, all powers of x_+^n have the same order of magnitude $O(1)$: Taylor expansion of $\phi(x_+)$ near $x_+ = 0$ leads to no hierarchy in the corresponding expansion of the B-decay amplitude. We need to keep the full x_+-dependence of the operator $\phi(x_+)$ on the light cone ($x^2 = 0$).

So, the leading term of the expansion of the B-decay amplitude related to the x-vertex corresponds to the expansion near $x_- = 0, x_\perp = 0$ and has the form

$$\int dx_+ dx_- dx_\perp d\kappa_+ d\kappa_- d\kappa_\perp \frac{1}{\mu^2 - 2(\kappa_+ - q_+)k_- + k_\perp^2} e^{i\kappa_+ x_- + i\kappa_- x_+ - ik_\perp x_\perp}\langle 0|...\phi(x_+)...|B(p)\rangle,$$

The x_- and $x_\perp$ integrals may be taken and lead to $\delta(\kappa_\perp)\delta(\kappa_+)$. Integrating these δ-functions, we obtain:

$$\int d\tau d\kappa_- \frac{1}{\mu^2 + 2q_+\kappa_-} e^{i\kappa_- \tau}\langle 0|\phi(x_+ = \tau)...|B(p)\rangle.$$

The x'-vertex

Now, $q'_- \sim M_B$ is the only nonzero component of the vector q'. The propagator has the form $m^2 - 2\kappa'_-(\kappa'_- - q'_-) + {\kappa'_\perp}^2 \sim \Lambda_{\rm QCD} M_B$. Obviously, we can perform Taylor expansion of $\phi'(x')$ near $x'_+ = 0$ and $x'_\perp = 0$ but have to keep its full dependence on the variable x_-. Taking into account this property and denoting $\tau' = x'_-$, the dominant contribution of the x'-vertex reads

$$\int d\tau' d\kappa'_+ \frac{1}{m^2 + 2q'_-\kappa'_+} e^{i\kappa'_+\tau'} \langle 0|...\phi'(x_- = \tau')...|B(p)\rangle.$$

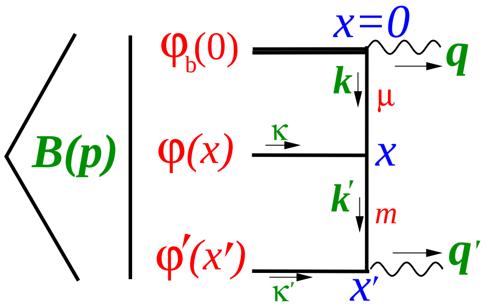
The amplitude of B-decay of FCNC topology

Making use of the leading contributions of the x- and x'-vertices we obtain:

$$A(p|q,q') = \int d\tau d\kappa_- \frac{1}{\mu^2 + 2q_+\kappa_-} e^{i\kappa_-\tau} \int d\tau' d\kappa'_+ \frac{1}{m^2 + 2q'_-\kappa'_+} e^{i\kappa'_+\tau'} \langle 0|\phi(x_+ = \tau)\phi_b(0)\phi'(x_- = \tau')|B(p)\rangle.$$

This is *factorization theorem*: The dominant contribution to a B-decay amplitude of FCNC topology is given by the convolution of the hard kernel composed of propagators of light degrees of freedom and the 3BS in the "double collinear" kinematical configuration: the upper (above b-quark line) and the lower (below b-quark line) parts of the diagram are aligned along different LC directions.

- **3BS contributions to amplitudes of SL B-decays**



$$A_{\rm SL}(p\,|q,q') = \int \frac{dx'\,dx\,dk\,dk'}{(\mu^2-k^2)(m^2-k'^2)} e^{-ikx-ik'(x'-x)+iq'x'}\langle 0|\varphi_b(0)\varphi(x)\varphi'(x')|B(p)\rangle.$$

Introduce Feynman parameter v to combine two propagators and redefine variables

$$\tilde{k} = k - v\kappa, \qquad x = x'v + z.$$

Here κ is the momentum transferred in the *central* **point x; z measures the deviation of x from the straight line joining the end points 0 and x'. This gives**

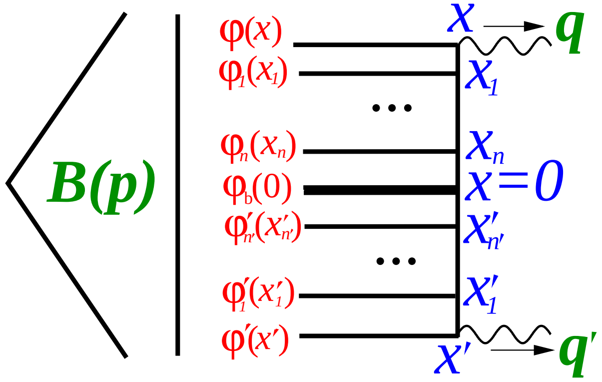
$$A_{\rm SL}(p|q,q') \;=\; \int\limits_0^1 dv \int \frac{d\kappa\, d\tilde{k}\, dz\, dx' \;\; e^{ix'(q'-\tilde{k})+i\kappa z}}{\left[m^2(1-v)+\mu^2 v-\tilde{k}^2-v(1-v)\kappa^2\right]^2}\langle 0|\varphi_b(0)\varphi(x'v+z)\varphi'(x')|B(p)\rangle,$$

κ is soft compared to propagator virtualities $O(m_b\Lambda_{\rm QCD})$, $\kappa\to 0$ in denominator, $\to\delta(z)$.

