РОСАТОМ

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ВНИИЭФ

ГОСУДАРСТВЕННАЯ КОРПОРАЦИЯ ПО АТОМНОЙ ЭНЕРГИИ «РОСАТОМ»

Quantum mechanics of stationary states of particles in external singular fields of black holes with event horizons of zero thickness

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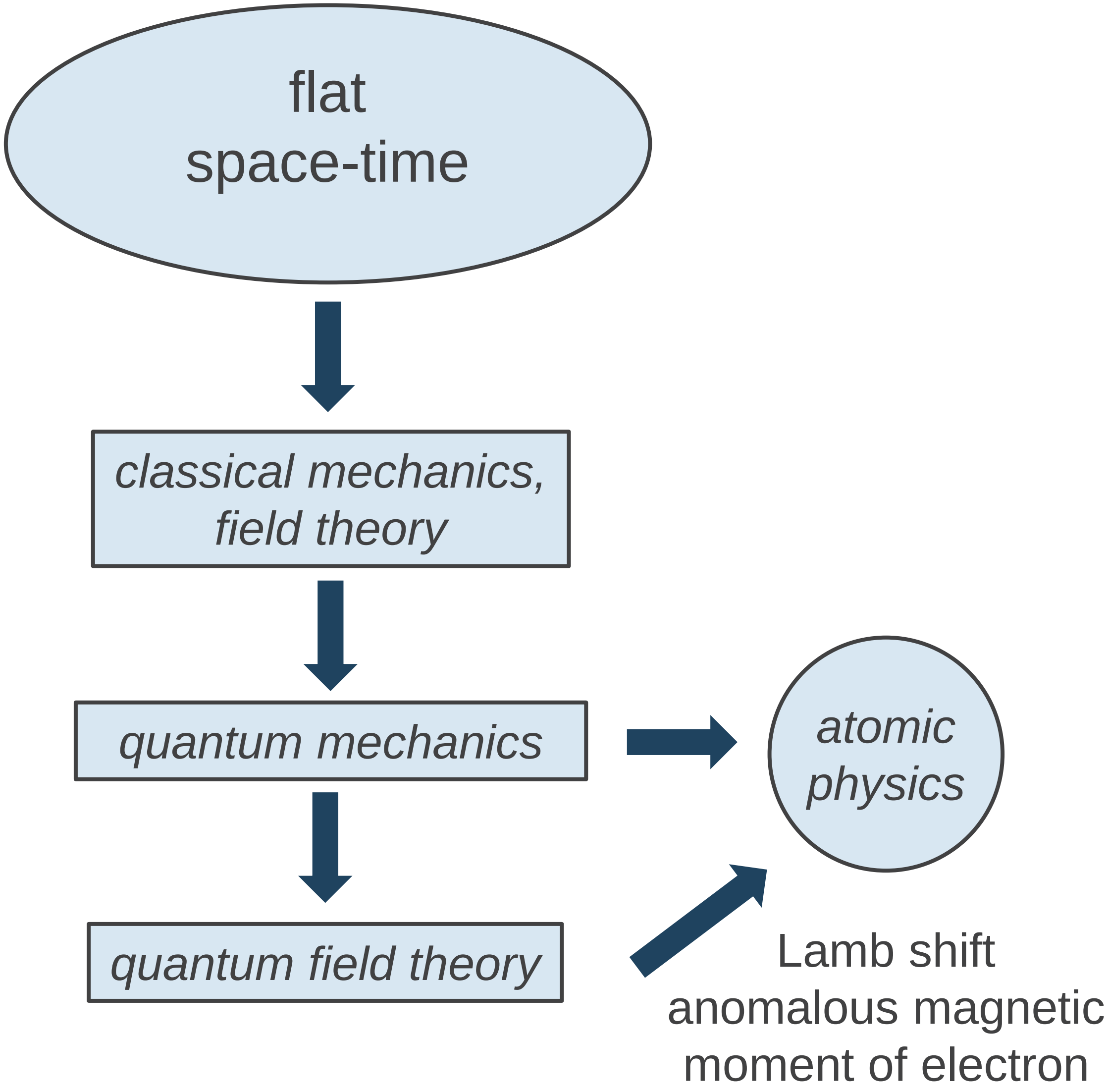
Plan of talk

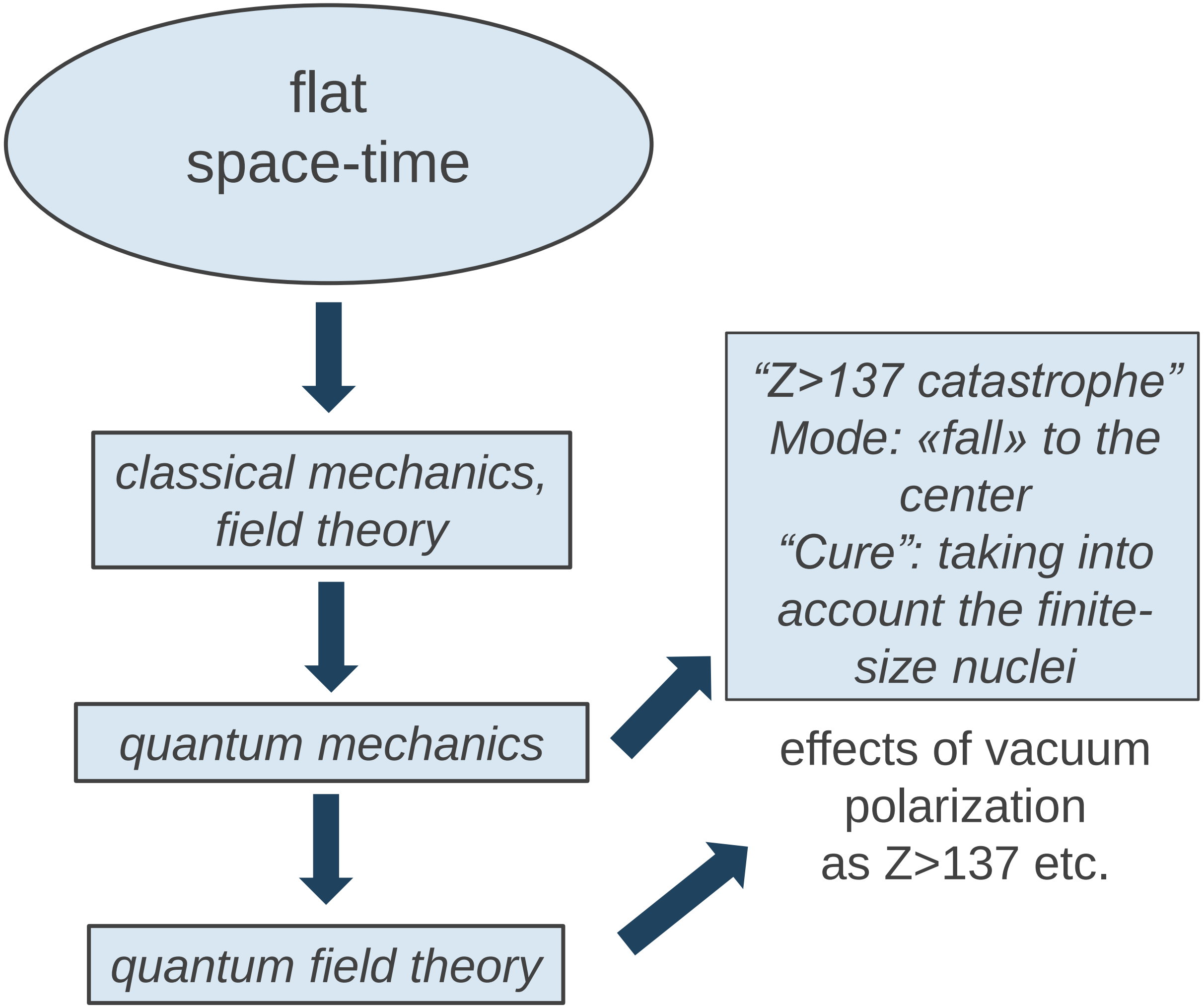
1. *Introduction*
2. *Quantum mechanical hypothesis of cosmological censorship*
3. *Hydrogen-like atom in the strong Coulomb field*
4. *Effective potentials of the relativistic Schrödinger-type equations in the geometries of classical solutions of the general relativity theory with zero and non-zero cosmological constant for:*
    - *bosons (S=0)*
    - *photons (S=1)*
    - *fermions (S=1/2)*
5. *Asymptotics of effective potentials in neighborhoods of event horizons*
6. *Incompatibility of quantum theory with the classical conception of black holes with event horizons of zero thickness*
7. *Discussion and conclusion*
8. *Myths of classical solutions of the general relativity theory*

