Applicability limits for gluon fragmentation into J/ψ mesons

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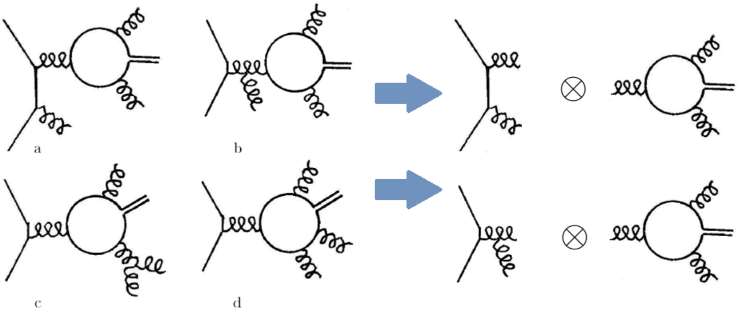
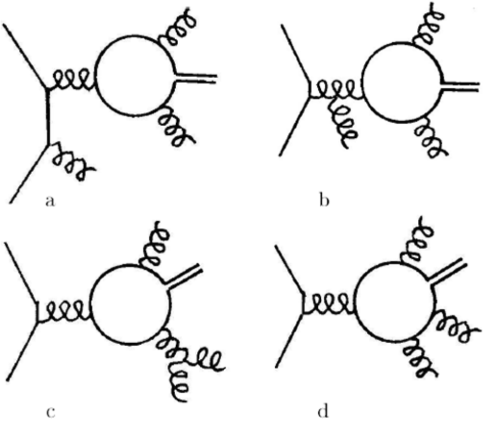
The main idea of the fragmentation approach using the example of the $q\bar{q} \to ggg\, J/\psi$ process



The diagrams of types (a) and (b) can be factorized (cut) into pieces representing the quark-antiquark annihilation $q\,\bar{q} \to g\,g$ and gluon fragmentation $g \to \psi\,g\,g$. The other diagrams bring non-factorizable contributions which probably spoil the fragmentation pattern.

Full set of diagrams



There are 36 diagrams of type (a); 18 diagrams of type (b); 18 diagrams of type (c); and 24 diagrams of type (d). So, we end up with 96 Feynman diagrams.

The phase space

$$\sigma = \frac{2}{3}|\mathcal{R}(0)|^2\alpha_s^5 \int d^4\xi d^3(\ln r)\frac{(r_1 r_2 r_3)^2}{x_1 x_2 E^4}\mathcal{F}(x_1)\mathcal{F}(x_2)|\mathcal{M}|^2$$

where r_i - transverse momenta, ξ - rapidity

$$p_i = (E_i, p_i^x, p_i^y, q_i) \qquad r = (p_x^2 + p_y^2)^{1/2}$$

$$\xi = \frac{1}{2}\ln\frac{E+q}{E-q} = \ln\frac{E+q}{m^\perp} = \ln\frac{m^\perp}{E-q}$$

Boundary conditions:

$$-1 \le \xi_J \le 1 \qquad -2.5 \le \xi_g \le 2.5$$

$$5 \le r_J \le 150 \qquad 0.2 \le r_g \le 150$$

The Method of orthogonal amplitudes

Decomposition by basis.

$$\bar{v}Ru = C_1\,\bar{v}u + C_2\,\bar{v}\hat{K}u + C_3\,\bar{v}\hat{Q}u + C_4\,\bar{v}\hat{K}\hat{Q}u +$$
$$+C_1'\,\bar{v}\,t^x u + C_2'\,\bar{v}\hat{K}t^x u + C_3'\,\bar{v}\hat{Q}t^x u + C_4\,\bar{v}\hat{K}\hat{Q}t^x u$$

$$C_1 = \frac{\bar{v}Ru \times \bar{v}u}{(\bar{v}u)^2} \qquad C_1' = \frac{\bar{v}Ru \times \bar{v}\,t^x u}{(\bar{v}\,t^x u)^2}$$

$$C_2 = \frac{\bar{v}Ru \times \bar{v}\hat{K}u}{(\bar{v}\hat{K}u)^2} \qquad C_2' = \frac{\bar{v}Ru \times \bar{v}\hat{K}t^x u}{(\bar{v}\hat{K}t^x u)^2}$$

