Tetraquark – adequate formulation of QCD sum rules

Dmitri Melikhov

SINP, M. V. Lomonosov Moscow State University

For correlators involving tetraquark currents, both sides of conventional QCD sums rules à la Shifman-Vainshtein-Zakharov split into two non-overlapping classes of contributions, such that each of these classes satisfy its own QCD sum rule. The contribution of the tetraquark pole appears only in one of these sum rules which therefore represent tetraquark-adequate QCD sum rules. We demonstrate that the appropriate OPE side of the tetraquark-adequate QCD sum rules starts with specidic diagrams of order $O(\alpha_s^2)$.

In collaboration with W. Lucha and H. Sazdjian

Based on:

Narrow exotic tetraquark mesons in large-N_c QCD, **PRD96, 014022, 2017;**

Tetraquark and two-meson states at large-N_c, **EPJC77, 866, 2017;**

Tetraquark-adequate formulation of QCD sum rules, **PRD100, 2019;**

Tetraquark-adequate QCD sum rules for quark-exchange diagrams, **arXiv:1909.06324, 2019;**

Choice of local tetraquark interpolating current

• For the sake of simplicity consider flavour-exotic currents involving quark fields of four different flavours.

• For color structure of the tetraquark current it is sufficient to consider singlet-singlet structure [any global color singlet composed of colored bilinears (octet-octet, triplet-antitriplet) may be reduced to singlet-singlet by Fierz].

• Choosing any "internal" color structure of the globally singlet interpolating tetraquark current does not "preselect" any structure of the possible tetraquark bound state which we want to study (molecular meson-meson or confined $\bar{D}D$).

• For currents consisting of quark fields of four different flavours we consider two different flavour structures (color singlet-singlet)

$$\theta_{abcd} = \bar{q}_a q_b \bar{q}_c q_d$$

and

$$\theta_{adcb} = \bar{q}_a q_d \bar{q}_c q_b.$$

For all Green functions we must consider "direct"

$$\langle j_{ab} j_{cd} j^{\dagger}_{ab} j^{\dagger}_{cd} \rangle$$

and "recombination"

$$\langle j_{ab} j_{cd} j^{\dagger}_{ad} j^{\dagger}_{cb} \rangle$$

contributions.

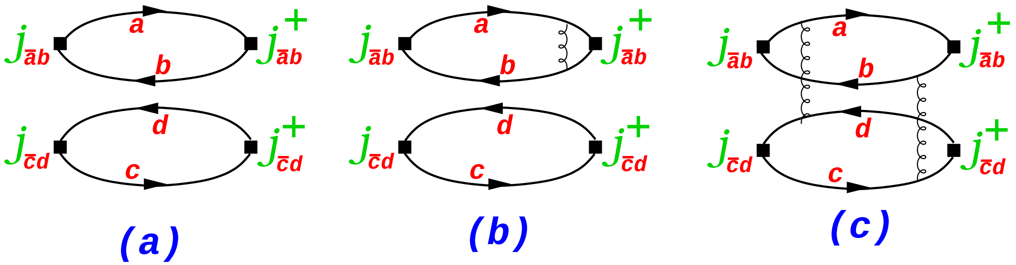
Direct Green functions

• *Four-point functions of bilinear quark currents*:

We start with four-point Green functions of bilinear quark currents, depend on 6 variables p_1^2, p_2^2, $p_1'^2$, $p_2'^2$, $p = p_1 + p_2 = p_1' + p_2'$, and the two Mandelstam variables $s = p^2$ and $t = (p_1 - p_1')^2$.