What we do


flat
space-time
classical mechanics,
field theory
quantum mechanics
quantum field theory
atomic
physics
Lamb shift
anomalous magnetic
moment of electron

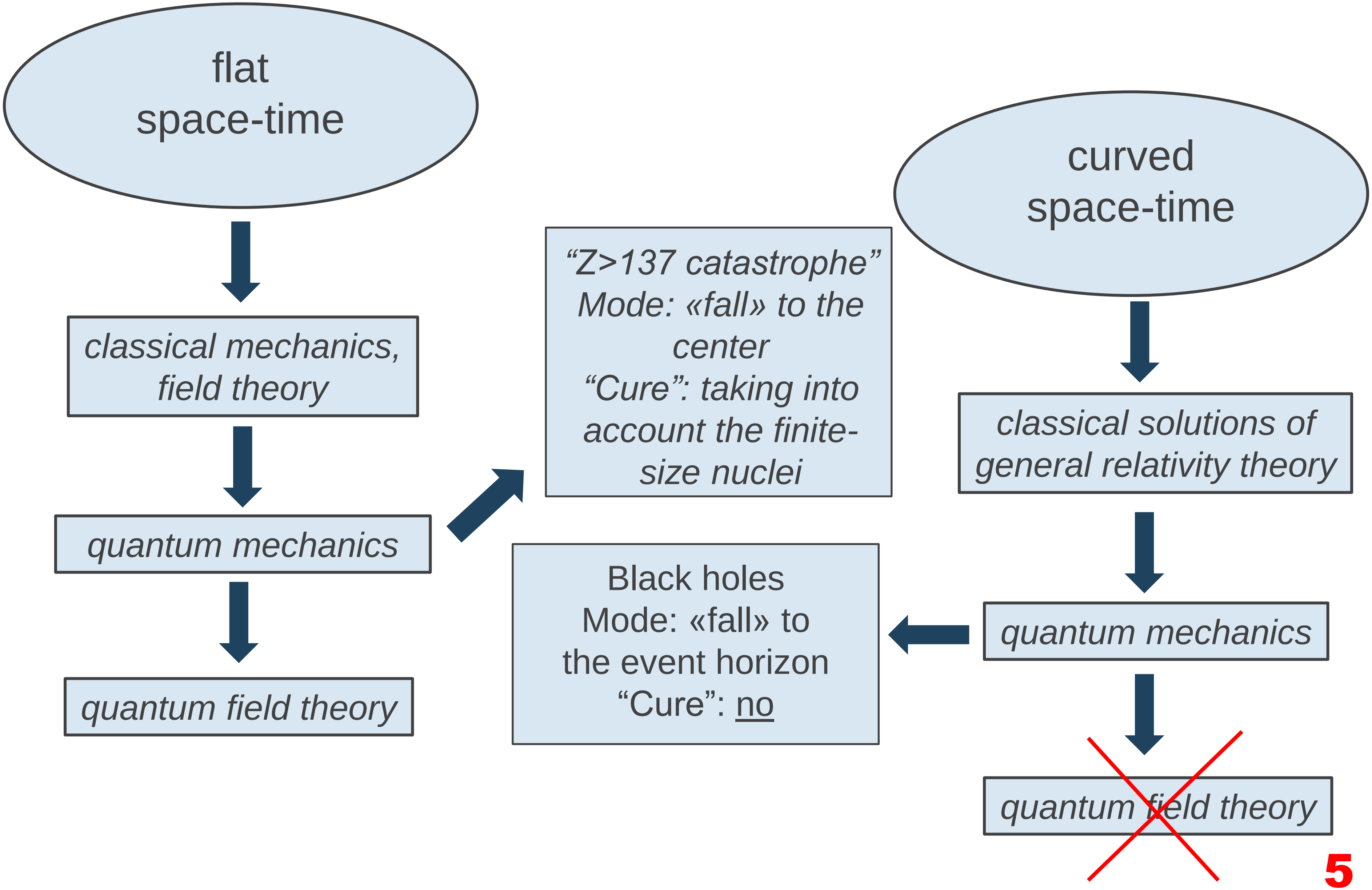
Introduction

What we do



Introduction

What we do



Introduction

The report considers the interaction of scalar particles, photons and fermions with the gravitational and electromagnetic Schwarzschild, Reissner-Nordström, Kerr and Kerr-Newman fields. The behavior of effective potentials in the relativistic Schrödinger-type second-order equations is analyzed. It was found that the quantum theory is incompatible with the hypothesis of the existence of classical black holes with event horizons of zero thickness that were predicted based on solutions of the general relativity with zero and non-zero cosmological constant Λ.

The alternative may be presented by compound systems, i.e., collapsars with fermions in stationary bound states.

Introduction

The following papers are the base of report

1. Stationary solutions of second-order equations for point fermions in the Schwarzschild gravitational field // V.P.Neznamov and I.I.Safronov, J. Exp. Theor. Phys. (2018) ***127****: 647.*

2. Stationary solutions of second-order equations for fermions in Reissner–Nordström space-time // V.P.Neznamov, I.I.Safronov, and V.E.Shemarulin, J. Exp. Theor. Phys. (2018) ***127****: 684.*

3. Stationary solutions of the second-order equation for fermions in Kerr–Newman space-time // V.P.Neznamov, I.I.Safronov, and V.E.Shemarulin, J. Exp. Theor. Phys. (2019) ***128****:64.*

4. Second-order equations for fermions on Schwarzschild, Reissner-Nordström, Kerr and Kerr-Newman space-times // V.P.Neznamov, Theor. Math. Phys. (2018) ***197****: 1823.*

5. Quantum mechanical equivalence of the metric of a centrally symmetric gravitational field // M.V.Gorbatenko and V.P.Neznamov, Theor. Math. Phys. (2019) ***198:****425.*

6. Stationary solutions of the second-order equation for fermions in the external Coulomb field // V.P.Neznamov and I.I.Safronov, J. Exp. Theor. Phys. (2019) ***155****: 792.*

Introduction

The following papers are ready for publication

1. Quantum mechanics of particle stationary states in external singular spherically and axially symmetric gravitational fields // M.V.Gorbatenko and V.P.Neznamov .

2. Quantum mechanics of particle stationary states in external singular spherically and axially symmetric gravitational fields with non-zero cosmological constant // M.V.Gorbatenko and V.P.Neznamov .

3. Quantum mechanics of stationary states of particles interacting with non-extreme rotating charged black holes in minimal five-dimensional gauged supergravity // M.V.Gorbatenko and V.P.Neznamov .