$$C_3 = \frac{\bar{v}Ru \times \bar{v}\hat{Q}u}{(\bar{v}\hat{Q}u)^2} \qquad C_3' = \frac{\bar{v}Ru \times \bar{v}\hat{Q}t^x u}{(\bar{v}\hat{Q}t^x u)^2}$$

$$C_4 = \frac{\bar{v}Ru \times \bar{v}\hat{K}\hat{Q}u}{(\bar{v}\hat{K}\hat{Q}u)^2} \qquad C_4' = \frac{\bar{v}Ru \times \bar{v}\hat{K}t^x\hat{Q}u}{(\bar{v}\hat{K}\hat{Q}t^x u)^2}$$

The squared Matrix element

$$|\mathcal{M}|^2 = \frac{1}{12}\frac{1}{3}\frac{1}{24}4\frac{|\mathcal{M}\bar{u}(q_1)\hat{X}v(q_2)|^2}{(q_1\cdot q_2)} + \frac{1}{2}\frac{1}{3}\frac{1}{24}\frac{|\mathcal{M}\bar{u}(q_1)t^x\hat{X}v(q_2)|^2}{(q_1\cdot q_2} =$$

$$= \frac{5}{81}\frac{A_{22}^2}{(q_1\cdot q_2)} + \frac{5}{18}\frac{1}{(q_1\cdot q_2)}\Bigg(2\left(\mathrm{Amp}_1^2 + \mathrm{Amp}_2^2 + \mathrm{Amp}_3^2\right) +$$

$$+\frac{5}{9}\left(A_{19}^2 + A_{20}^2 + A_{21}^2\right) - \frac{1}{3}\left(A_{19}A_{20} + A_{19}A_{21} + A_{20}A_{21}\right)\Bigg)$$

where

$$\mathrm{Amp}_1 = A_1 + \frac{1}{2}(A_{13} - A_{15} + A_{191} - A_{192} + A_{16} + A_{18} - A_{211} + A_{212})$$

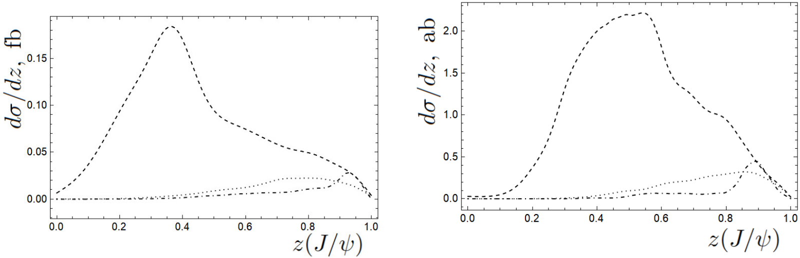
$$\mathrm{Amp}_2 = A_2 + \frac{1}{2}(A_{13} - A_{14} + A_{17} - A_{201} + A_{202} - A_{18} + A_{211} - A_{212})$$

$$\mathrm{Amp}_3 = A_3 + \frac{1}{2}(A_{14} - A_{15} + A_{191} - A_{192} - A_{16} + A_{17} - A_{201} + A_{202})$$

Program implementation

The analytical manipulations have been done using the system FORM and the package FEYNCALC for WOLFRAM MATHEMATICA. The gauge invariance of the output expression has been explicitly tested by substituting gluon momenta for their polarization vectors.

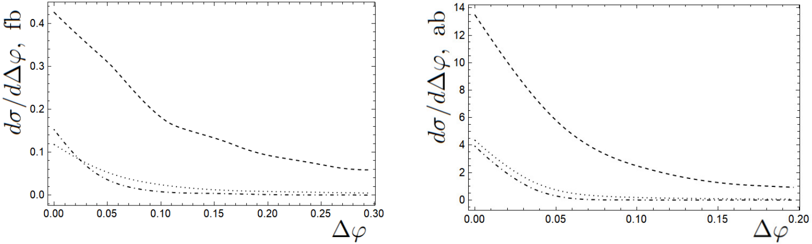
Distributions in the fragmentation variable z (meson's light-cone momentum fraction) as seen under the different kinematic constraints



Left: dashed, $p_{\psi T} > 5$ GeV, $p_T^* > 20$ GeV; dotted, $p_{\psi T} > 20$ GeV, $p_T^* > 20$ GeV; dash-dotted, $p_{\psi T} > 5$ GeV, $p_T^* > 20$ GeV, $m^* < E^*/3$.

