

Charmonium production in the TMD factorisation using the Improved Color Evaporation Model

Kirill Shilyaev¹, Vladimir Saleev^{1,2}

¹Samara University, Samara

²JINR, Dubna

QFTHEP'270, MSU, Moscow

June 30, 2025

Outline

- ▶ Improved Color Evaporation model (ICEM)
- ▶ TMD factorisation and Soft Gluon Resummation approach
- ▶ InEW scheme for matching CPM and TMD predictions
- ▶ J/ψ production at small and large p_T within current data
- ▶ Extracting of the ICEM $F^{J/\psi}$ parameter
- ▶ ICEM predictions for SPD NICA energy

Improved Color Evaporation model

- ▶ Improved Color Evaporation Model, scale hierarchy $m_c \gg \lambda \gg \Lambda_{\text{QCD}}$ [Y.-Q. Ma, R. Vogt (2016)]:
 - $c\bar{c}$ -pair production on $\mathcal{O}(m_c)$ scale
 - soft gluon exchange and emission on $\mathcal{O}(\lambda)$ scale
 - $\lambda \sim q_T$, q_T is intrinsic transverse momentum of parton
- ▶ Momentum shift for $c\bar{c} \rightarrow J/\psi$ transition (with invariant mass $M_{c\bar{c}}$ of the produced $c\bar{c}$ -quark pair):

$$\langle p^{J/\psi} \rangle = \frac{M_{J/\psi}}{M_{c\bar{c}}} p^{c\bar{c}} + \mathcal{O}(\lambda^2/m_c)$$

- ▶ General cross-section expression:

$$d\sigma(p^{J/\psi}) = F^{J/\psi} \times \int_{M_{J/\psi}}^{2m_D} dM_{c\bar{c}} \int d^4 p^{c\bar{c}} \frac{d\sigma(p^{c\bar{c}})}{dM_{c\bar{c}}} \delta^{(4)}\left(p^{J/\psi} - \frac{M_{J/\psi}}{M_{c\bar{c}}} p^{c\bar{c}}\right) + \mathcal{O}(\lambda^2/m_c^2)$$

with $F^{J/\psi}$ as a hadronisation factor assumed to be independent on the factorisation approach

General remarks on our calculations of prompt J/ψ production

- ▶ Only **prompt** production of J/ψ within the ICEM
 - no direct opportunity to estimate the feed-down ratio without the higher states production data
- ▶ Study of both **gluon-gluon fusion** $g + g \rightarrow c + \bar{c} + X$ and **quark-antiquark annihilation** $q + \bar{q} \rightarrow c + \bar{c} + X$
 - rise of quark-antiquark subprocess contribution with the decrease of center-of-mass energy \sqrt{s}
- ▶ Assumption of $c\bar{c}$ -quark pair production in color-octet state
 - followed by soft gluon interactions and hadronisation into some observable state
- ▶ Dependence of $F^{J/\psi}$ on center-of-mass energy \sqrt{s}
- ▶ Calculations in the LO approximation of the pQCD with respect to α_s
 - for $g + g (q + \bar{q}) \rightarrow c + \bar{c} + g$ processes at $p_T \gg M_{J/\psi}$

TMD factorisation and TMD PDFs

- ▶ **Transverse Momentum Dependent (TMD) factorisation:** $q_T, k_T \ll \mu_F \sim M$
- ▶ Partons' momenta, preserving $\mathcal{O}(q_T/M)$ terms:

$$q_1 \approx \left(\frac{x_1 \sqrt{s}}{2}, \mathbf{q}_{1T}, \frac{x_1 \sqrt{s}}{2} \right), \quad q_2 \approx \left(\frac{x_2 \sqrt{s}}{2}, \mathbf{q}_{2T}, -\frac{x_2 \sqrt{s}}{2} \right)$$

- ▶ TMD parton distribution functions $F(x, \mathbf{q}_T, \mu_F, \zeta) \Rightarrow$ two-scale **Collins-Soper** equations
- ▶ General formula of TMD factorisation [[TMD Handbook](#), [arXiv:2304.03302](#)]:

$$\frac{d\sigma}{d\mathbf{p}_T} = \sigma_0 \int d\mathbf{q}_{1T} d\mathbf{q}_{2T} F(x_1, \mathbf{q}_{1T}, \mu_F, \zeta_1) F(x_2, \mathbf{q}_{2T}, \mu_F, \zeta_2) \delta(\mathbf{q}_{1T} + \mathbf{q}_{2T} - \mathbf{p}_T)$$