Criteria for selecting diagrams which potentially contribute to the tetraquark pole at $s = M_T^2$:

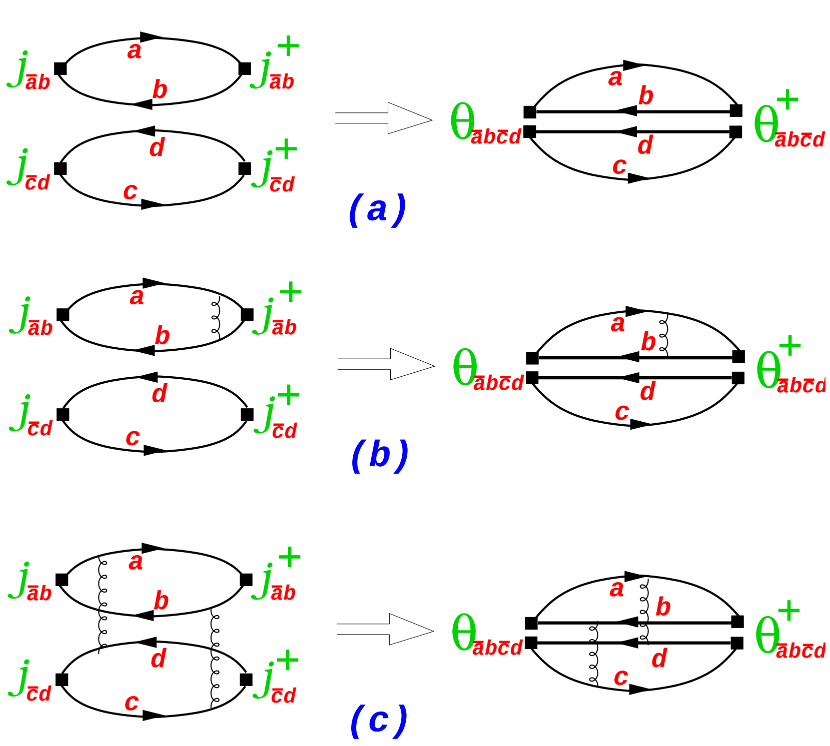
1. The diagram should have a nontrivial (i.e., non-polynomial) dependence on the variable s.
2. The diagram should have a four-particle cut (i.e. threshold at $s = (m_a + m_b + m_c + m_d)^2$), where m_i are the masses of the quarks forming the tetraquark bound state. The presence or absence of this cut is established by solving the Landau equations for the corresponding diagram.

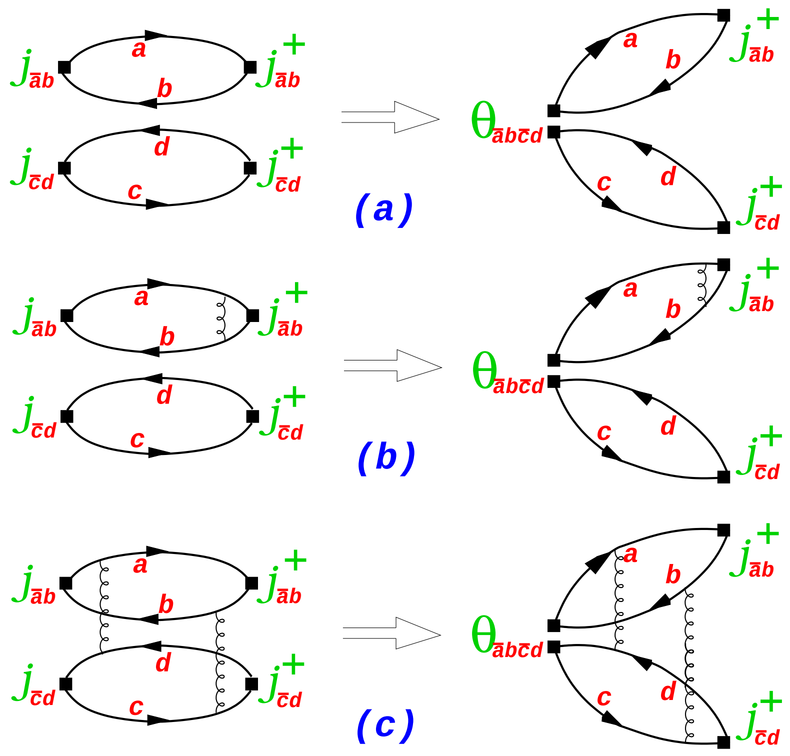


Diagrams satisfying these criteria are "T-phile" diagrams. Only diagram (c) is T-phile diagram.

