Charming loops in exclusive rare FCNC B – decays

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Rare B-decays induced by flavour-changing neutral currents (FCNC) is one of the promising candidates for probing physics beyond the Standard model. However, for identifying potential new physics from the data, reliable control over QCD contributions is necessary. I focus on one of such QCD contributions - the charming loops that provide difficulties in disentangling new physics and discuss the possibility to gain control over them.

1. **Motivation: tensions with SM predictions in FCNC $b \to s, d$ decays**
2. **$H_{\rm eff}$ for $b \to s, d$ and the $\langle \gamma l^+ l^- | H_{\rm eff} | B \rangle$ amplitude**
3. **Charming loops**
4. **Conclusions and outlook**

FCNC $b \to s$ and $b \to d$ transitions do not occur at the tree level in SM and proceed via loops. As the result, BRs of FCNC decays are small; on the other hand, new particles may show up virtually in the loops. Therefore, FCNC decays are most popular candidates for indirect search of physics BSM.

Tensions between SM predictions and observations in FCNC $b \to s$ transitions:

In SL decays:

- $\mathcal{R}_K = \frac{\mathcal{B}(B^+\to K^+\mu^+\mu^-)}{\mathcal{B}(B^+\to K^+e^+e^-)} = 0.745^{+0.090}_{-0.074}(\text{stat}) \pm 0.036(\text{syst})$ (2.6σ) **range of** $q^2 = [1,6]$ **GeV^2;**
- $\mathcal{B}(B^+ \to K^+\mu^+\mu^-)_{\text{SM}} = (1.75^{+0.60}_{-0.29}) \times 10^{-7}$**;**
  $\mathcal{B}(B^+ \to K^+\mu^+\mu^-)_{\text{exp}} = (1.19 \pm 0.03 \pm 0.06) \times 10^{-7}$ **range of** $q^2 = [1,6]$ **GeV^2.**
- $\mathcal{R}_{K^{*0}} = 0.69\,^{+\,0.11}_{-\,0.07}\,(\text{stat}) \pm 0.05\,(\text{syst})$ **for ,** $q^2 = [1,6]$ **GeV^2**
- **Same for** $\mathcal{B}(B^+ \to \phi\mu^+\mu^-)$ $(> 3\sigma)$

In leptonic decays:

- $\frac{\mathcal{B}(B_s^0\to\mu^+\mu^-)_{\text{exp}}}{\mathcal{B}(B_s^0\to\mu^+\mu^-)_{\text{SM}}} = 0.76^{+0.20}_{-0.18}$ (1.2σ)

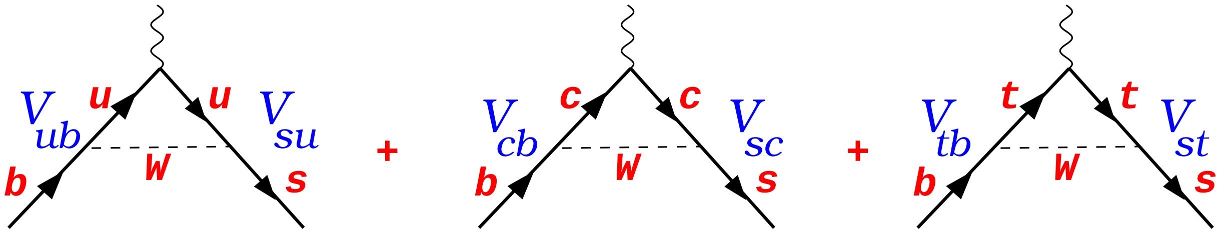
Effective Hamiltonian for FCNC B – decays

At the tree level, the SM allows the following transitions between quarks:

- **charged current transitions** $b \to W^- q$**,** $q = u, c, t$
- **neutral current transitions:** $b \to b\gamma$**,** $b \to bZ^0$**.**

FCNC transitions $b \to s, d$ **are forbidden in SM at the tree level, and proceed via loops (boxes and penguins).**

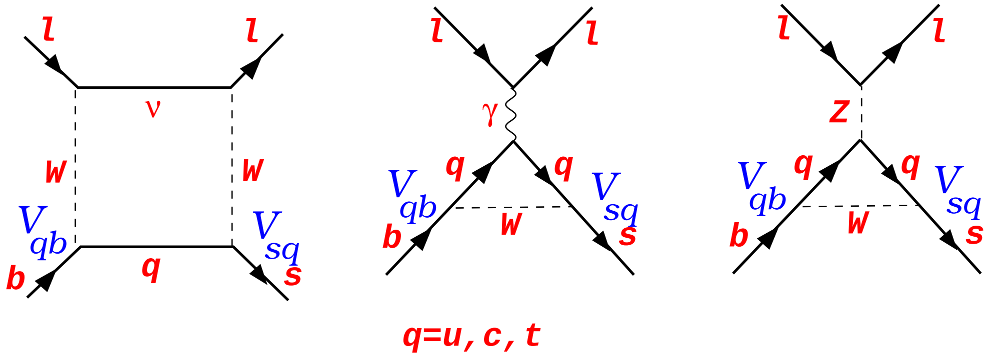
- **Example:** $b \to s\gamma$ **vertex**



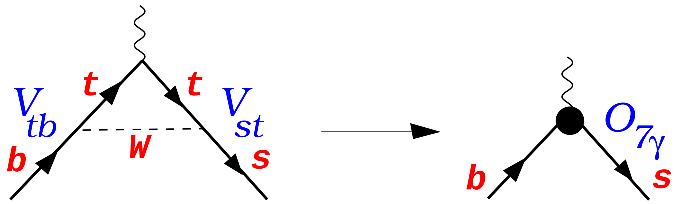
CKM Unitarity: $V^*_{us} V_{ub} + V^*_{cs} V_{cb} + V^*_{ts} V_{tb} = 0$

Important feature: due to CKM unitarity, leading UV divergences cancel (GIM mechanism).

- **Example:** $b \to sl^+l^-$ **vertex**



In the loops heavy and light particles propagate. For the description of B-decays, much heavier particles W, Z, t may be "integrated out". Example for $b \to s\gamma$:



The contribution of heavy degrees of freedom is described in terms of the effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{b\to s} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i(\mu) O_i(\mu),$$

$O_i(\mu)$ **- operators;** $C_i(\mu)$ **- WC,** $\mu_0 = 5$ **GeV:** $C_7(\mu_0) = 0.312$**,** $C_{9V}(\mu_0) = -4.21$**,** $C_{10A}(\mu_0) = 4.64$**.**

Contributions of top and W to $\mathcal{H}_{\text{eff}}$**:**

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{b\to s\ell^+\ell^-} &= \frac{G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{2\pi} V_{tb} V_{ts}^* [-2im_b \frac{C_{7\gamma}(\mu)}{q^2} \cdot \bar{s}\sigma_{\mu\nu} q^\nu (1+\gamma_5)\, b \cdot \bar{\ell}\gamma^\mu \ell + \\ &+ C_{9V}(\mu) \cdot \bar{s}\gamma_\mu (1-\gamma_5)\, b \cdot \bar{\ell}\gamma^\mu \ell + C_{10A}(\mu) \cdot \bar{s}\gamma_\mu (1-\gamma_5)\, b \cdot \bar{\ell}\gamma^\mu \gamma_5 \ell] \end{aligned}$$

