Colored excitations contribution to the dynamics of QGP

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Introduction

Basics

I System is far from equilibrium

II QGP is well described by hydrodynamics since $\tau \sim 1\,\text{fm}/c$

Problem issues

I Thermalization time

II Early stage of QGP dynamics

III Not clear which theory does work

Kinetic theory for QGP

Assumptions

- Small coupling $g_s \ll 1$
- QGP consists of free particles
- Large number of particles inside Debye sphere
- Impact of soft modes is negligible

Semiclassical theory[1]

Distribution function

$f = f(x, \mathbf{p}, \vec{Q})$

Color representation

Color is classical variable $\vec{Q} = Q_a,\ a \in \overline{1,8}$

Gluons

Gluonic modes are overpopulated. Classical field approximation is appropriate

[1] Elze and Heinz, "Quark-Gluon transport theory".

Vlasov-Boltzmann equation

$$p^{\mu}(\partial_{\mu} + gQ_a F^a_{\mu\nu}\partial^{\nu}_p + gf_{abc}A^b_{\mu}Q^c\partial^a_Q)f(x,\mathbf{p},\vec{Q}) = \mathcal{C}[f]$$

Moments expansion in color space

$$f(x,\mathbf{p}) = \int DQ\, f(x,\mathbf{p},\vec{Q}) \qquad f_a(x,\mathbf{p}) = \int DQ\, Q_a f(x,\mathbf{p},\vec{Q})$$

$$f_{ab}(x,\mathbf{p}) = \int DQ\, Q_a Q_b f(x,\mathbf{p},\vec{Q}) \qquad \dots$$

Moments equation

$$p^\mu \partial_\mu f(x,\mathbf{p}) = g p^\mu F^a_{\mu\nu}(x)\partial^\nu_{\mathbf{p}} f_a(x,\mathbf{p}) + \int C(x,p,Q)Dq,$$

$$p^\mu[\delta_{ac}\partial_\mu + g f_{abc}A^b_\mu(x)]f_c(x,\mathbf{p}) = g p^\mu F^b_{\mu\nu}(x)\partial^\nu_{\mathbf{p}} f_{ab}(x,\mathbf{p}) + \int Q_a C(x,p,Q)DQ$$

Truncation of moments expansion

From color commutators:

$$f_{ab} = \frac{1}{6}\delta_{ab} f - \frac{1}{2} d_{abc} f_c$$

$$p^\mu[\delta_{ac}\partial_\mu + g f_{abc}A^b_\mu(x) + \frac{g}{2} d_{abc} F^b_{\mu\nu}(x)\partial^\nu_{\mathbf{p}}]f_c(x,\mathbf{p}) = \frac{g}{6} p^\mu F^a_{\mu\nu}(x)\partial^\nu_{\mathbf{p}} f(x,\mathbf{p}) + \int q_a C(x,p,Q)DQ$$

Moments interpretation

$f(x,\mathbf{p})$ — colorless excitations as {u,u,u}, {s,s,s}, {c,c,c}
$f_c(x,\mathbf{p})$ — colored excitation as u, s, c

Connection with EKT[2]

Assume $F^a_{\mu\nu} = 0$ then

$$p^\mu \partial_\mu f(x,\mathbf{p}) = \int C(x,p,Q) DQ$$

[2] Arnold, Moore, and Yaffe, "Effective kinetic theory for high temperature gauge theories".