T-phile diagrams appear at order $O(\alpha_s^2)$ and higher.

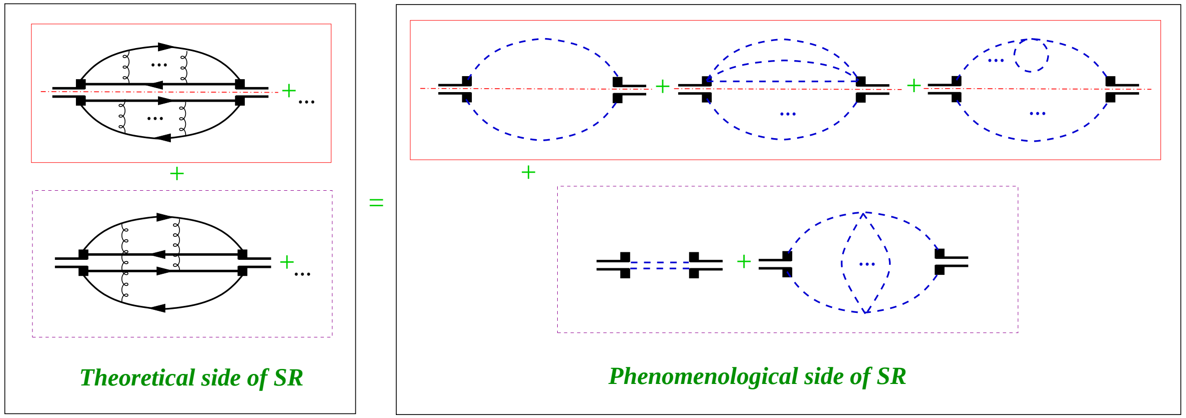
• *Direct two- and three-point Green function involving tetraquark currents*.



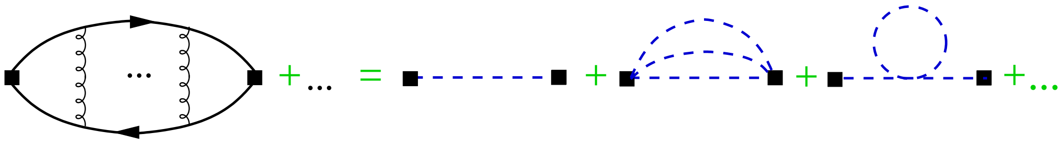


One expects diagrams (a) and (b) should not contribute to QCD sum rules describing T properties.

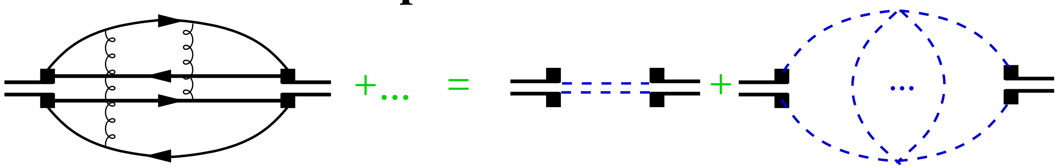
Sum rule for direct two – point function of tetraquark currents



Sum rule for two-point function of bilinear quark currents:



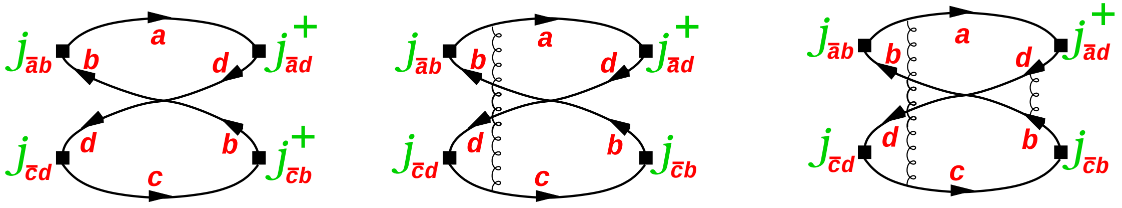
Final T-adequate sum rule for direct two-point function:



Only those QCD diagrams obtained by merging vertices in T-phile diagrams of 4-point functions of bilinear quark currents contribute to properly formulated QCD sum rules for T-states.