Don't forget:

b, c, u, d, s**-quarks are dynamical!**

To calculate any amplitude of B**-decay, one needs to calculate the amplitude of** $H_{\rm eff}$ **(describes top and** W**, and** Z**) and add contributions of loops with dynamical light degrees of freedom (masses** $\ll M_W$**).**

In the following I concentrate on $B \to \gamma l^+ l^-$ **decays.**

$\langle \gamma l^+ l^- | H_{\rm eff} | B \rangle$ **+ add contributions of** c **and** u **quarks.**

Top – quark contributions

We need to calculate $\langle \gamma l^+ l^- | H_{\rm eff} | B \rangle$ with

$$\mathcal{H}_{\rm eff}^{b\to s\ell^+\ell^-} = \frac{G_F}{\sqrt{2}}\frac{\alpha_{\rm em}}{2\pi} V_{tb}V_{ts}^* [-2im_b \frac{C_{7\gamma}(\mu)}{q^2}\cdot \bar{s}\sigma_{\mu\nu}q^\nu (1+\gamma_5)\, b \cdot \bar{\ell}\gamma^\mu \ell + \\ + C_{9V}(\mu)\cdot \bar{s}\gamma_\mu (1-\gamma_5)\, b\cdot \bar{\ell}\gamma^\mu\ell + C_{10A}(\mu)\cdot \bar{s}\gamma_\mu (1-\gamma_5)\, b\cdot \bar{\ell}\gamma^\mu\gamma_5\ell]$$

The $\langle \gamma l^+ l^- | H_{\rm eff} | B \rangle$ amplitude can be parameterized via form factors:

$$\begin{aligned}
\langle \gamma(k,\,\epsilon)|\bar{s}\gamma_\mu\gamma_5 b|B_s(p)\rangle &= i\,e\,\epsilon^*_\alpha \left(g_{\mu\alpha}\,pk - p_\alpha k_\mu\right)\frac{F_A(k'^2,k^2)}{M_B},\\
\langle \gamma(k,\,\epsilon)|\bar{s}\gamma_\mu b|B_s(p)\rangle &= e\,\epsilon^*_\alpha\,\epsilon_{\mu\alpha\xi\eta}p_\xi k_\eta \frac{F_V(k'^2,k^2)}{M_B},\\
\langle \gamma(k,\,\epsilon)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|B_s(p)\rangle\,(p-k)^\nu &= e\,\epsilon^*_\alpha \left[g_{\mu\alpha}\,pk - p_\alpha k_\mu\right] F_{TA}(k'^2,k^2),\\
\langle \gamma(k,\,\epsilon)|\bar{s}\sigma_{\mu\nu} b|B_s(p)\rangle\,(p-k)^\nu &= i\,e\,\epsilon^*_\alpha\epsilon_{\mu\alpha\xi\eta}p_\xi k_\eta\, F_{TV}(k'^2,k^2).
\end{aligned}$$

k' **momentum emitted from the FCNC vertex $b\to s$ (k'^2 - first variable of the ffs)**
k **momentum emitted from the e.-m. vertex (k^2 - second variable of the ffs)**

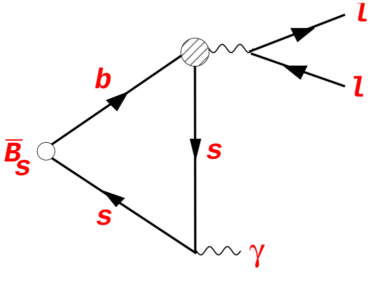
Electromagnetic gauge invariance imposes rigorous constraints on the form factors:

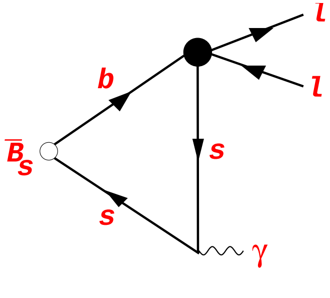
$$F_{TA}(0,q^2) = F_{TV}(0,q^2),\, F_{TA}(0,0) = F_{TV}(0,0)$$

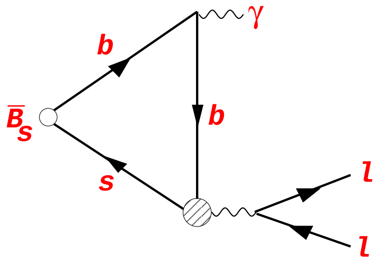
but

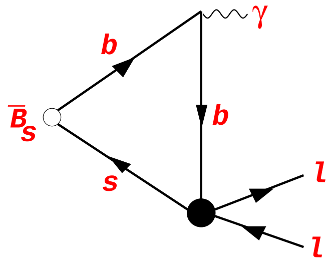
$$F_{TA}(q^2,0) \neq F_{TV}(q^2,0).$$

• **Diagrams with** *real* **photon emission from valence quarks are described via** $F(q^2, 0)$ **(no poles in the range** $0 < q^2 < M_B^2$**: poles in** q^2 **appear at** $q^2 = M_R^2$**):**

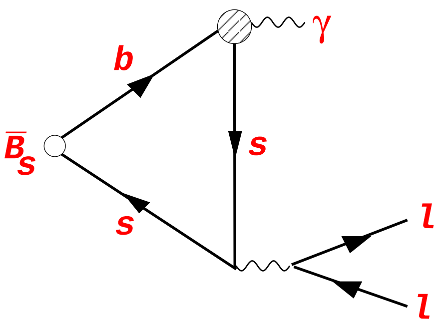


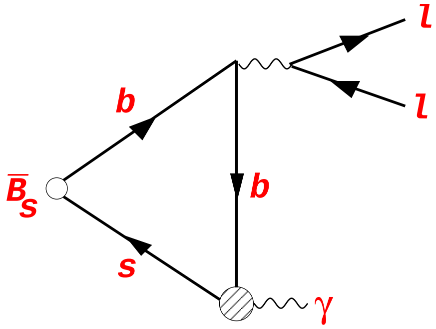




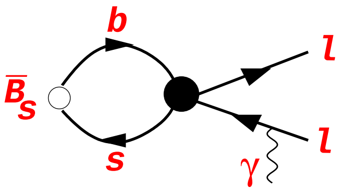


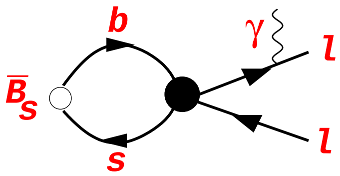
• **Diagrams with** *virtual* **photon emission from valence quarks are described by** $F(0, q^2)$ **(in the first diagram pole in the physical** q^2**-range):**





• **Bremsstrahlung (**$\sim C_{10A}$**)**





Dashed blob: penguin operator $O_{7\gamma}$**; full blob: four-fermion operators** O_{9V} **and** O_{10A}**.**

Form factor calculation

The methods are:

• Lattice QCD: for $B \to K, K^*$ form factors at large q^2; however in practice very difficult for $B \to \gamma$ form factors

• QCD sum rules (light-cone sum rules): at small q^2 (large recoil).