- ▶ To implement **CS** evolution, the transfer to impact parameter \mathbf{b}_T space by 2D Fourier transform is done:

$$\frac{d\sigma}{d\mathbf{p}_T} = \sigma_0 \int \frac{d\mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{p}_T \mathbf{b}_T} \hat{F}(x_1, \mathbf{b}_T, \mu_F, \zeta_1) \hat{F}(x_2, \mathbf{b}_T, \mu_F, \zeta_2)$$

Soft Gluon Resummation approach, perturbative evolution

- ▶ Soft and collinear gluon resummation approach by [J. Collins, D. Soper (1981)]:

$$\frac{d\sigma(J/\psi)}{d\mathbf{p}_T} = \sigma_0 \int_0^\infty db_T b_T J_0(p_T b_T) e^{-S_P(b_T^*, \mu_b'^*, Q)} e^{-S_{NP}(b_T, Q)} \hat{F}(x_1, \mu_b'^*, b_T^*) \hat{F}(x_2, \mu_b'^*, b_T^*)$$

- ▶ Sudakov factor in LL–LO perturbative calculations [J. Collins, D. Soper (1982)]:

$$S_P(b_T, \mu_F, Q) = \frac{C_A}{\pi} \int_{\mu_b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \alpha_s(\mu') \left[\ln \frac{Q^2}{\mu'^2} - \left(\frac{11 - 2N_f/C_A}{6} + \frac{1}{2} \right) \right] + \mathcal{O}(\alpha_s)$$

- ▶ Sudakov factor expression is valid only on region $b_0/Q \leq b_T \leq b_{T, \max}$ which is being controlled with [D. Boer, W. J. den Dunnen (2014); J. Collins, D. Soper, G. Sterman (1985)]

$$\mu_b \rightarrow \mu_b' = \frac{Qb_0}{Qb_T + b_0} \quad \text{and} \quad b_T^*(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{T, \max})^2}}$$

Soft Gluon Resummation approach, nonperturbative content

- ▶ Master formula for soft gluon resummation:

$$\frac{d\sigma(J/\psi)}{d\mathbf{p}_T} = \sigma_0 \int_0^\infty db_T b_T J_0(p_T b_T) e^{-S_P(b_T^*, \mu_{b^*}', Q)} e^{-S_{NP}(b_T, Q)} \hat{F}(x_1, \mu_{b^*}', b_T^*) \hat{F}(x_2, \mu_{b^*}', b_T^*)$$

- ▶ **Nonperturbative** quark factor obtained in SIDIS data fitting [S. Aybat, T. Rogers (2011)]:

$$S_{NP}(b_T, Q) = \left[g_1 \ln \frac{Q}{2Q_{NP}} + g_2 \left(1 + 2g_3 \ln \frac{10xx_0}{x_0 + x} \right) \right] b_T^2$$

- it should be Casimir-scaled by C_A/C_F for initial gluons
- ▶ In the leading order of α_s , the perturbative tail of TMD PDF is expressed with collinear PDF:

$$\hat{F}(x, \mu_{b^*}', b_T^*) = f(x, \mu_{b^*}') + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T \Lambda_{\text{QCD}})$$

Matching of small- p_T and high- p_T regions within Inverse-Error Weighting Scheme

- ▶ Matched cross-section as a weighted sum of CPM and TMD terms
[M. Echevarria, T. Kasemets, J.-P. Lansberg, C. Pisano, A. Signori (2018)]:

$$d\sigma = \mathcal{W} d\sigma^{\text{TMD}} + \mathcal{Z} d\sigma^{\text{CPM}}$$

- ▶ Normalised weights for each of the two terms:

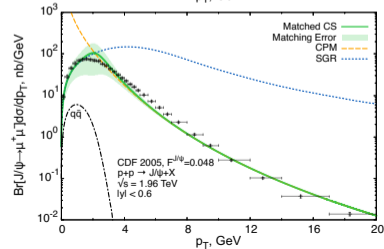
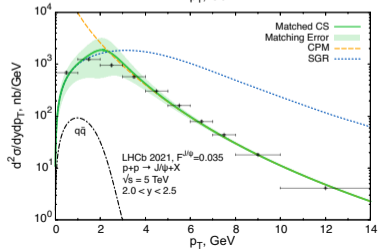
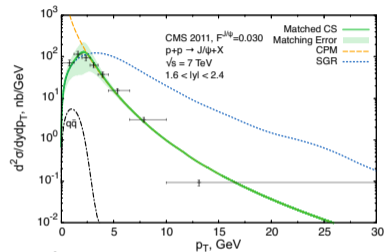
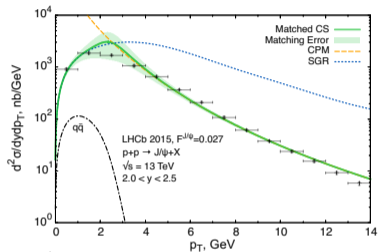
$$\mathcal{W} = \frac{\Delta\mathcal{W}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}, \quad \mathcal{Z} = \frac{\Delta\mathcal{Z}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}$$
$$\Delta\mathcal{W} = \left(\frac{p_T}{Q}\right)^2 + \left(\frac{m}{Q}\right)^2, \quad \Delta\mathcal{Z} = \left(\frac{m}{p_T}\right)^2 \left(1 + \ln^2 \frac{\sqrt{Q^2 + p_T^2}}{p_T}\right)$$

- ▶ Uncertainty due to the matching procedure:

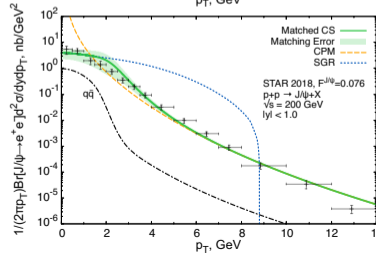
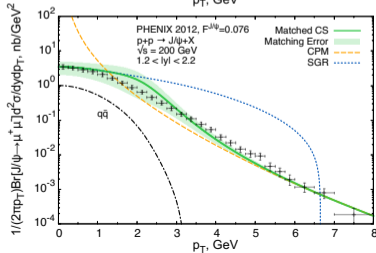
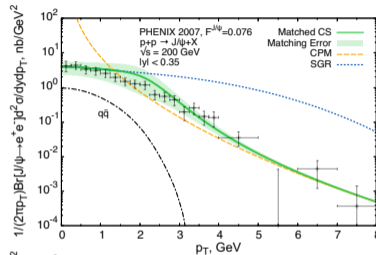
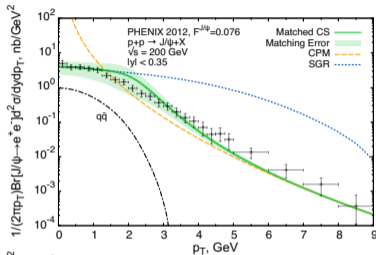
$$\Delta d\sigma = \frac{d\sigma}{\sqrt{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}} = \frac{\Delta\mathcal{W} \cdot \Delta\mathcal{Z}}{\sqrt{\Delta\mathcal{W}^2 + \Delta\mathcal{Z}^2}} d\sigma$$

Extraction of $F^{J/\psi}$ at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM)

$\sqrt{s} = 1.96 - 13$ TeV

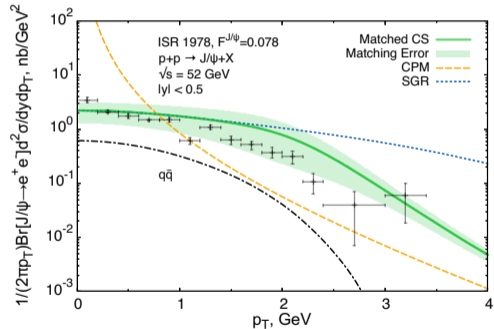
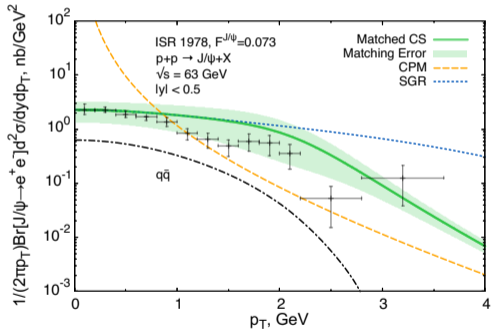


Extraction of $F^{J/\psi}$ at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM) $\sqrt{s} = 200$ GeV