Introduction

We also used the results of separation of variables:

1. in Klein-Gordon equation

- for Kerr-Newman metric - *V.B.Bezerra, H.S.Viera and André A.Costa, Class. Quantum Grav.* ***31****, 045003 (2014);*
- for Kerr-Newman-(A)dS metric - *G.V.Kraniotis, Class. Quantum Grav.* ***33****, 225011 (2016);*
- for five-dimensional geometry of Kerr-Newman-AdS - *S.Q.Wu, Phys. Rev. D* ***80*** *(2009) 084009, arxiv:0906.2049v4 [hep-th];*

2. in Maxwell equations

- for Kerr, Kerr-(A)dS metrics, five-dimensional geometry of Myers-Perry - *O.Lunin, J. High Energ. Phys. (2017)* ***2017****:138, arxiv: 1708.06766v2 [hep-th];*

3. in Dirac equation

- for Kerr-Newman metric - *S.Chandrasekhar, Proc. Roy. Soc. London ser. A* ***349****, 571 (1976);*

  D.Page, Phys. Rev. D ***14****, 1509 (1976); F.Finster, N.Kamran, J.Smoller and S.-T. Yau, Comm. Pure Appl. Math.* ***53****, 1201 (2000);*
- for Kerr-Newman-(A)dS metric - *C.V.Kraniotis, J. Phys. Commun.* ***3****, 035026 (2019), arxiv: 1801.03157;*
- for five-dimensional Kerr-Newman-AdS metric - *S.Q.Wu, Phys. Rev. D* ***80*** *(2009) 084009, arxiv: 0906.2049v4 [hep-th].*

Introduction

Hereafter, we use dimensionless variables for the second-order equations for particles with energy E, mass m and electrical charge q in the spice-time of considered metrics

$$\rho = \frac{r}{l_c};\ \ \varepsilon = \frac{E}{mc^2};\ \ \alpha = \frac{r_0}{2l_c} = \frac{GMm}{\hbar c} = \frac{Mm}{M_P^2}; \alpha_Q = \frac{r_Q}{l_c} = \frac{\sqrt{G}Qm}{\hbar c} = \frac{\sqrt{\alpha_{fs}}}{M_P} m \frac{Q}{e};$$

$$a = \frac{J}{Mc};\ \ \alpha_a = \frac{a}{l_c};\ \ \alpha_{em} = \frac{qQ}{\hbar c} = \alpha_{fs} \frac{qQ}{e^2};\ \ (r_0 = 2GM/c^2,\ r_Q = \sqrt{G}Q/c^2)$$

$l_c = \hbar/mc$ is the Compton wave length of a particle;

$M_P = \sqrt{\hbar c/G} = 2.2 \cdot 10^{-5}\ \text{g}\ \left(1.2 \cdot 10^{19}\,\text{GeV}\right)$ is the Planck mass;

$\alpha_{fs} = \dfrac{e^2}{\hbar c} \approx \dfrac{1}{137}$ is the electromagnetic constant of the fine structure;

α, α_{em} are the gravitational and electromagnetic coupling constants;

α_Q, α_a are the dimensionless constants characterizing the source of the electromagnetic field with the charge Q and the ratio of an angular momentum J to the mass M in the Kerr and Kerr-Newman metrics.

Quantum mechanical hypothesis of cosmological censorship

For classical physics, the hypothesis of cosmic censorship proposed by Penrose forbids the existence of singularities not covered by event horizons in nature.

«Nature has an aversion for naked singularity»

R.Penrose

«Природа питает отвращение к голой сингулярности»

Р.Пенроуз

Quantum mechanical hypothesis of cosmological censorship

In the quantum mechanics, G.T.Horowitz and D.Marolf *(Phys. Rev. D* ***52****, 5670 (1995))* actually proposed a quantum mechanical hypothesis of cosmic censorship. They write in the Introduction: “We will say that a system is nonsingular, when the evolution of any state is uniquely defined for all time. If this is not the case, then there is some loss of predictability and we will say that the system is singular” and incompatible with the quantum theory.

Similarly to Penrose, we should add that such singular systems cannot exist in nature.

Quantum mechanical hypothesis of cosmological censorship

For the radial second-order equations reduced to the relativistic Schrödinger-type equations with the effective potentials $U_{eff}(\rho)$, the behavior of these potentials in the neighborhood of event horizons is essential.

For all the considered metrics, the behavior of effective potentials in the neighborhood of the event horizons has often the form of an infinitely deep potential well $U_{eff}(\rho)\Big|_{\rho\to\rho_\pm} = -K_1 \Big/ (\rho - \rho_\pm)^2 .$

Quantum mechanical hypothesis of cosmological censorship

If $K_1 \geq 1/8$, a so-called "fall" of a particle to the event horizon occurs. In this case the system is singular. The behavior of the radial function from the Schrödinger-type equation has the following form

$$R(\rho)\Big|_{\rho\to\rho_\pm} \sim (\rho-\rho_\pm)^{1/2}\sin\left(\sqrt{K_2}\ln(\rho-\rho_\pm)+\delta\right),$$

where $K_2 = 2\left(K_1 - (1/8)\right)$.

As $\rho \to \rho_\pm$, the functions of stationary states of discrete and continuous spectrum $R(\rho)$ have an unlimited number of zeros, discrete levels of energy " dive" into the area of negative continuum. The functions $R(\rho)$ do not have the defined values as $\rho = \rho_\pm$.

Also, the system will be singular, if the exponent of the denominator in the expression for the effective potential higher than two. In this case

$$R_s(\rho)\Big|_{\rho\to\rho_\pm} \sim (\rho-\rho_\pm)^{s/4}\sin\left(\frac{2}{s-2}\sqrt{\frac{2K_1}{(\rho-\rho_\pm)^{s-2}}}+\delta_s\right),$$

where δ, δ_s are arbitrary phases, $s > 2$ is the exponent in the effective potential.

Quantum mechanical hypothesis of cosmological censorship

In the Hamiltonian formulation, the mode of a "particle fall" to the event horizons means that the Hamiltonian H has nonzero deficiency indexes. For elimination of this mode, it is necessary to choose the additional boundary conditions on the event horizons. The self-conjugate extension of the Hermitian operator H is determined by this choice.

Hydrogen-like atom in the strong Coulomb field

In the quantum mechanics there is an example confirming the quantum mechanical hypothesis of cosmic censorship. For hydrogen-like atoms, the Sommerfeld formula for the fine structure of the energy levels has the form

$$\varepsilon = \left(1 + \frac{\alpha_{fs}^2 Z^2}{\left(n - |\kappa| + \sqrt{\kappa^2 - \alpha_{fs}^2 Z^2}\right)}\right)^{-1/2} .$$

As $Z > 137|\kappa|$ the expression for energy becomes imaginary («the $Z > 137$ catastrophe»).

Let us consider solutions of the relativistic Schrödinger-type equation with the effective potential U_{eff}^C for fermions in the Coulomb field. The asymptotics of the effective potential as $\rho \to 0$ has the form

$$U_{eff}^C \Big|_{\rho \to 0} = -\frac{\left(Z\alpha_{fs}\right)^2 - \frac{3}{4} + \left(1 - \kappa^2\right)}{2\rho^2}.$$

Hydrogen-like atom in the strong Coulomb field

In the asymptotics, dependently on Z it is possible to single out three characteristic areas. For example, we consider these areas for the bound states $1S_{1/2}\left(\kappa=-1\right)$, $2P_{1/2}\left(\kappa=+1\right)$. In the first area $1\leq Z<\sqrt{3}/2\alpha_{fs}$ as $\rho\to 0$ there exists the positive barrier $\sim 1/\rho^2$ with the following potential well. As $Z=Z_{cr}=\sqrt{3}/2\alpha_{fs}\approx 118.7$ the potential barrier disappears; as for $Z>Z_{cr}$ as $\rho\to 0$ the potential well $-K/\rho^2$ remains.