Also LEET (large energy effective theory) gives constraints on the behavior of the form factors in this region. E.g. for $B \to \gamma$ form factors $F(E) \sim 1/E$.

• Phenomenological relativistic quark models in the full range of q^2, however, difficult to gain control over systematic uncertainties.

A schematic calculation of some of the contributions in "QCD":

$$F(q^2, q'^2) \;=\; \int dx e^{iqx} \langle 0|T(\bar b(x)s(x), \bar s(0)s(0))|B_s(p)\rangle = \int dx e^{iqx} dk e^{-ikx} \frac{\langle 0|\bar b(x)s(0)|B_s(p)\rangle}{m_s^2 - k^2 - i0}.$$

$p = q + q'$ **and** $p^2 = M_B^2$**.** B**-meson 2DA depends on 2 variables** x^2 **and** xp

$$\langle 0|\bar b(x)s(0)|B_s(p)\rangle = \int_0^1 d\xi e^{-ipx\xi} \left\{\phi_0(\xi) + x^2\phi_1(\xi) + \dots\right\}$$

- **The LC contribution** $x^2 = 0$ **is easy**

$$F(q^2, q'^2) = \int \frac{dx e^{iqx}\phi_0(\xi) d\xi e^{-i\xi px} e^{-ikx} dk}{m_s^2 - k^2 - i0} = \int_0^1 \frac{d\xi\, \phi_0(\xi)}{m_s^2 - (q - \xi p)^2}$$

Taking into account that $(p - q)^2 = q'^2$**, and thus** $2qp = p^2 - q^2 - q'^2$**, we obtain** $k^2 = q^2(1-\xi) - \xi(1-\xi)M_B^2 + q'^2\xi$**. Important:** $\phi_0(\xi)$ **is peaked near** $\xi \sim \Lambda_{\rm QCD}/m_b$**, so** $k^2 \sim -\Lambda_{\rm QCD} m_b$**. The propagating quark is highly virtual, so perturbative expression for its propagator is ok.**

$$F(q^2, q'^2) = \int_0^1 \frac{\phi_0(\xi) d\xi}{m_s^2 - \left(q^2(1-\xi) - \xi(1-\xi)M_B^2 + q'^2\xi\right)}$$

- x^2 **terms in 2DA: write** $x_\alpha = -i\partial_\alpha e^{ikx}$**, parts integration.** $x^2 \to \Lambda_{\rm QCD}/m_b$ **compared to LC term.**

Contributions to $F(q^2, q'^2)$ from powers of x^2 in 2DA are suppressed.

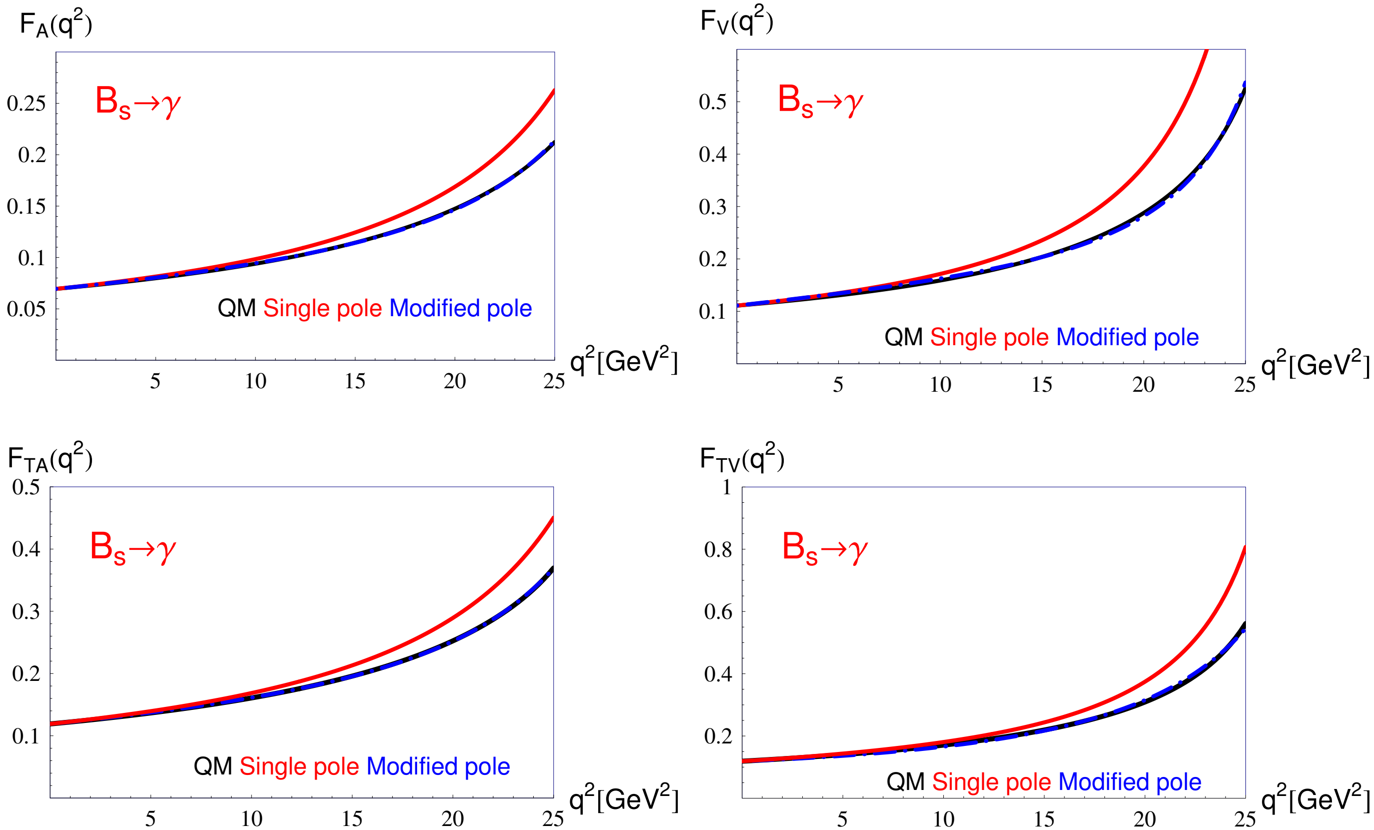
• Form factors $F(q^2, 0)$:

(i) Single-pole suggested by LEET:

$$F_i(q^2, 0) = \frac{F_i(0)}{1 - q^2/M_R^2}.$$

(ii) Modified pole; parametrizes our results in a broader range $0 < q^2 < 20$ GeV^2