Recombination Green functions

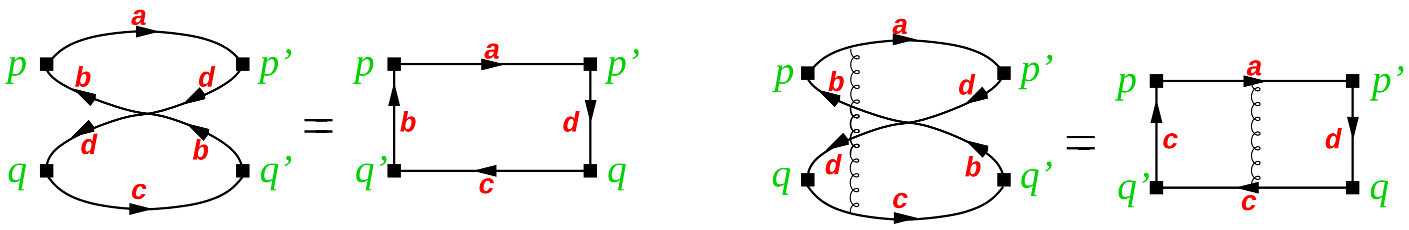
Recombination 4-point functions:



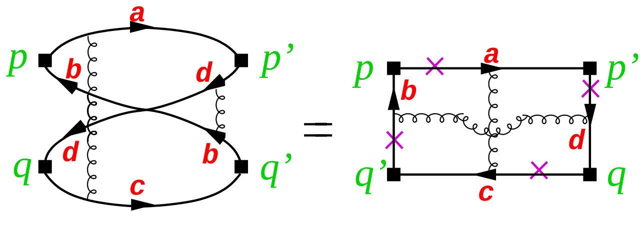
(a) (b) (c)

Which QCD diagrams contain four-quark s-channel cut?

To answer this question (i) unfold diagrams (ii) solve Landau equations



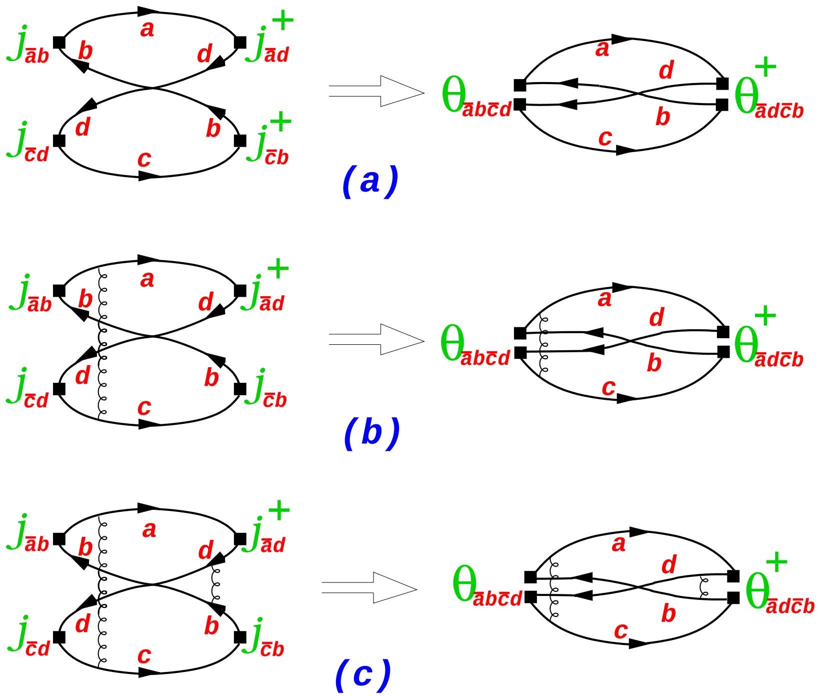
(a) (b)