In the second area $119\leq Z<137$ the coefficient is $K<1/8$, which permits the existence of the fermion stationary bound states.

In the third area $Z\geq 137$ as $\rho\to 0$ there is the potential well with $K\geq 1/8$, which is indicative of the implementation of the regime “fall” to the center. In Fig. 1 for $Z=1;119;140$ the dependencies $U_{eff}^{F}\left(\rho\right)$ as $\kappa=-1$ $\left(1S_{1/2}\right)$ are shown. There for comparison there are shown the dependence of the Coulomb potential $V\left(\rho\right)=-\left(Z\alpha_{fs}/\rho\right)$.

Hydrogen-like atom in the strong Coulomb field

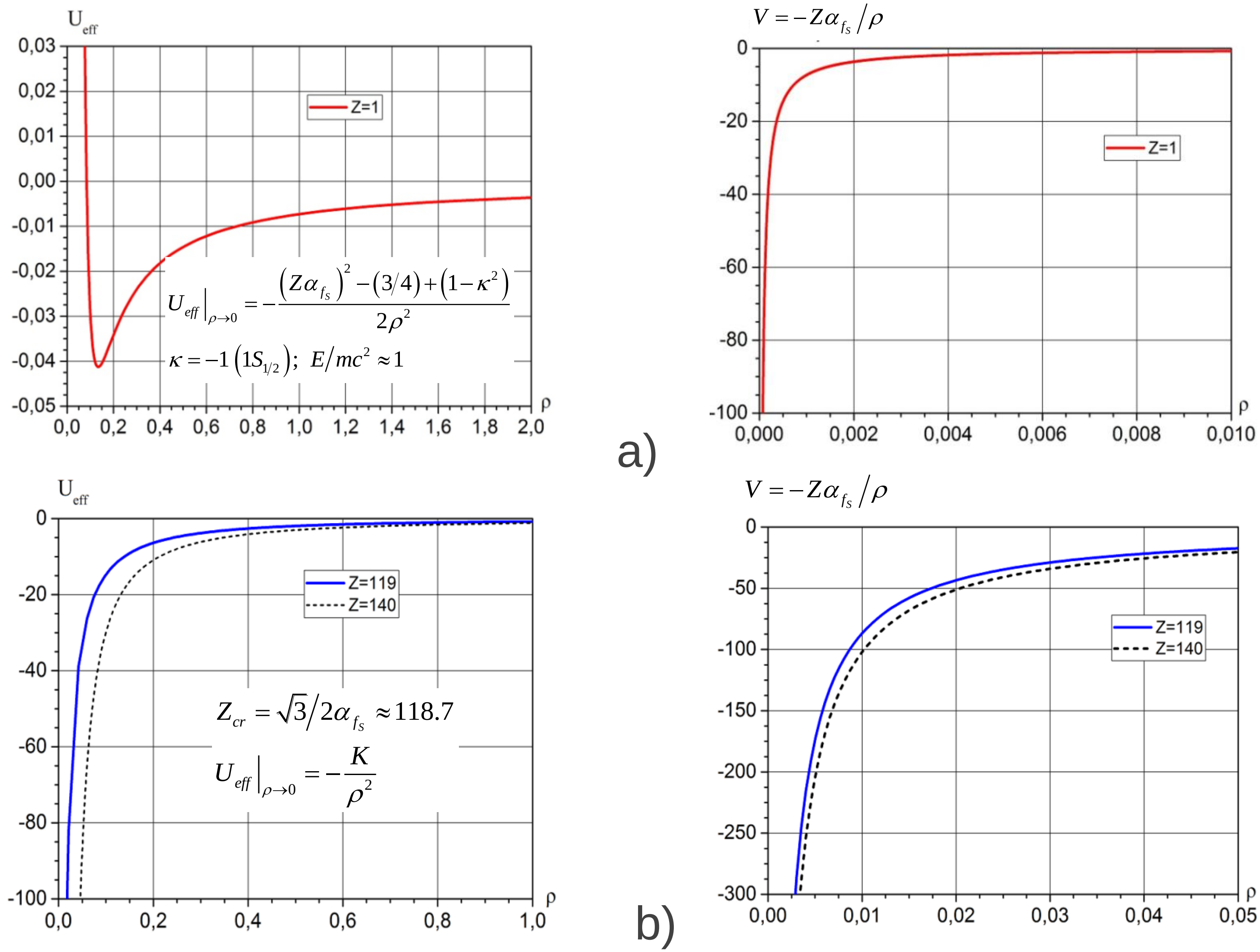


Fig. 1. The dependencies $U_{eff}(\rho)$ and $V(\rho)$.

Hydrogen-like atom in the strong Coulomb field

In the third area with $\left(Z\alpha_{f_S}\right)^2 \geq \kappa^2$ the system "a fermion in the Coulomb field" is singular, and incompatible with the quantum theory. For elimination of mode "fall" to the center, it was offered to take into account the finite sizes of nuclei *(I.Ya.Pomeranchuk and A.Smorodinsky (1945). W.Paper and W.Griener (1969))*. As a result, at the characteristic lengths of the nuclei sizes it is cut either the Coulomb potential, or the effective one (see Fig.1 b). Presently, there are ~ 30 such cutting methods (*D. Andrae, Physics Reports **336**, 413 (2000)*).

The system "an electron in the Coulomb field of the finite-size atomic nucleus" is nonsingular.