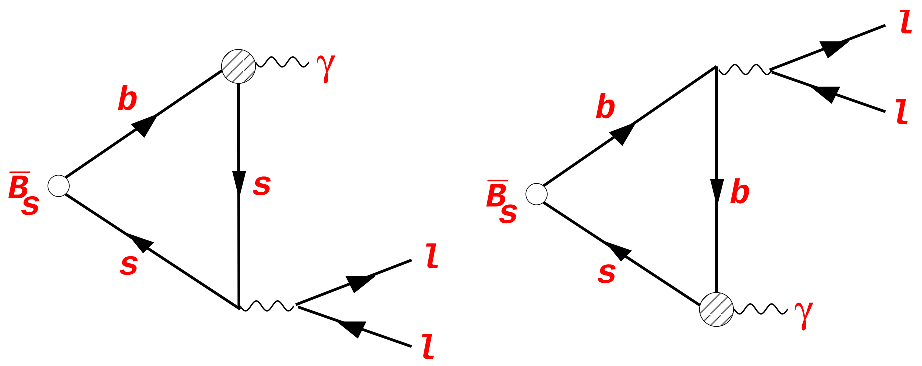
$$F_i(q^2, 0) = \frac{F_i(0)}{(1 - q^2/M_R^2)(1 - \sigma_1 q^2/M_R^2 - \sigma_2 (q^2/M_R^2)^2)}$$



• **Form factors $F(0, q^2)$:**

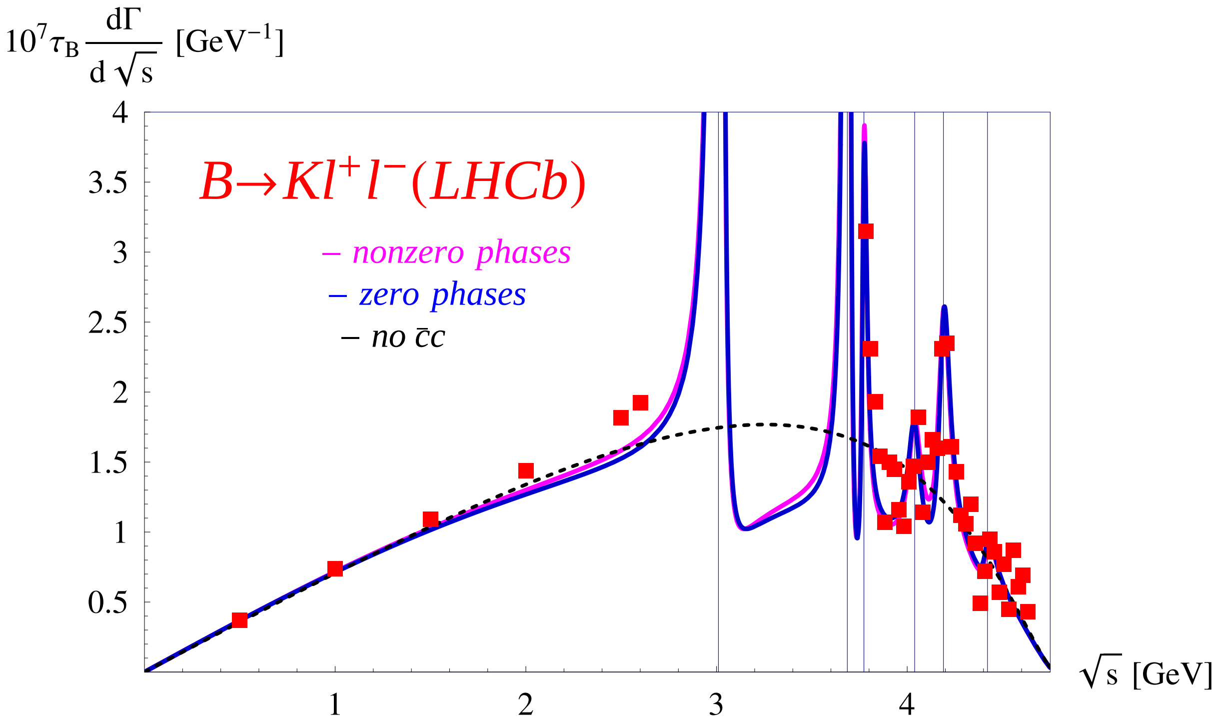
For subprocesses with resonances in the physical q^2-range, we have calculated form factors at q^2 below the resonances via dispersion approach. For larger values of q^2 we make use of the vector-meson dominance

$$F(0, q^2) = F(0, 0) + q^2 \frac{f_V / M_V}{M_V^2 - q^2 - i\Gamma_V M_V}.$$



Charm – loop contributions

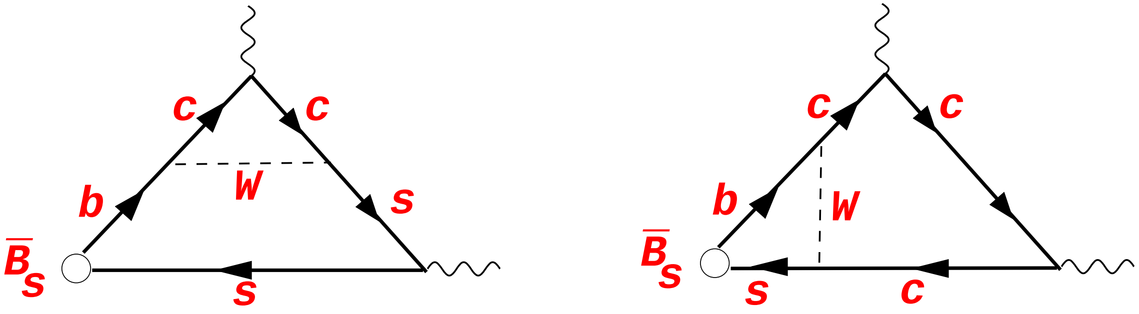
Illustration: $B \to K l^+ l^-$ **decay** $0 < \sqrt{s} < (M_B - M_K)$**,** s **- momentum squared of** $l^+ l^-$ **pair.**



• Charmonia appear in the kinematical decay region. In the charmonia region, charm contribution dynamically enhanced and dominates.

• Far from the charmonia region, top dominates (black dashed). Still, to study possible NP effects,
Need to gain theoretical control over charm contributions

Charm generates two different topologies: (a) penguin topology (b) weak annihilation topology



(a) (b)

• **Account of hard gluon exchanges lead to the four-quark operators**

$$H_{\rm eff}^{b\to s\bar{c}c} = -\frac{G_F}{\sqrt{2}} V_{cb}V_{cs}^* \left\{C_1(\mu)\mathcal{O}_1 + C_2(\mu)\mathcal{O}_2\right\}$$

with

$$\mathcal{O}_1 = \bar{s}^j\gamma_\mu(1-\gamma_5)c^i\,\bar{c}^i\gamma^\mu(1-\gamma_5)b^j, \qquad \mathcal{O}_2 = \bar{s}^i\gamma_\mu(1-\gamma_5)c^i\,\bar{c}^j\gamma^\mu(1-\gamma_5)b^j,$$