Collision operator

Inner product

$$\sum_{species} \nu_s \int_p dp\, f_0(p)[1 \pm f_0(p)] g(p) h(p) = \langle g, h \rangle$$

Requirements

- Entropy is rising
- Detailed balance (T-invariance)

Transport coefficients

η — viscosity

τ_π — shear pressure relaxation time

$\mathcal{P} + \epsilon$ — enthalpy

$$\pi_{ij} = -\eta(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k) + \zeta\delta_{ij}\partial_k u_k$$

$$\tau_\pi \partial_t \pi^{ij} = \pi^{ij} - \pi_0^{ij}$$

Evaluation method[3]

$$\delta f(x,\mathbf{p}) = f_0[1 \pm f_0]\, a X_{i\ldots j} I_{i\ldots j}(\mathbf{p}) \chi(p)$$

$$\eta = G_\eta(u, T, \ldots)\langle 1, \chi \rangle$$
$$\eta\tau_\pi = G_\tau(u, T, \ldots)\langle \chi, \chi \rangle$$
$$\mathcal{P} + \epsilon = G_\epsilon(u, T, \ldots)\langle 1, 1 \rangle$$

Cauchy-Bunyakowsky-Schwartz inequality

$$\frac{\tau_\pi}{\eta/(\epsilon + \mathcal{P})} = \frac{G_\tau G_\epsilon}{G_\eta^2} \frac{\langle \chi, \chi \rangle \langle 1, 1 \rangle}{\langle 1, \chi \rangle^2} \geq \frac{G_\tau G_\epsilon}{G_\eta^2}$$

[3] Ghiglieri, Moore, and Teaney, “Second-Order Hydrodynamics in Next-to-Leading-Order QCD”.

Comparison with other theories

Our result

$$\left.\frac{\tau_\pi}{\eta/(\epsilon+\mathcal{P})}\right|_{0,\,c} \geq 5$$

AdS/CFT ($\mathcal{N}=4$ SYM)

$$\frac{\tau_\pi}{\eta/(\epsilon+\mathcal{P})} = 4 - 2\ln 2$$

Application in numerical studies

Parameters under control

- Total energy
- Number of particles

Additional parameters

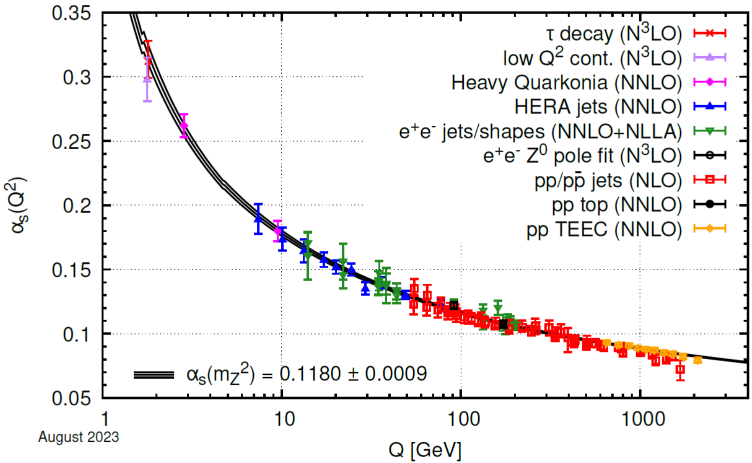
- Transport coefficients relations

Summary

- Limitation on transport coefficients is obtained
- Results might be used to distinguish theories
- Transport coefficient relation might be used in numerical studies

- Outlook
  - More transport coefficients are required
  - Impact of non monotonically increasing entropy

Thank you for attention



0.35
0.3
0.25
0.2
0.15
0.1
0.05
$\alpha_s(Q^2)$
τ decay (N^{3}LO)
low Q^2 cont. (N^{3}LO)
Heavy Quarkonia (NNLO)
HERA jets (NNLO)
e$^+$e$^-$ jets/shapes (NNLO+NLLA)
e$^+$e$^-$ Z^0 pole fit (N^{3}LO)
pp/p$\bar{p}$ jets (NLO)
pp top (NNLO)
pp TEEC (NNLO)
$\alpha_s(m_Z^2)$ = 0.1180 ± 0.0009
1
10
100
1000
August 2023
Q [GeV]

For Further Reading I

Mrówczyński, Stanisław and Schenke, Björn and Strickland, Michael
Color instabilities in the quark–gluon plasma.
Physics Reports, 682:1–97, 2017.