Hydrogen-like atom in the strong Coulomb field

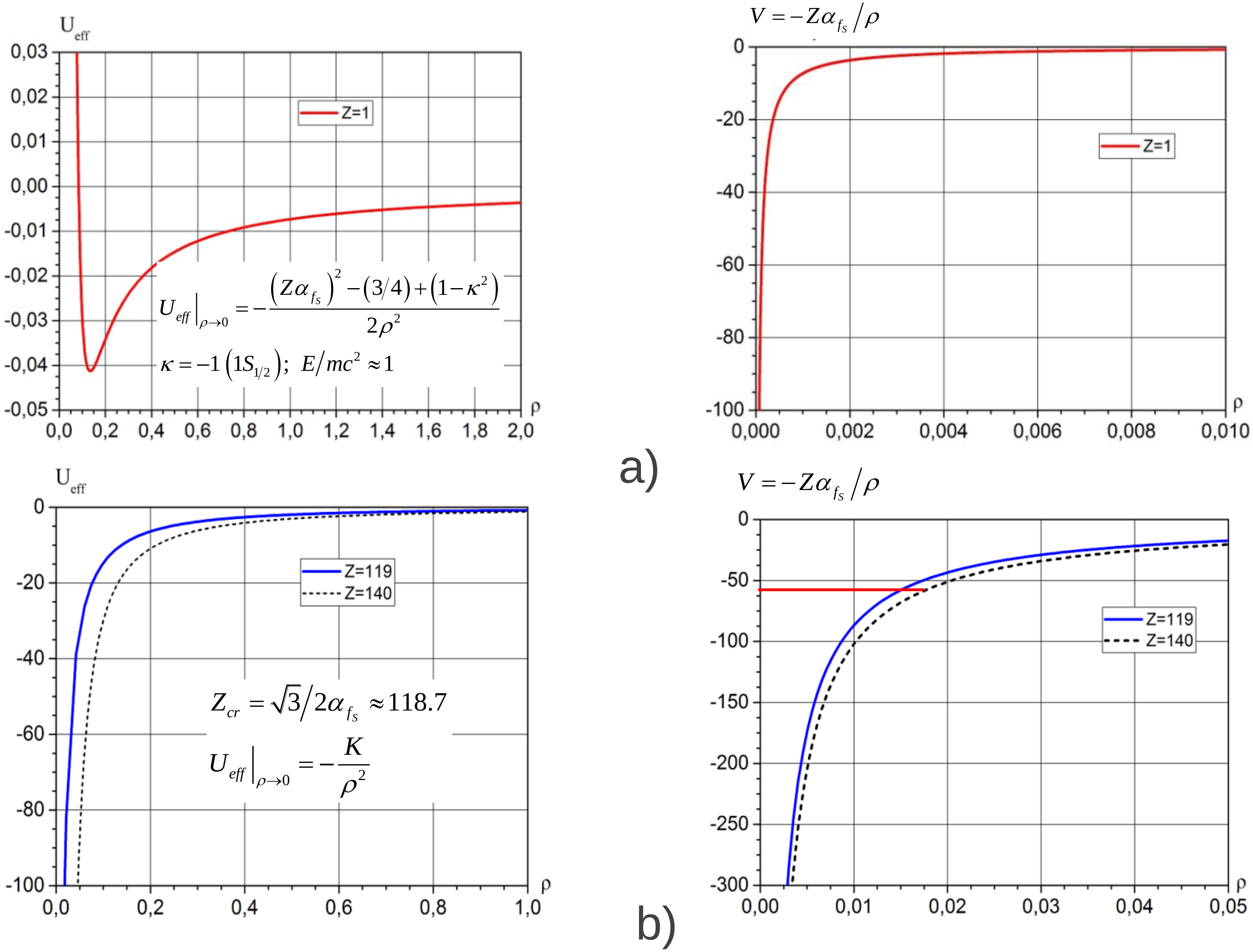


Fig. 1. The dependencies $U_{eff}(\rho)$ and $V(\rho)$.

Hydrogen-like atom in the strong Coulomb field

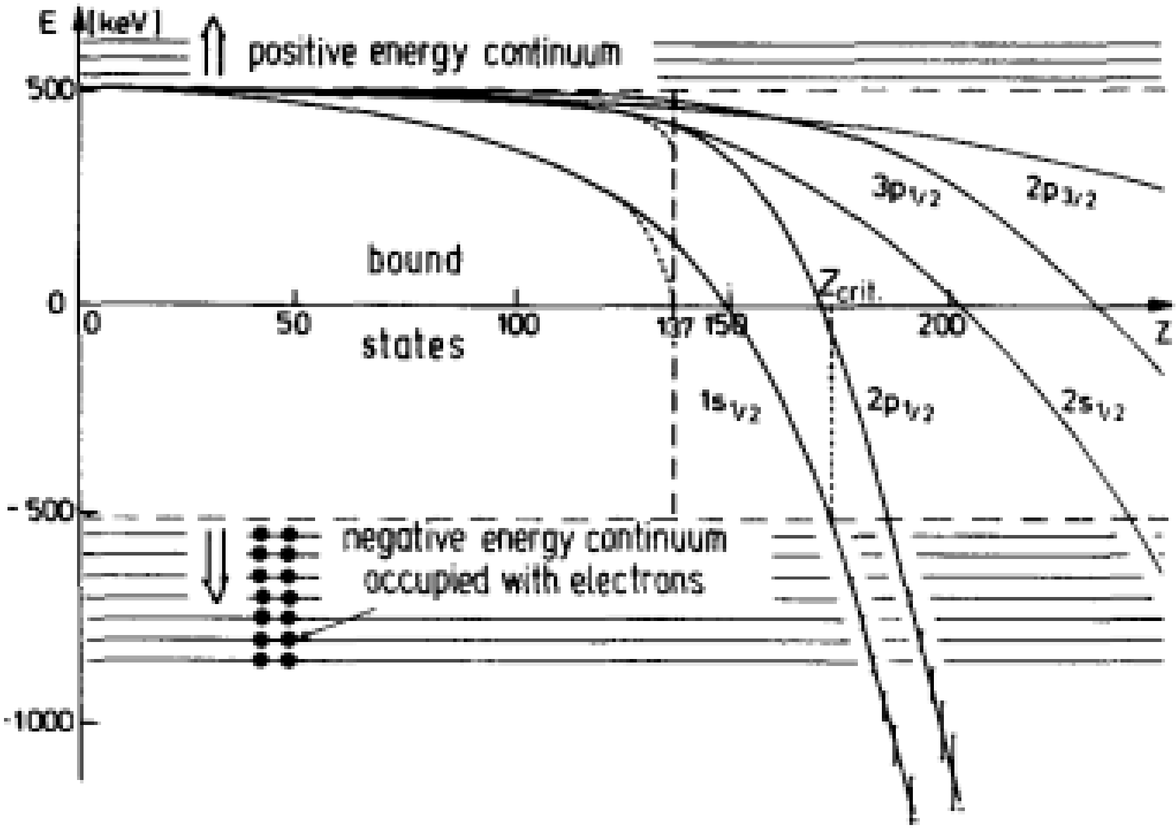


Fig. 1.4. Lowest bound states of the Dirac equation for nuclei with charge Z. While the Sommerfeld fine-structure energies (– – –) for $\varkappa = -1$ (s states) end at $Z = 137$, the solutions for extended Coulomb potentials (———) can be traced down to the negative-energy continuum reached at the critical charge Z_{cr} for the $1s$ state. The bound states entering the continuum obtain a spreading width as indicated [Mü 72a]

[W.Greiner, B.Mueller, J.Rafelski. *Quantum Electrodynamics of Strong Fields.* Springer-Verlag Berlin Heidelberg New York Tokyo, 1985)]

Effective potentials

For a closed system of “a particle in the external force field”, quantum mechanics allows existence of stationary states with definite real energies of particle. The stationary states involve both the states of discrete spectrum (bound states) and states of continuous spectrum (scattering states). In this case, the wave function of the particle is written as

$$\psi(\mathbf{r},t)=\psi(\mathbf{r})e^{-iEt},$$

where E is a particle energy. Here and hereafter, we use the system of units $\hbar=c=1$.

We will explore in closed systems the existence possibility of stationary states at interaction of scalar particles $(S=0)$, photon $(S=1)$, fermions $(S=1/2)$ with the Schwarzschild, Reissner-Nordström, Kerr and Kerr-Newman black holes.

We consider the four-dimensional geometries with zero and non-zero cosmological constant Λ and also AdS black hole geometry in five-dimensional gauged supergravity.

Effective potentials

Scheme of our analysis

I. Scalar uncharged particles $(S=0)$

1. The Klein-Gordon equation : $(-g)^{-1/2}\dfrac{\partial}{\partial x^{\mu}}\left[(-g)^{1/2}g^{\mu\nu}\dfrac{\partial}{\partial x^{\nu}}\Phi\right]+m^2\Phi=0$

2. Separation of variables : $\Phi(\mathbf{r},t)=\sum\limits_{l,m_\varphi} R_l^{S,RN}(r)Y_{lm_\varphi}(\theta,\varphi)e^{-iEt}$

(for the spherically symmetric Schwarzschild, Reissner-Nordström metrics, $Y_{lm_\varphi}(\theta,\varphi)$ are the spherical harmonics)

$$\Phi_{KN}(\mathbf{r},t)=\sum_{l,m_\varphi} R_l^{KN}(r)S(\theta)e^{-iET}e^{im_\varphi\varphi},$$

(for the axially symmetric Kerr, Kerr-Newman metrics,

$S(\theta)$ are the oblate spheroidal harmonic functions,

$S_{lm_\varphi}(ic\cos\theta)$ where $c=a^2\left(\left(E^2/m^2\right)-1\right))$

3. Second-order equation for radial functions $R_l(\rho)$:

$$\frac{d^2R_l}{d\rho^2}+A(\rho)\frac{dR_l}{d\rho}+B(\rho)R_l=0$$

Effective potentials

4. Reduction to form of the Schrödinger equation with the effective potential $U_{eff}(\rho)$:

$$\bar{R}_l(\rho) = R_l(\rho)\exp\frac{1}{2}\int A(\rho')d\rho',$$

$$\frac{d^2\bar{R}_l(\rho)}{d\rho^2} + 2\left(E_{Schr} - U_{eff}(\rho)\right)\bar{R}_l(\rho) = 0,$$

$$U_{eff}(\rho) = E_{Schr} + \frac{1}{4}\frac{dA}{d\rho} + \frac{1}{8}A^2 - \frac{1}{2}B,$$

$$E_{Schr} = \left(\varepsilon^2 - 1\right)/2.$$

In the second equation the summand E_{Schr} is singled out and at the same time added to the third equation. On the one hand, it is done, in order to impart to equation to form of the Schrödinger-type equation, and, on the other hand, ensure the classical asymptotics of the effective potential as $\rho \to \infty$.