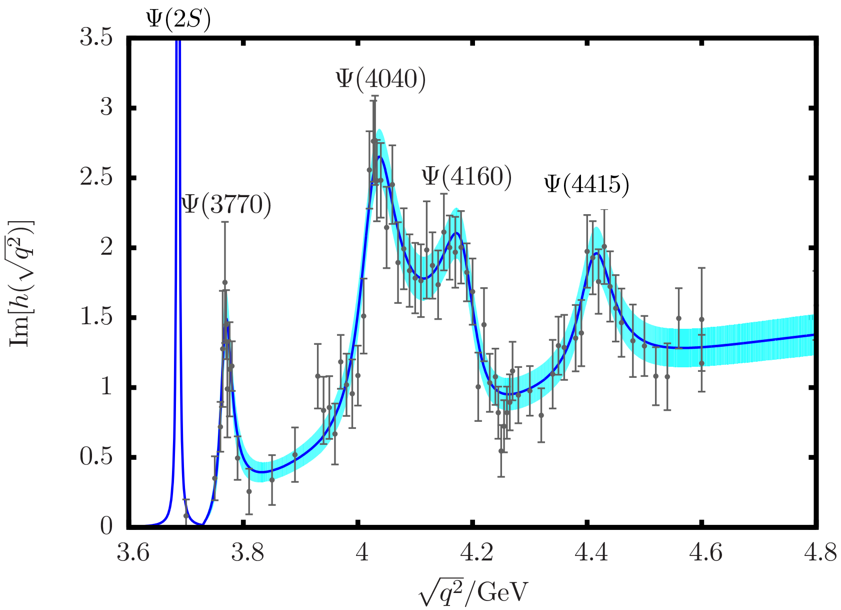
and the similar terms with $c \to u$ (i, j color indices). The SM Wilson coefficients at the scale $\mu_0 = 5$ GeV [corresponding to $C_2(M_W) = -1$]: $C_1(\mu_0) = 0.241$, $C_2(\mu_0) = -1.1$.

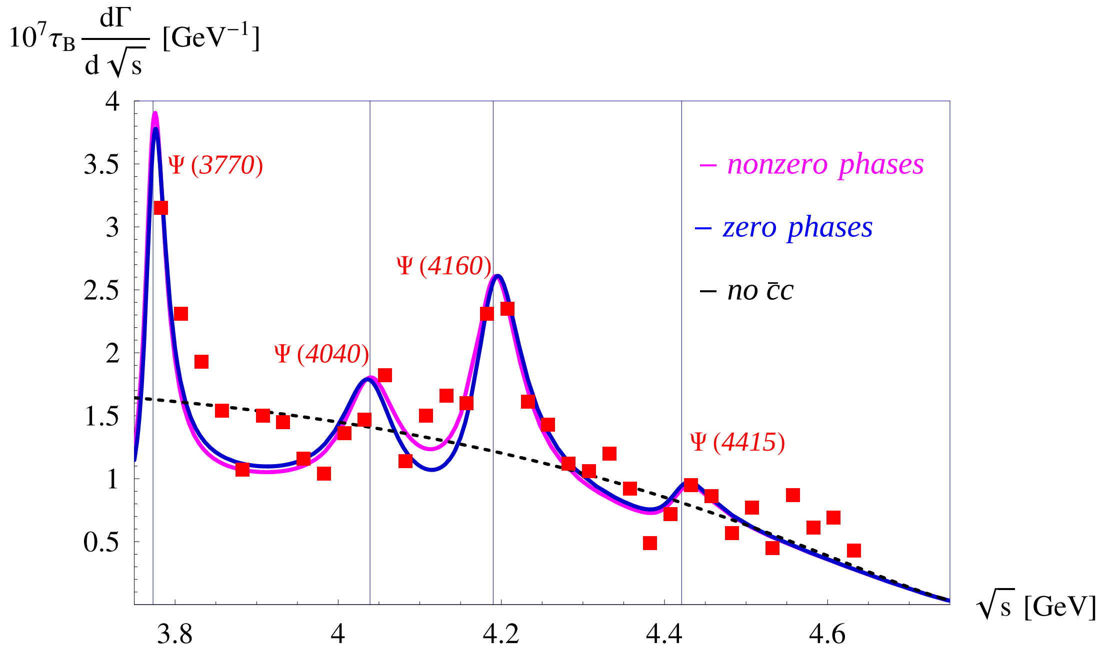
These operators lead to factorizable contributions to the amplitudes of exclusive FCNC B-decays.

• **Soft gluon exchanges between the charm-quark loop and the B-meson loop lead to nonfactorizable contributions to the amplitudes.**

• **Nonfactorizable charm contributions are comparable with factorizable contributions**

How do we know that? Compare charmonia in l^+l^--annihilation and in FCNC B-decays:



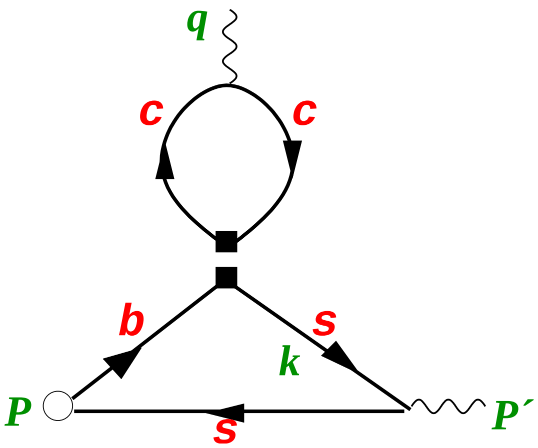


The patterns of charmonia in charm contribution to vacuum polarization (left) and in $B \to Kl^+l^-$ (right) are different. The difference is due to nonfactorizable contributions.

• **In some cases, factorizable charm contribution vanishes and thus only nonfactorizable charm contributes (e.g in $B \to K^*\gamma$)**

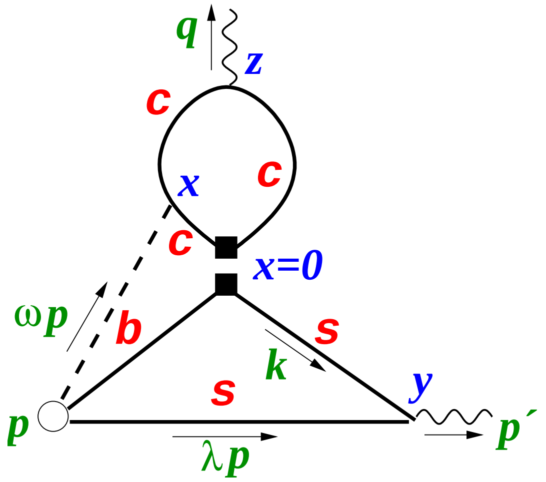
We need formalism to calculate nonfactorizable charm effects in QCD.

Factorizable part



Product of $B \to \gamma$ form factor and the charm polarization function.
At $q^2 \ll 4m_c^2$, the charm loop may be calculated in pQCD, and has been measured in a broad range of q^2.

Nonfactorizable part (illustration for scalar "quarks" and scalar "gluon")



$$A(q,p) = \frac{1}{(2\pi)^8}\int \frac{dk}{m_s^2-k^2}\int dy e^{-i(k-p')y}\int dx e^{-i\kappa x} d\kappa\, \Gamma_{cc}(\kappa,q)\,\langle 0|\bar{s}(y)G(x)b(0)|B_s(p)\rangle .$$

• **The 3DA depends on 5 variables xp, yp, x^2, y^2, xy ($p^2=M_B^2$) and may be parametrized as follows:**

$$\langle 0|s^\dagger(y)G(x)b(0)|B_s(p)\rangle = \int d\lambda e^{-i\lambda yp}\int d\omega e^{-i\omega xp}\Phi(\omega,\lambda)\left[1+O\left(\Lambda_{\rm QCD}^2 x^2, \Lambda_{\rm QCD}^2 y^2, \Lambda_{\rm QCD}^2 (x-y)^2\right)\right].$$

$\Phi(\omega,\lambda)$ **is peaked at** $\lambda,\omega\sim\Lambda_{\rm QCD}/m_b$ **[b-quark carries the major part of the B-meson momentum].**

• **Charm-quark loop:**

In the charm triangle diagram $\Gamma_{cc}(\kappa, q)$, for $q^2 \ll 4m_c^2$, all external virtualities $\kappa^2 = \omega^2 M_B^2 = O(\Lambda_{\rm QCD}^2)$ and $q'^2 = (\omega p - q)^2 \sim -\omega M_B^2$ are far below the $\bar{c}c$ thresholds,

Charm loop is perturbative as soon as $q^2 < 4m_c^2$.