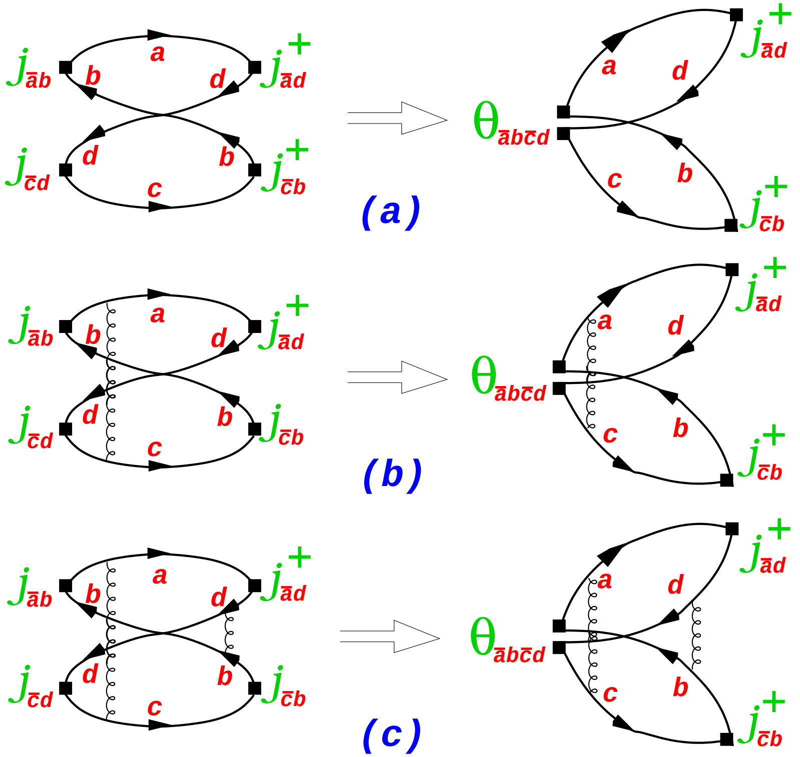


(c)

Result: (a) and (b) do not have 4-quark cut; one needs two gluon exchanges [diagram (c)].

Recombination two- and three-point function involving tetraquark currents:





One expects that diagrams (a) and (b) do not contribute to the final *T*-appropriate QCD sum rule.

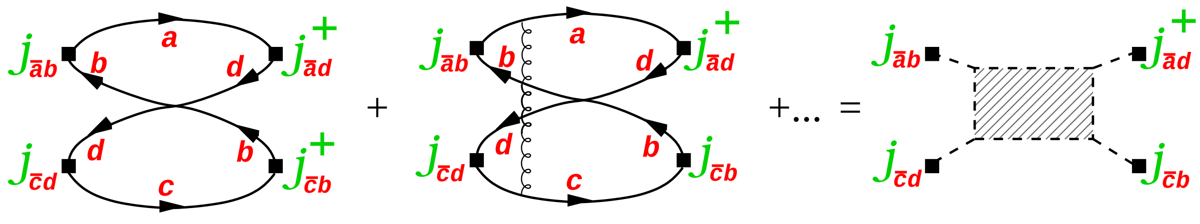
Go back to 4-pt recombination Green functions: QCD diagrams for 4-pt function may be classified according to the structure of their s-channel singularities: let us consider separately

(a) the subset of diagrams on the OPE side that do not contain 4-quark s-channel cut and