5. Study of behavior of effective potential in the neighborhood of the event horizons.

Effective potentials

II. Photons $(S=1)$

The Maxwell equations for the spherically symmetric Schwarzschild and Reissner-Nordstrom metrics were written in three-dimensional form (in three-dimensional space with metric $\gamma_{ik}=-g_{ik}+\left(g_{0i}g_{0k}/g_{00}\right)$. Separation of the variables was performed via expansion of electric $\mathbf{E}(\mathbf{x},t)$ and magnetic $\mathbf{H}(\mathbf{x},t)$ fields in terms of the vector harmonics of the electric, magnetic and longitudinal types. For the axially symmetric Kerr, Kerr-Newman metrics, the variables separation procedure of O.Lunin was used. As a result, the effective potentials of the Schrodinger-type relativistic equations were obtained.

III. Fermion with charge $(S=1/2)$

For metrics under consideration, the effective potentials of the Schrodinger-type equation were obtained in our papers *(V.P.Neznamov, I.I.Safronov, V.E.Shemarulin, J. Exp. Theor. Phys.* ***127*** *(2018),* ***128*** *(2019)).* These papers also contain the solutions for the stationary bound states with energies $E=E^{st}$.

Asymptotics of effective potentials in neighborhoods of event horizons

Schwarzschild field

scalar particle photon fermion

$E \neq E^{st}$

$$U_{eff}\Big|_{r\to r_0} = -\frac{1}{\left(r-r_0\right)^2}\left(\frac{1}{8}+\frac{r_0^2E^2}{2}\right)$$

fermion

$E = E^{st} = 0$

$$U_{eff}\Big|_{r\to r_0} = -\frac{3}{32}\frac{1}{\left(r-r_0\right)^2}$$

Reissner-Nordström field

scalar particle fermion

$\varepsilon \neq \varepsilon^{st}$

$$U_{eff}\Big|_{\rho\to(\rho_\pm)_{RN}} = -\frac{1}{\left(\rho-(\rho_\pm)_{RN}\right)^2}\left[\frac{1}{8}+\frac{\left(\varepsilon-\frac{\alpha_{em}}{(\rho_\pm)_{RN}}\right)^2(\rho_\pm)_{RN}^4}{2\left[\left((\rho_+)_{RN}-(\rho_-)_{RN}\right)\right]^2}\right]$$

photon

$$U_{eff}\Big|_{r\to(r_\pm)_{RN}} = -\frac{1}{\left(r-(r_\pm)_{RN}\right)^2}\left[\frac{1}{8}+\frac{\omega^2(r_\pm)_{RN}^4}{2\left[(r_+)_{RN}-(r_-)_{RN}\right]^2}\right]$$

fermion

$\varepsilon = \varepsilon^{st} = \alpha_{em}/(\rho_\pm)_{RN}$

$$U_{eff}\Big|_{\rho\to(\rho_\pm)_{RN}} = -\frac{3}{32}\frac{1}{\left(\rho-(\rho_\pm)_{RN}\right)^2}$$

Asymptotics of effective potentials in neighborhoods of event horizons

Kerr, Kerr-Newman fields

scalar particle

fermion

$\varepsilon \neq \varepsilon^{st}$

$$U_{eff}^{KN}\Big|_{\rho\to(\rho_\pm)_{KN}} = -\frac{1}{\left(\rho-(\rho_\pm)_{KN}\right)^2}\left[\frac{1}{8}+\frac{\left(\varepsilon-\varepsilon_{KN}^{st}\right)^2\left((\rho_\pm)_{KN}^2+\alpha_a^2\right)^2}{2\left[(\rho_+)_{KN}-(\rho_-)_{KN}\right]^2}\right]$$

photon

fermion

$$\varepsilon = \varepsilon^{st} = \frac{\alpha_a m_\varphi + \alpha_{em}(\rho_\pm)_{KN}}{\alpha_a^2 + (\rho_\pm)_{KN}^2}$$

$$U_{eff}^{KN}\Big|_{\rho\to(\rho_\pm)_{KN}} = -\frac{3}{32}\frac{1}{\left(\rho-(\rho_\pm)_{KN}\right)^2}$$

$$U_{eff}^{KN}\Big|_{\rho\to(\rho_\pm)_{KN}} = -\frac{1}{\left(r-(r_\pm)_K\right)^2}\left[\frac{1}{8}+\frac{\left(\omega-\dfrac{am_\varphi}{(r_\pm)_{KN}^2+a^2}\right)^2\left((r_\pm)_{KN}^2+a^2\right)^2}{2\left((r_+)_{KN}-(r_-)_{KN}\right)^2}\right]$$

Asymptotics of effective potentials in neighborhoods of event horizons

Non-zero cosmological constant

We have proved that Kerr-Newman-(A)dS, Kerr-(A)dS, Reissner-Nordström-(A)dS and Schwarzschild-(A)dS black holes have the same nature of the divergence of effective potentials near the event horizons as in case of $\Lambda = 0$.

For example, for the most general Kerr-Newman-(A)dS field, for the de Sitter solution $(\Lambda > 0)$ at $r \to r_+$

scalar particle

fermion

$\Omega_+ \neq 0$

photon

$$U_{eff}^{KN-dS}\Big|_{r\to r_+} = -\frac{1}{(r-r_+)^2}\left\{\frac{1}{8} + \frac{\Omega_+^2}{2\left[(r_+ - r_-)(r_+ - r_\Lambda^+)(r_+ - r_\Lambda^-)\right]^2}\right\}$$

$$\Omega_+ = \Xi\left(E\left(r^2 + a^2\right) - m_\varphi a - \frac{qQr}{\Xi}\right), \quad \Xi = 1 + a^2\frac{\Lambda}{3}$$

$$U_{eff}^{KN-dS}\Big|_{r\to r_+} = -\frac{1}{(r-r_+)^2}\times$$
$$\times\left\{\frac{1}{8} + \frac{\left(\omega\left(r^2+a^2\right) - m_\varphi a\right)^2}{2\left[(r_+ - r_-)(r_+ - r_\Lambda^+)(r_+ - r_\Lambda^-)\right]^2}\right\}$$

We also proved that the Kerr-Newman-AdS black hole in the minimal five-dimensional gauged supergravity has the same nature of the divergence of effective potentials near the event horizons as in the four-dimensional case.

AdS black hole $\longrightarrow$ CFT

Asymptotics of effective potentials in neighborhoods of event horizons

Coordinate transformations of the Schwarzschild metric

For the scalar particles, our analysis can be supplemented with the Eddington-Finkelstein and the Painlevé-Gullstrand metrics, for which the leading singularity in the neighborhood of event horizon remains at the real axis and it is similar to those for the Schwarzschild metrics.