Right: dashed, $p_{\psi T} > 20$ GeV, $p_T^* > 50$ GeV; dotted, $p_{\psi T} > 50$ GeV, $p_T^* > 50$ GeV; dash-dotted, $p_{\psi T} > 5$ GeV, $p_T^* > 50$ GeV, $m^* < E^*/10$.

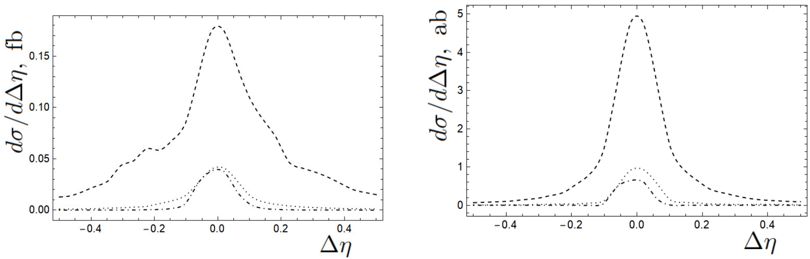
Azimuthal angle difference between the fragmenting gluon momentum and the momentum of the final state meson as seen under the different kinematic constraints



Left: dashed, $p_{\psi T} > 5$ GeV, $p_T^* > 20$ GeV; dotted, $p_{\psi T} > 20$ GeV, $p_T^* > 20$ GeV; dash-dotted, $p_{\psi T} > 5$ GeV, $p_T^* > 20$ GeV, $m^* < E^*/3$.

Right: dashed, $p_{\psi T} > 20$ GeV, $p_T^* > 50$ GeV; dotted, $p_{\psi T} > 50$ GeV, $p_T^* > 50$ GeV; dash-dotted, $p_{\psi T} > 5$ GeV, $p_T^* > 50$ GeV, $m^* < E^*/10$.

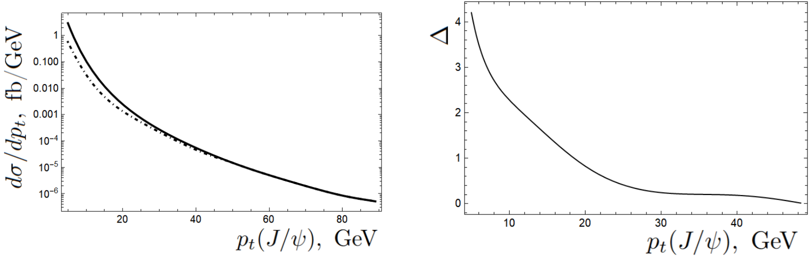
Pseudorapidity difference between the fragmenting gluon and final state meson as seen under the different kinematic constraints



Left: dashed, $p_{\psi T} > 5$ GeV, $p_T^* > 20$ GeV; dotted, $p_{\psi T} > 20$ GeV, $p_T^* > 20$ GeV; dash-dotted, $p_{\psi T} > 5$ GeV, $p_T^* > 20$ GeV, $m^* < E^*/3$.

Right: dashed, $p_{\psi T} > 20$ GeV, $p_T^* > 50$ GeV; dotted, $p_{\psi T} > 50$ GeV, $p_T^* > 50$ GeV; dash-dotted, $p_{\psi T} > 5$ GeV, $p_T^* > 50$ GeV, $m^* < E^*/10$.

Transverse momentum distributions for color-singlet J/ψ production at middle rapidities in *pp* collisions.



Solid curve, full $\mathcal{O}(\alpha_s^5)$ calculation; dash-dotted curve, fragmentation approximation.

Left: the differential cross sections $d\sigma/dp_{\psi T}$;

Right: the relative size of the differential cross sections $\Delta = (d\sigma_{\text{full}} - d\sigma_{\text{frag}})/d\sigma_{\text{frag}}$.

We find that the non-fragmentation contributions in the color-singlet channel remain important up to J/ψ transverse momenta as large as approximately 30 or 40 GeV