• ***s*-quark propagator**

$$m_s^2 - (\lambda p - p')^2 = m_s^2 - \lambda q^2 + (1-\lambda)(\lambda M_B^2 - p'^2) \sim -\Lambda_{\rm QCD} m_b.$$

***s*-quark is highly virtual**

• **The contribution of the LC part of the B-meson 3DA $\Phi(\omega, \lambda)$ to $A(q, p)$ is easy to calculate:**

$$A(q,p) = \frac{1}{(2\pi)^8} \int \frac{dk}{m_s^2 - k^2} \int dy e^{-i(k-p')y} \int dx e^{-i\kappa x} d\kappa\, \Gamma_{cc}(\kappa, q) \int d\lambda e^{-i\lambda y p} \int d\omega e^{-i\omega x p} \Phi(\omega, \lambda)$$

$$\int dx \to \delta(\kappa + \omega p), \qquad \int dy \to \delta(k + \lambda p - p').$$

and integrate over κ and k we get

$$A(q,p) = \int_0^\infty d\lambda \int_0^\infty d\omega\, \Phi(\lambda, \omega) \Gamma_{cc}(-\omega p, q) \frac{1}{m_s^2 - (\lambda p - p')^2}.$$

• **Contributions of x^2, y^2, xy-terms in 3DA to the amplitude $A(q,p)$ relative to the $\Phi(\omega,\lambda)$ term:**

$$\Lambda_{\rm QCD}^2 y^2 \sim \frac{\Lambda_{\rm QCD}}{m_b}, \qquad \Lambda_{\rm QCD}^2 x^2 \sim \frac{\Lambda_{\rm QCD}^3 m_b}{m_c^4}, \qquad \Lambda_{\rm QCD}^2 xy \sim \frac{m_b \Lambda_{\rm QCD}}{m_c^2}.$$

Summary for nonfactorizable corrections:

Nonfactorizable corrections are expressed via

$$\langle 0|s^\dagger(y)G(x)b(0)|B_s(p)\rangle = \int d\lambda e^{-i\lambda yp} \int d\omega e^{-i\omega xp} \Phi(\omega,\lambda) \left[1 + O\left(\Lambda_{\rm QCD}^2 x^2, \Lambda_{\rm QCD}^2 y^2, \Lambda_{\rm QCD}^2 (x-y)^2\right)\right],$$

The new recent result is that the knowledge of its functional dependence on $(x-y)^2$ is essential for a proper resummation of large $\Lambda_{\rm QCD} m_b/m_c^2$ corrections.

Previosuly, it way believed that the 3DA with aligned arguments
$x_\mu = u y_\mu$**, on the LC** $x^2 = 0$**,** $y^2 = 0$ **and** $(x-y)^2 = 0$
is sufficient to calculate nonfactorizabe contributions.

One needs the off-LC contributions. A challenge for future calculations

Numerical results for $q^2 < 4m_c^2$:

- $B \to K^*\gamma$

Is described via single number $T_1(q^2 = 0)$.

Charm-loop correction is purely nonfactorizable (factorizable part vanished for $q^2 = 0$).

$C_{7\gamma}(5\,GeV) = 0.312$, $\quad \delta C_{7\gamma}(5\,GeV) = 0.012^{+0.016}_{-0.009}$

- $B \to K l^+ l^-$ **at the level of 5% at $q^2 = 3 - 4$ GeV^2**

- $B \to K^* l^+ l^-$ **at the level of 10-15% at $q^2 = 3 - 4$ GeV^2**

We need a wide range $0 < q^2 < M_B^2$, including the region of charmonium resonances. QCD-based calculation cannot be applied at q^2 in the resonance region, where nonperturbative approaches based on hadron degrees of freedom are necessary,

Charm-loop contributions to the $B_s \to \gamma l^+ l^-$ amplitude may be parametrized as follows:

$$H_{\mu\alpha}(k',k) = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* e \left[\epsilon_{\mu\alpha k' k} H_V - i \left(g_{\alpha\mu}\, kk' - k'_\alpha k_\mu \right) H_A - i \left(k'_\alpha - \frac{kk'}{k^2} k_\alpha \right) \left(k_\mu - \frac{kk'}{k'^2} k'_\mu \right) H_3 \right],$$

with the invariant form factors H_i depending on two variables, $H_i(k'^2, k^2)$. At $q^2 < 4m_c^2$ one calculates nonfactrizable contributions to these functions via B-meson 3DA. To go to larger q^2 it was proposed to use dispersion representation in q^2 [Khodjamirian et al, 2010]

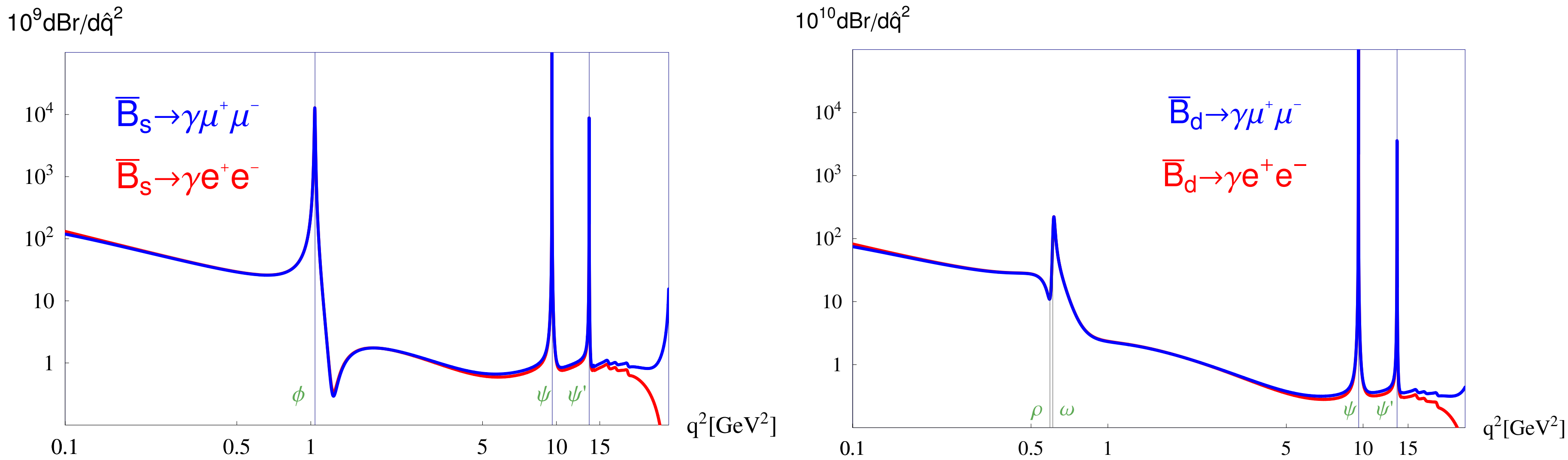
$$H_i(q^2) = a_i + b_i q^2 + (q^2)^2 \left\{ \sum_{\psi = J/\psi, \psi'} \frac{f_\psi \mathcal{A}^i_{B\psi\gamma}}{m_\psi^3 (m_\psi^2 - q^2 - i m_\psi \Gamma_\psi)} + h_i(q^2) \right\}, \qquad i = A, V, 3$$