For fermions, for the Schwarzschild space-time in the isotropic coordinates, as well as the Eddington-Finkelstein, Painlevé-Gullstrand, Lemaitre-Finkelstein and Kruskal-Szekeres metrics, it is proved that in case of the stationary bound state $\varepsilon^{st} = 0$ the leading singularity in the event horizon neighborhood is preserved the same as in the initial Schwarzschild metrics.

Incompatibility of quantum theory with the classical conception of black holes with event horizons of zero thickness

The analysis shows that for $\varepsilon \neq \varepsilon^{st}$ in the considered gravitational fields the existence of the stationary particle states is impossible. Systems "a particle in the gravitational and electromagnetic fields" are singular and incompatible with the quantum theory.

Incompatibility of quantum theory with the classical conception of black holes with event horizons of zero thickness

Existence of the stationary discrete states with $\varepsilon = \varepsilon^{st}$ does not change the previous conclusion because to attain the values ε^{st} it is necessary to have the quantum transitions with emission or absorption of the photons with defined energy. However, quantum mechanical stationary states of photons with real energy ω do not exist in the considered gravitational and electromagnetic fields.

For all the considered metrics and particles with different spins it is characteristic the universal nature of the divergence of the effective potentials near to the event horizons. Uncovered singularities doesn't allow using the quantum theory in full measure that leads to the necessity of changing the initial formulation of the physical problem.

Discussion and conclusion

1. *Is it possible to cure the concerned GR solutions from the standpoint of quantum mechanics?*

The answer to this question is negative. Actually, the uniqueness theorem for the black holes affirms that the most general asymptotically flat solution of the GR equations is the Kerr metrics with the monopole mass M and the angular momentum J (*M. Heusler, Cambrige, UK (1996)*). Any deviation from the spherically symmetric mass distribution leads to the event horizon disappearance and appearance of series of naked singularities instead (see the static and stationary q-metrics).

Discussion and conclusion

1.1 Deformed distribution of the collapsar mass: prolate or oblate along the axis Z

the Schwarzschild metric:

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \frac{1}{1 - \frac{2m}{r}} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

The static q-metric (*Quevedo, Int. J.Mod. Phys. D (2011)*)

$$ds^2 = \left(1 - \frac{2m}{r}\right)^{1+q} dt^2 - \frac{1}{\left(1 - \frac{2m}{r}\right)^q} \left[\left(1 + \frac{m^2 \sin^2\theta}{r^2\left(1 - \frac{2m}{r}\right)}\right)^{-q(2+q)} \left(\frac{dr^2}{\left(1 - \frac{2m}{r}\right)} + r^2 d\theta^2\right) + r^2 \sin^2\theta d\varphi^2\right]$$

$q \in (-1, 0)$ is the prolate mass distribution

$q \in (0, \infty)$ is the oblate mass distribution

Discussion and conclusion

As a result, we have “naked singularity” instead of a black hole as $r \to r_0$ for any $q \neq 0$.

One can build a stationary q-metric (*Quevedo at al., arxiv: 1510.04155v1*).

The similar conclusion: a black hole transforms to naked singularity on the surface covering the Schwarzschild event horizon.

1.2 Nonpoint collapsar and linkage.

If, similarly to the Coulomb potentiasl as $Z \geq 137$ $(\kappa = -1)$, we perform the linkage of the GR external vacuum solutions with the internal solution options, preserving the continuity of the metric tensor and its first derivatives, then the linkage radius will larger than the radius of the external event horizon. This is not a black hole!!!

As a result, in the mentioned above cases we deviate from the classical black holes with the event horizons.

Discussion and conclusion

2. Is it possible to use the concerned classical GR solutions from the standpoint of quantum mechanics?

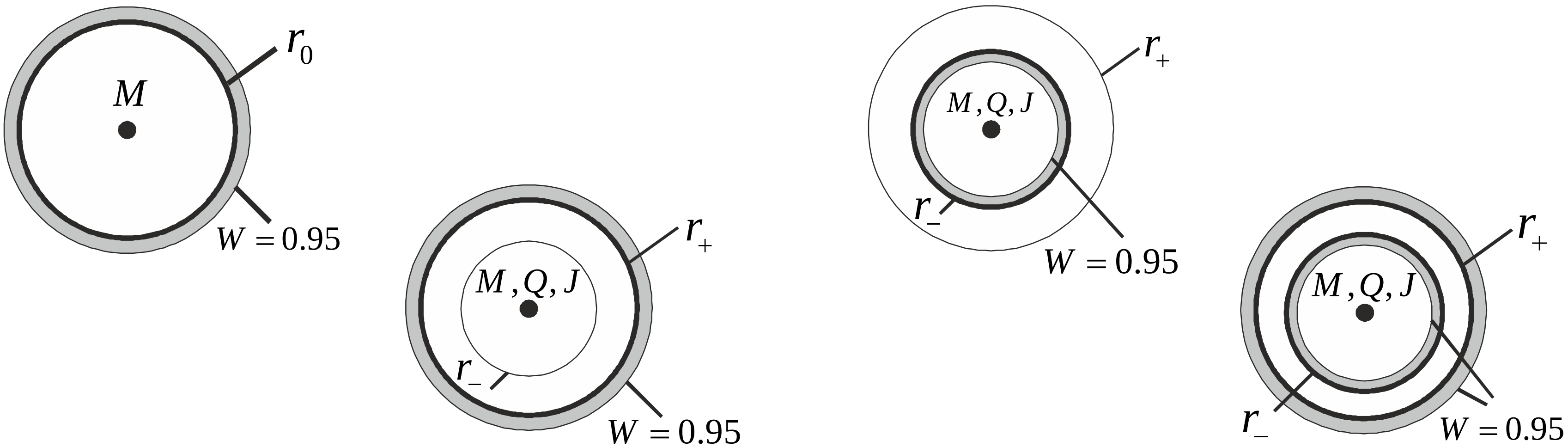
Is it possible to blur the event horizons in a natural way?

Our answer to these questions is positive and we propose to supplement the gravitational collapse mechanism.

Let us, at the last collapse stage the gravitational field captures the half-spin particles, which after the formation of the event horizons will get into the stationary bound states with $\varepsilon = \varepsilon^{st}$ both under the inner and above external event horizons. For the next particles interacting with such compound systems, the self-consistent gravitational and electromagnetic fields will be determined both by mass and charge of collapsar , and by the masses and charges of the fermions in the stationary bound states with $\varepsilon = \varepsilon^{st}$ in the neighborhood of the event horizons.

Discussion and conclusion

Obviously, such system may be nonsingular. To provide a strong proof, there are required exact calculations of the self-consistent gravitational and electromagnetic fields, as well as the proof that the stationary states of the quantum-mechanical probe particles exist in these fields. The discussed compound systems may be considered as the dark matter carriers. On the other hand, these systems may be the building blocks for joining new particles and formation of the macroscopic objects in the long run.



Discussion and conclusion

CONCLUSION

The quantum theory is incompatible with the existence of the Schwarzschild, Reissner-Nordström, Kerr, Kerr-Newman black holes with event horizons of zero thickness that were predicted based on the GR solutions with zero and non-zero cosmological constant Λ .