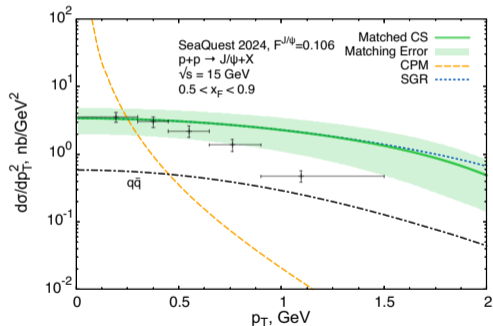
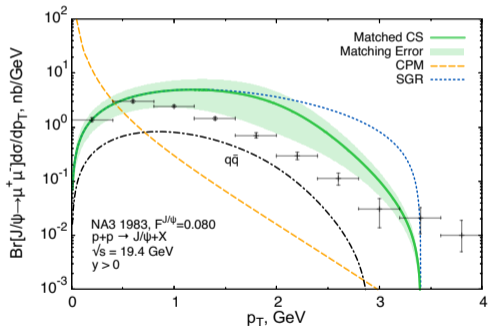
Extraction of $F^{J/\psi}$ at $p_T < 1$ GeV (SGR)

$\sqrt{s} = 52 - 63$ GeV

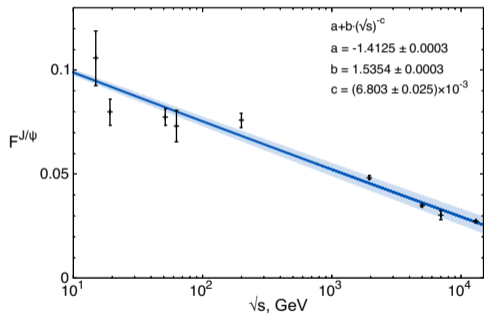


Extraction of $F^{J/\psi}$ at $p_T < 0.5$ GeV (SGR)

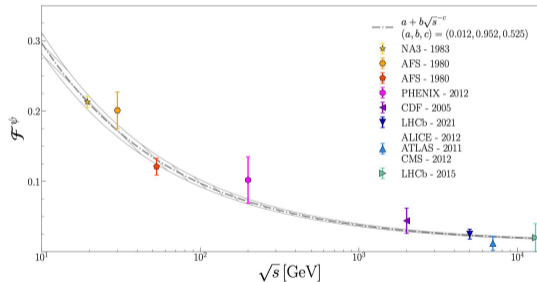
$\sqrt{s} = 15 - 19.4$ GeV



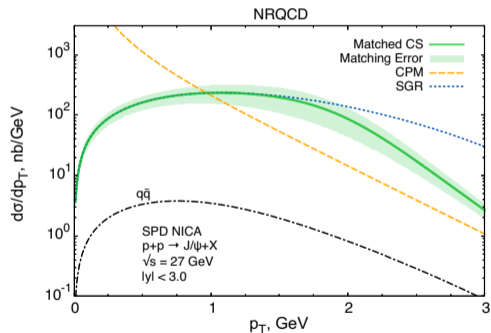
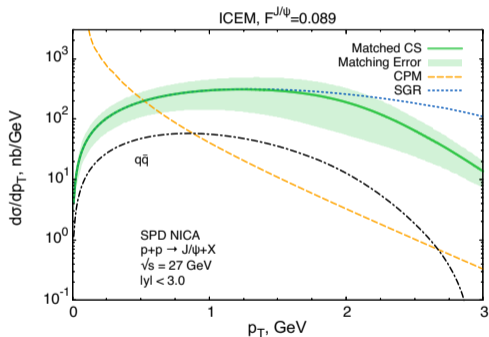
Dependence of $F^{J/\psi}$ on center-of-mass energy \sqrt{s}



[A. Chernyshev, V. Saleev (2022)]



Predictions for SPD NICA, comparison with the previous NRQCD calculations



Summary

- ▶ We have used the Soft Gluon Resummation approach to calculate small- p_T J/ψ production in the TMD factorisation
- ▶ ICEM's hadronisation factor $F^{J/\psi}$ crucially depends on the center-of-mass energy \sqrt{s} , showing decrease when \sqrt{s} increases
- ▶ Soft Gluon Resummation approach for gluon and quark TMD PDF satisfyingly describes experimental data for unpolarised J/ψ production at \sqrt{s} from 15 GeV up to 13 TeV in the TMD domain of $p_T < 1$ GeV
- ▶ We have tested the factorisation matching scheme InEW for description of intermediate p_T domain in the ICEM, calculation agrees with experimental data within the uncertainty band
- ▶ We estimate the perspective region for extraction of gluon TMD PDF in J/ψ production as $p_T \leq 1$ GeV

THANK YOU FOR ATTENTION!