where a_i and b_i are the (unknown) subtraction constants and the functions $h_i(q^2)$ describe the hadron continuum including the broad charmonium states lying above the DD threshold.

The amplitudes $|\mathcal{A}^i_{B\psi\gamma}|$ may be taken from the data, but different resonances may have nonzero relative phases generated by nonfactorizable corrections.

Numerical results

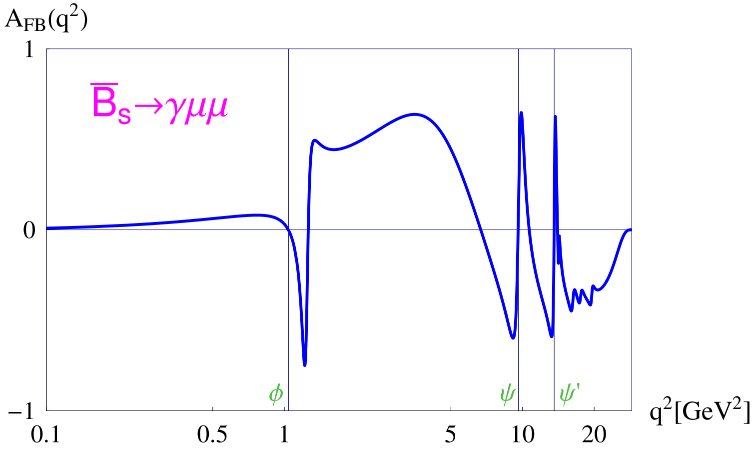
Differential distributions in $B \to \gamma l^+ l^-$ vs $q^2[GeV^2]$.

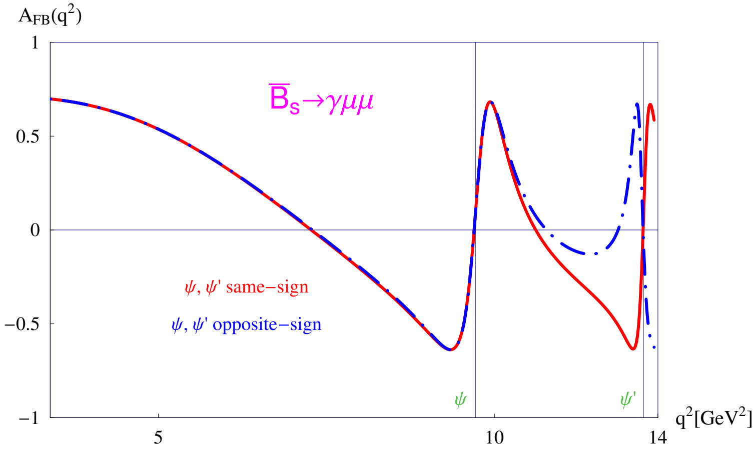


Light vector resonances and charmonia have completely different origins: Light vector resonances appear as resonance contribution in the same quark loop as *B*-meson (i.e. they contain one valence quark). Charmonia emerge in a different quark loop (after going to effective EW theory).

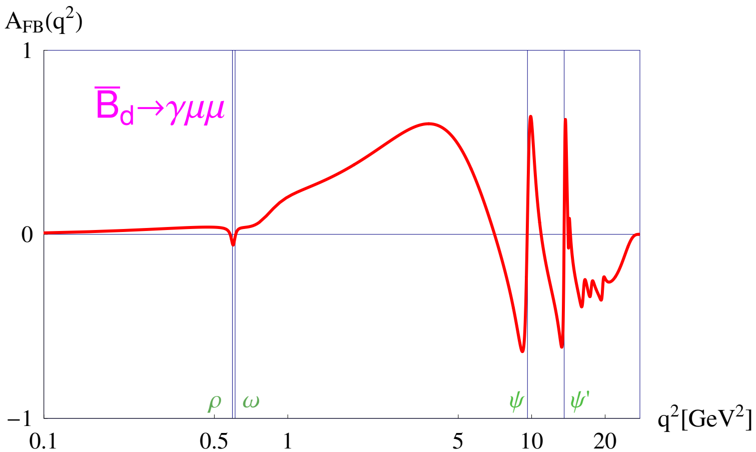
Forward-backward asymmetry

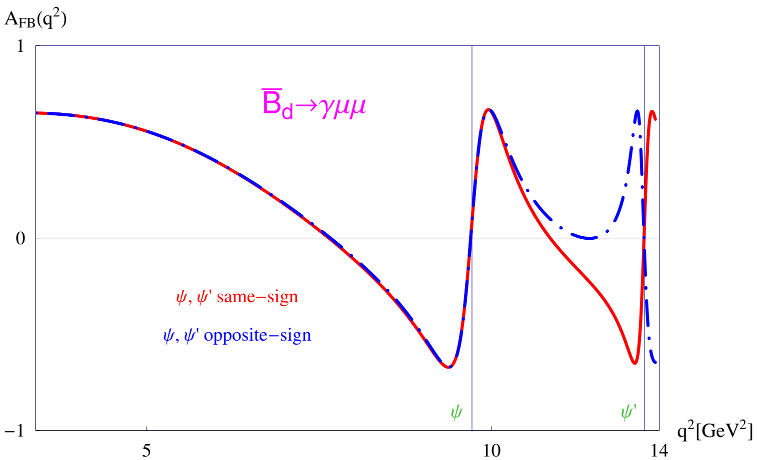
$B_s \to \mu^+\mu^-\gamma$:





$B_d \to \mu^+\mu^-\gamma$:





Conclusions and outlook

• FCNC B-decays remain a promising candidate for indirect NP searches. A few tensions with the SM predictions (B-physics anomalies) have been observed experimentally.