Myths of classical solutions of the general relativity theory

№	classical theory	quantum theory	opinion of physical community
1	*unstable system electron + proton*	*stable atomic systems*	+
2	*ergoregion in Kerr, Kerr-Newman metrics for rotating system* $\left(r^2 - r_0 r + a^2\cos^2\theta + r_Q^2\right) = 0$	*ergoregion does not appear in the quantum Klein-Gordon-Fock, Maxwell, Dirac equations and in effective potentials*	-
3	*ring singularity in Kerr, Kerr-Newman metrics* $\left(r^2 = a^2\right)$	*ring singularity does not appear in quantum equations and in effective potentials*	-
4	*hypothesis of Penrose's cosmic censorship*	*there is not confirmation for Reissner-Nordström, Kerr and Kerr-Newman metrics in fermion Schrödinger-type equations with effective potentials*	+ -
5	*black holes with event horizons of zero thickness*	*quantum theory is incompatible with the conception of black holes with event horizons of zero thickness*	-
6	-	*phenomenology: firewalls, generalized principle of uncertainty etc.* *compound systems: collapsars with fermions in stationary bound states*	+ - -

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Particle Creation by Black Holes

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Abstract. In the classical theory black holes can only absorb and not emit particles. However it is shown that quantum mechanical effects cause black holes to create and emit particles as if they were hot bodies with temperature $\frac{\hbar\kappa}{2\pi k} \approx 10^{-6}\left(\frac{M_\odot}{M}\right)$ °K where κ is the surface gravity of the black hole. This thermal emission leads to a slow decrease in the mass of the black hole and to its eventual disappearance: any primordial black hole of mass less than about 10^{15} g would have evaporated by now. Although these quantum effects violate the classical law that the area of the event horizon of a black hole cannot decrease, there remains a Generalized Second Law: $S+\frac{1}{4}A$ never decreases where S is the entropy of matter outside black holes and A is the sum of the surface areas of the event horizons. This shows that gravitational collapse converts the baryons and leptons in the collapsing body into entropy. It is tempting to speculate that this might be the reason why the Universe contains so much entropy per baryon.

Discussion and conclusion

In order to calculate this asymptotic form it is more convenient to decompose the ingoing and outgoing solutions of the wave equation into their Fourier components with respect to advanced or retarded time and use the continuum normalization. The finite normalization solutions can then be recovered by adding Fourier components to form wave packets. Because the space-time is spherically symmetric, one can also decompose the incoming and outgoing solutions into spherical harmonics. Thus, in the region outside the collapsing body, one can write the incoming and outgoing solutions as

$$f_{\omega' lm} = (2\pi)^{-\frac{1}{2}} r^{-1} (\omega')^{-\frac{1}{2}} F_{\omega'}(r) e^{i\omega' v} Y_{lm}(\theta, \phi)\,, \tag{2.11}$$

$$p_{\omega lm} = (2\pi)^{-\frac{1}{2}} r^{-1} \omega^{-\frac{1}{2}} P_{\omega}(r) e^{i\omega u} Y_{lm}(\theta, \phi)\,, \tag{2.12}$$

where v and u are the usual advanced and retarded coordinates defined by

$$v = t + r + 2M \log\left|\frac{r}{2M} - 1\right|, \tag{2.13}$$

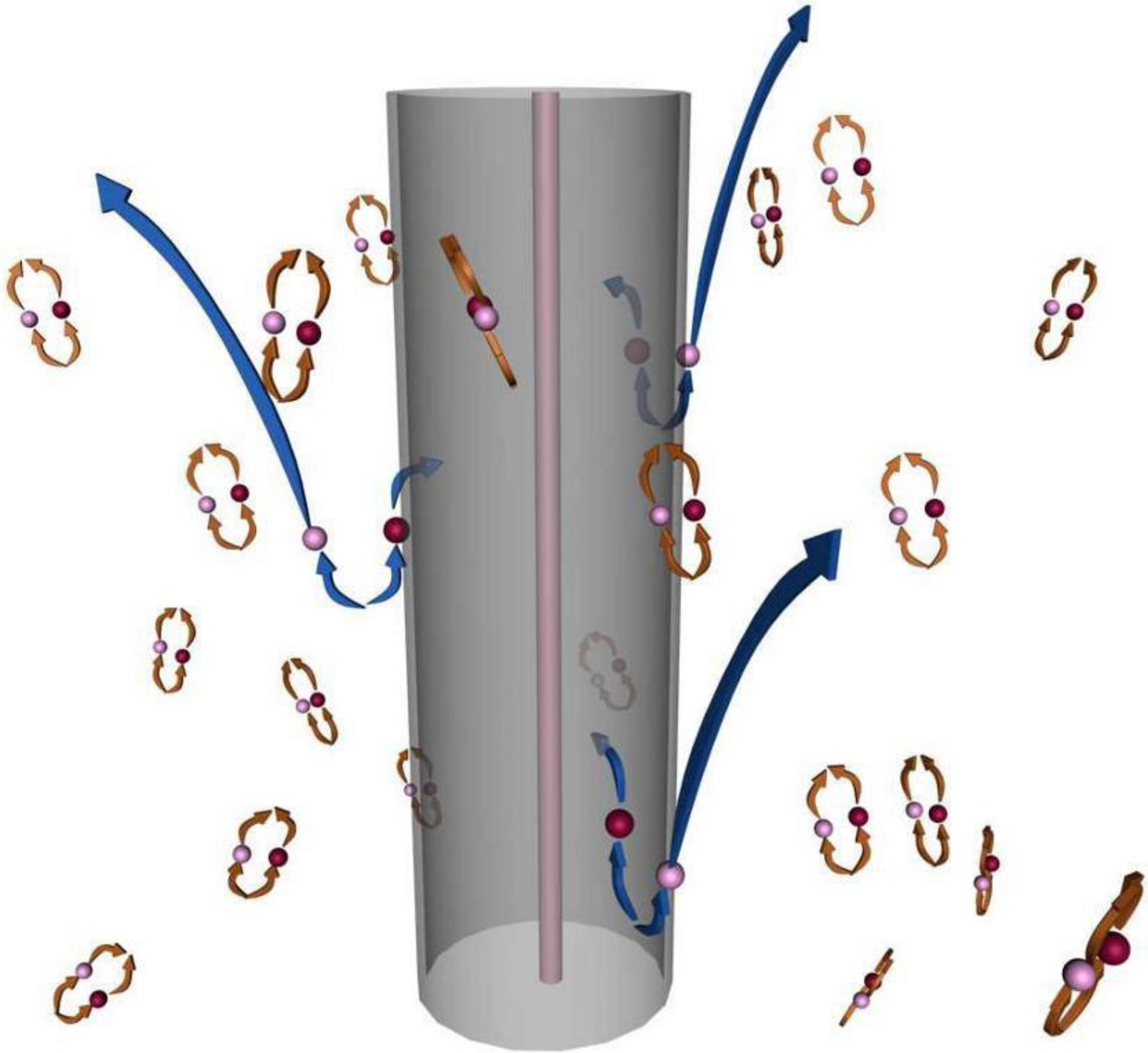
$$u = t - r - 2M \log\left|\frac{r}{2M} - 1\right|. \tag{2.14}$$

Each solution $p_{\omega lm}$ can be expressed as an integral with respect to ω' over solutions $f_{\omega' lm}$ and $\bar{f}_{\omega' lm}$ with the same values of l and $|m|$ (from now on I shall drop the suffices l, m):

$$p_{\omega} = \int_0^{\infty} (\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} \bar{f}_{\omega'}) d\omega'\,. \tag{2.15}$$

To calculate the coefficients $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$, consider a solution p_{ω} propagating backwards from $\mathscr{I}^+$ with zero Cauchy data on the event horizon. A part $p_{\omega}^{(1)}$ of the solution p_{ω} will be scattered by the static Schwarzchild field outside the collapsing body and will end up on $\mathscr{I}^-$ with the same frequency ω. This will give a $\delta(\omega' - \omega)$ term in $\alpha_{\omega\omega'}$. The remainder $p_{\omega}^{(2)}$ of p_{ω} will enter the collapsing body

Discussion and conclusion



Thank you for your attention!