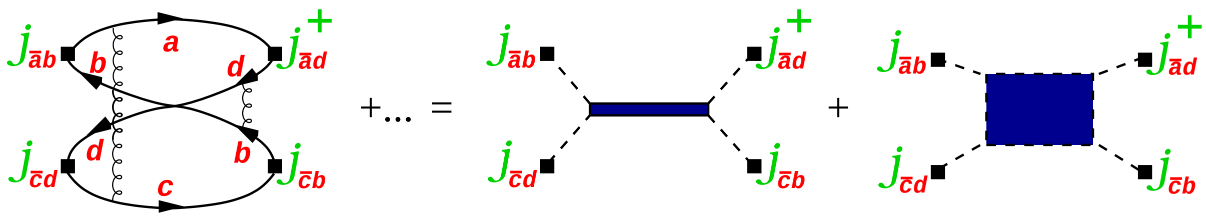
(b) subset of those diagrams on the OPE side that do contain such a cut

For each subset we can write quark-hadron duality relations. Crucial observation:

- **on the hadron side of (a) there are no hadron diagrams with s-channel two-meson cut;**
- **Two-meson s-channel intermediate states with quark flavour content $ac\bar{b}\bar{d}$ and the possible s-channel T-pole are contained in subset (b) only.**

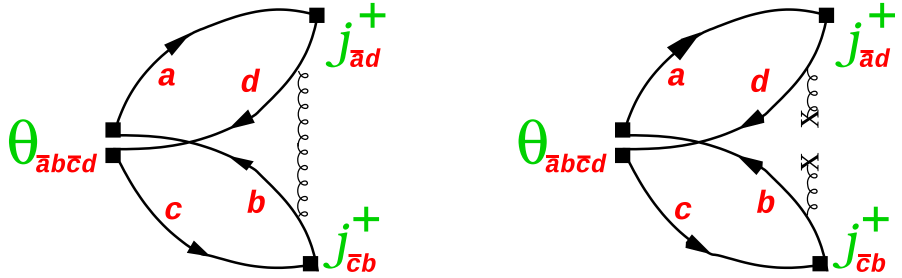


(a)



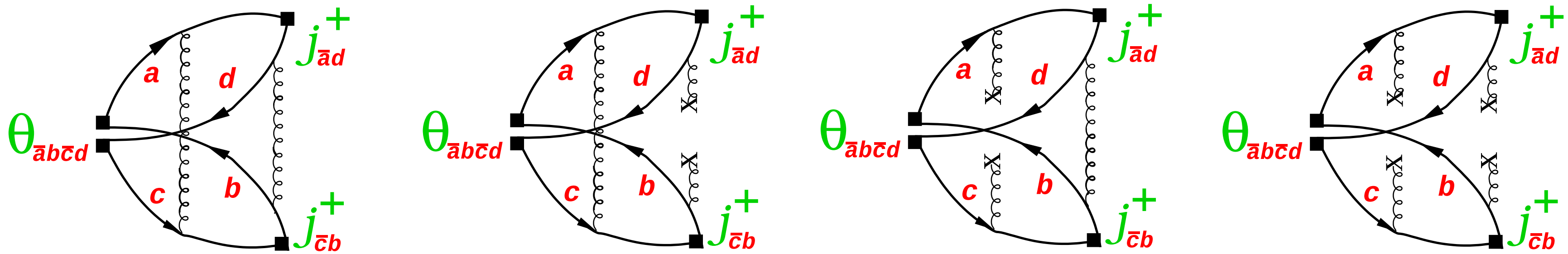
(b)

Diagrams that DO NOT contribute to T properties:



(a) (b)

Diagrams of lowest order that DO CONTRIBUTE to T properties



(a) (b) (c) (d)

Conclusions

• The ultimate QCD sum rules that contain tetraquark contribution on the hadron side (T-adequate QCD sum rules), have one crucial property: on the OPE side of this relation one has contributions of specific "nonfactorizable" QCD diagrams, that are obtained by merging the appropriate vertices in the T-phile diagrams of the 4-point function. These nonfactorizable diagrams start at order $O(\alpha_s^2)$.

Emphasize that $O(1)$ and $O(\alpha_s)$ diagrams are not related to T-properties and cannot appear in the T-adequate QCD sum rules.

• The same arguments apply to pentaquark states.