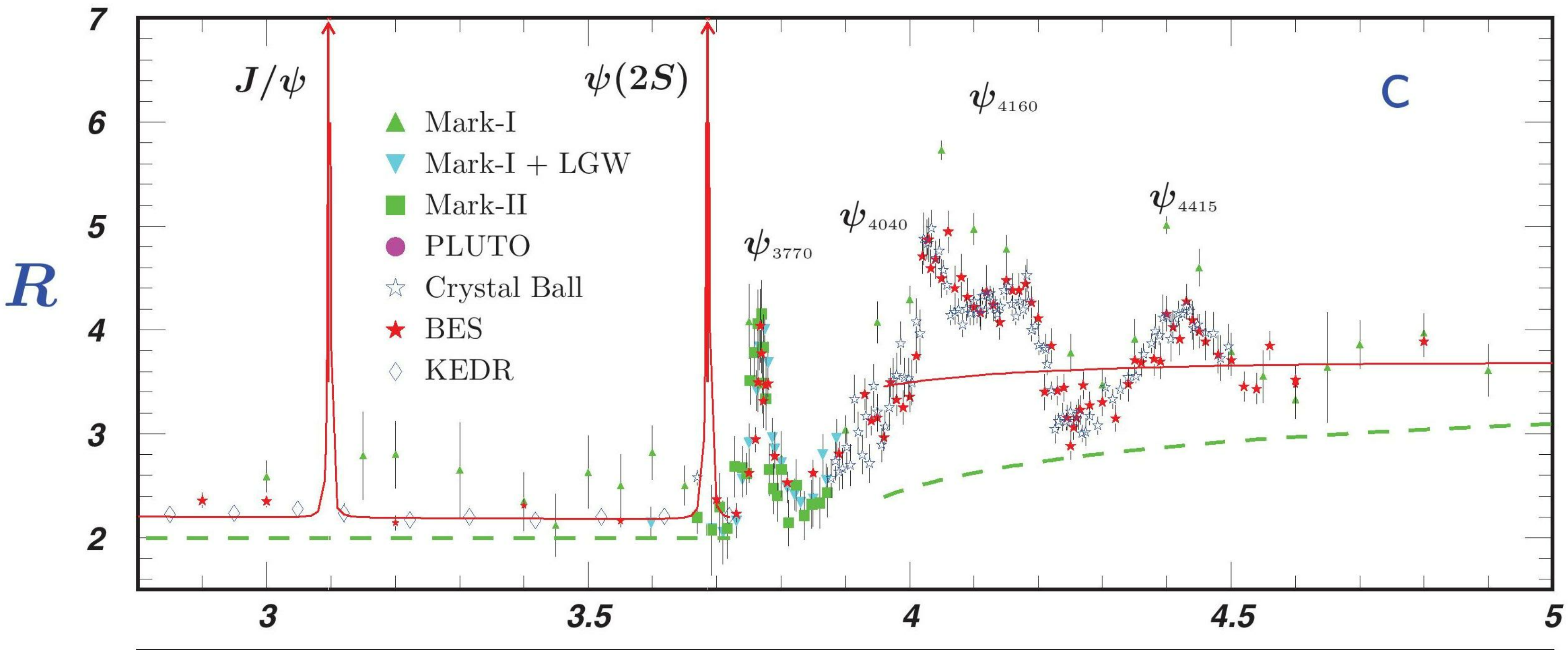
• Charm provides sizeable contributions to the amplitudes of FCNC decays. In the charmonia regions charm contributions dominate the amplitudes of FCNC B-decays. Moreover, nonfactorizable charm effects are comparable in size with factorizable charm effects.

• What happens with the charm contributions beyond the resonance regions, is a serious open theoretical problem in FCNC B-decays. According to some estimates, charm contributes to the differential distributions, including the asymmetries, at the level of 5-10% at q^2 also far beyond J/ψ, ψ'.

• At $q^2 \ll 4m_c^2$, a consistent description of nonfactorizable charming loops requires the knowledge of off-LC 3DAa.

• If charm effects are controlled well, the distributions (in particular, A_{FB}) potentially have the sensitivity to the precise values of the Wilson coefficients, i.e. have sensitivity to new physics.

• Can we use "duality" to predict charm contribution in exclusive FCNC *B*-decays?



• Including Lorentz structures in the B-meson three-particle DA:

In QCD, new functions emerge when one wants to generalize terms of the following type:

$$\langle 0|\bar{s}(x)G_{\alpha\beta}(ux)b(0)|B(v)\rangle = \int d\lambda e^{-i\lambda xp}\int d\omega e^{-i\omega uxp}\left[\frac{x_\alpha v_\beta}{xv} - \frac{x_\beta v_\alpha}{xv}\right]\Phi(\lambda,\omega). \quad (1)$$

How to generalize them for a non-aligned case? Obviously, new structures and new amplitudes arise:

$$\langle 0|\bar{s}(y)G_{\alpha\beta}(x)b(0)|B(v)\rangle = \int d\lambda e^{-i\lambda xv}\int d\omega e^{-i\omega yv}$$
$$\times\frac{1}{2}\left[\left(\frac{x_\alpha v_\beta}{xv} - \frac{x_\beta v_\alpha}{xv} + \frac{y_\alpha v_\beta}{yv} - \frac{y_\beta v_\alpha}{yv}\right)\Phi_S(\lambda,\omega) + \left(\frac{x_\alpha v_\beta}{xv} - \frac{x_\beta v_\alpha}{xv} - \frac{y_\alpha v_\beta}{yv} + \frac{y_\beta v_\alpha}{yv}\right)\Phi_A(\lambda,\omega)\right]. \quad (2)$$

$\Phi_S = \Phi$ from (1), whereas Φ_A is new. If the contributions induced by Φ_A are not suppressed, a consistent calculation of the decay amplitude A needs further inputs.

• Local vs light-cone OPE

Local OPE corresponds to power expansion of $G(ux)$: $G(ux) = G(0) + ux_\alpha \partial^\alpha G + \ldots$.

1. The leading contribution comes from $G(0)$ term:

$$\langle 0|\bar{s}(y)G(0)b(0)|B(p)\rangle|_{x=0} = \int d\lambda e^{-i\lambda yp} \int d\omega \left[\Phi(\lambda,\omega) + O(y^2)\right]. \quad (3)$$

2. Let us consider the contribution of $x_\alpha \partial^\alpha G(0)$.

• The x_α term can be generated by $\partial_\alpha e^{iqx}$ and after taking the integrals leads to $\frac{q^\alpha}{m_c^2}$.

• The $\partial^\alpha G(0)$ term reads

$$\begin{aligned}\frac{\partial}{\partial x_\alpha}\langle 0|\bar{s}(y)G(ux)b(0)|B(p)\rangle|_{x=0} &= -iup_\alpha \int d\lambda e^{-i\lambda yp} \int d\omega\, \omega\Phi(\lambda,\omega) + C_2\Lambda_{\rm QCD} x_\alpha \\ &= C_1 p_\alpha \frac{\Lambda_{\rm QCD}}{m_b} + C_2 \Lambda_{\rm QCD} x_\alpha \end{aligned} \quad (4)$$

The term C_2 arises when the derivative acts on x^2 and xy terms in the full off-LC 3DA.

3. The leading part in the ratio of the $\partial_\alpha G(0)$ over the $G(0)$ contributions to the amplitude arises when q_α contracts with the term $\sim p_\alpha$ and reads

$$\frac{qp\Lambda_{\rm QCD}}{m_b m_c^2} \sim \frac{M_B \Lambda_{\rm QCD}}{m_c^2} \sim 1. \quad (5)$$

For the realistic case of $c-$ and b-quarks, the "suppression" factor is around 1. So no real hierarchy of the contributions within the local OPE. Summation is necessary, this is done by LC OPE.