The dominant contribution to $A_{\rm SL}(p|q,q')$ comes from the collinear kinematical configuration of 3BS when the points $x=0$, x, and x' lie on the straight line with x between the end points $x=0$ and x': $x_\mu = vx'_\mu$, $0<v<1$.

• Multi-particle BS contribution to amplitudes of B-decays of FCNC topology

Generalized to multi-particle contributions to B-decay amplitudes:



The diagram involves a multi-particle BS of the B-meson (ϕ_b heavy, other fields light):

$$\langle 0|\phi(x)\phi_1(x_1)\dots\phi_n(x_n)\phi_b(0)\phi'_{n'}(x'_{n'})\dots\phi'_1(x'_1)\phi'(\tau')|B(p)\rangle.$$

In the reference frame where q along $(+)$-direction and q' along $(-)$-direction,

$$q_+ \propto (M_B,0,0),\; q_- \propto (0,M_B,0) \quad [\,a_\mu=(a_+,a_-,a_\perp),\; a^2=2a_+a_- - a_\perp^2\,],$$

the dominant contribution comes from the following configuration (proof by induction):

$$\begin{array}{llll} x=(\tau,0,0), & x_1=(\tau u_1,0,0), & \dots, & x_n=(\tau u_n,0,0), \quad 0<u_n<\dots<u_1<1, \\ x'=(0,\tau',0), & x'_1=(0,\tau' u'_1,0), & \dots, & x'_{n'}=(0,\tau' u'_{n'},0), \quad 0<u'_{n'}<\dots<u'_1<1. \end{array}$$

The coordinates $x,x_1,\dots,x_n$ are ordered and lie on the $(+)$-axis of the LC, and the coordinates $x',x'_1,\dots,x'_{n'}$ are ordered and lie on the $(-)$-axis of the LC. That is why we refer to this configuration as to the double collinear LC configuration.

• 3BS: Properties of the 3BS wave function of the B-meson

Consider heavy B-meson as bound state of scalar fields, one of which, $\phi_b(x)$, is heavy, the 3BS of the B-meson may be parametrized as follows:

$$\langle 0|\phi(x)\phi_b(0)\phi'(x')|B(p)\rangle \;=\; \int D(\omega,\omega')e^{-ipx\omega-ipx'\omega'}\Psi(\omega,\omega',x^2,x'^2,(x-x')^2),$$

where

$$D(\omega,\omega')\equiv d\omega\, d\omega'\,\theta(\omega)\theta(\omega')\theta(1-\omega-\omega').$$

For $\Psi(\omega,\omega',x^2,x'^2,(x-x')^2)$ one can write down Taylor expansion in its variables x^2, x'^2, $(x-x')^2$:

$$\Psi(\omega,\omega',x^2,x'^2,(x-x')^2) \;=\; \Psi_0(\omega,\omega')+x^2\Psi_{12}(\omega,\omega')+x'^2\Psi_{23}(\omega,\omega')+(x-x')^2\Psi_{13}(\omega,\omega')+\ldots,$$

where $\ldots$ stand for higher powers of $x^2, x'^2, (x-x')^2$. The distribution amplitudes (DAs) Ψ_i have support in the region $0<\omega$, $0<\omega'$, $\omega+\omega'<1$, and peak at small values $(\omega,\omega')=O(\Lambda_{\rm QCD}/M_B)$, reflecting the fact that the heavy b-quark carries almost the full momentum of the heavy B-meson, whereas the light degrees of freedom carry its small fraction $O(\Lambda_{\rm QCD}/M_B)$.

Collinear LC configuration (dominant contribution to SL amplitude)

$x'_\mu=ux_\mu$, $x^2=0$, $x'^2=0$, $(x-x'^2)=0$, $\Psi_0(\omega,\omega')$ is sufficient to obtain SL B-decay amplitude.

Double-Collinear LC configuration (dominant contribution to FCNC amplitude)

$x'_\mu\neq ux_\mu$, $x^2=0$, $x'^2=0$, $(x-x'^2)\neq 0$, so in the Taylor expansion for 3BS all powers of $(x-x'^2)$ should be taken into account to obtain the FCNC B-decay amplitude.

• SUMMARY

We studied a B-decay amplitude of a generic "FCNC topology" (an amplitude given by diagrams in which the heavy field may hit any point of the line along which light degrees of freedom propagate) and obtained the following results:

(i) The leading contribution to $A_{\rm FCNC}$ calculated in the reference frame where

$$q_\mu \propto n_\mu, \quad q'_\mu \propto n'_\mu,$$

where n_μ and n'_μ are two light-like vectors, $n^2 = 0$, $n'^2 = 0$, $nn' = 2$, is given by the convolution of the hard kernel (hard propagators) and 3BS of the B-meson in double collinear LC configuration

$$\langle 0|\bar q(\tau n_\mu) b(0) G_{\mu\nu}(\tau' n'_\mu)|B(p)\rangle.$$

Corrections to this contribution are suppressed by powers of m_b.

(ii) This makes an essential difference with 3DA contributions to SL B-decays: In the reference frame $q'_\mu \propto n'_\mu$, the leading 3DA contribution to $A_{\rm SL}$ is given by the convolution of a hard kernel and 3BS in collinear LC configuration

$$\langle 0|\bar q(\tau n'_\mu) b(0) G_{\mu\nu}(v\tau n'_\mu)|B(p)\rangle, \; 0 < v < 1.$$

Corrections are suppressed by powers of m_b.

*Further details: PRD***106***,054022(2022); PRD***108***,034007; 094022(2023); PRD***109***,114012(2024).*