Investigation of the properties of the Higgs fields basis

Mikhail Dolgopolov, V. Makeeva (Zagorovskaya),
E.N. Rykova, A. Gurskaya
Introduction
Outline

Introduction

Minimal supersymmetric standard model
  Effective potential of MSSM
  N-point loop integrals at vanishing external momenta
Outline

Introduction

Minimal supersymmetric standard model
  Effective potential of MSSM
  N-point loop integrals at vanishing external momenta

Effective Potential parameters. One-loop temperature corrections
In the simple isoscalar model the standard-like Higgs potential is

\[ U(\varphi) = -\frac{1}{2} \mu^2 \varphi^2 + \frac{1}{4} \lambda \varphi^4. \]

The thermal Higgs boson mass:

\[ m_h^2 = -\mu^2 + \lambda \frac{T^2}{4}. \]

Two solutions

\[ v(0) = 0 \quad \text{and} \quad v^2(T) = \frac{\mu^2}{\lambda} - \frac{T^2}{4}, \]

demonstrate the second order phase transition at the critical temperature

\[ T_c = \frac{2\mu}{\sqrt{\lambda}} = 2v(0), \]
Temperature loop corrections from $\tilde{t}$ and other scalar states could be large and lead to the first order phase transition. The intensity depends on

\[ \xi = \frac{v(T_c)}{T_c}, \]

where $v(T_c) = \sqrt{v_1^2(T_c) + v_2^2(T_c)}$ is the vacuum expectation value at the critical temperature $T_c$. The electroweak baryogenesis could be explained if

\[ \frac{v(T_c)}{T_c} > 1 \]

- the case of strong first order phase transition.
Introduction

In a number of analyzes the MSSM finite-temperature effective potential is taken in the representation

\[ V_{eff}(v, T) = V_0(v_1, v_2, 0) + V_1(m(v), 0) + V_1(T) + V_{ring}(T), \]  

where

- \( V_0 \) is the tree-level MSSM two-doublet potential at the SUSY scale
- \( V_1 \) is the (non-temperature) one-loop resumed Coleman-Weinberg term, dominated by \( \tilde{t} \) and \( \tilde{b} \) contributions
- \( V_1(T) \) is the one-loop temperature term
- \( V_{ring} \) is the correction of re-summed leading infrared contribution from multi-loop ring (or daisy) diagrams
Effective potential of MSSM

In two-doublet model there are two identical $SU(2)$ doublets of complex scalar fields $\Phi_1$ and $\Phi_2$

$$\Phi_1 = \begin{pmatrix} \phi_1^+(x) \\ \phi_1^0(x) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+(x) \\ \phi_2^0(x) \end{pmatrix}$$

with nonzero vacuum expectation values

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}.$$ 

Neutral components of doublets

$$\phi_1^0(x) = \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1), \quad \phi_2^0(x) = \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2).$$
Effective potential of MSSM

The most general renormalizable hermitian $SU(2) \times U(1)$ invariant potential: [Akhmetzyanova E.N., M.V.D, Dubinin M.N. Soft SUSY Breaking and Explicit CP Violation in the THDM // CALC 2003 & SQSÓ3 Proc.]

\[
U(\Phi_1, \Phi_2) = -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - \mu_{12}^2(\Phi_1^\dagger \Phi_2) - \mu_{12}^* (\Phi_2^\dagger \Phi_1) + \\
+ \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \\
+ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \frac{\lambda_5^*}{2} (\Phi_2^\dagger \Phi_1)(\Phi_2^\dagger \Phi_1) + \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \\
+ \lambda_6^* (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \lambda_7^* (\Phi_2^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)
\]

with effective real parameters $\mu_1^2, \mu_2^2, \lambda_1, \ldots, \lambda_4$ and complex parameters $\mu_{12}^2, \lambda_5, \lambda_6, \lambda_7$. 

Extensions of Higgs sector
Effective potential of MSSM

**Supersymmetry** boundary conditions
In the tree approximation on the energy scale $M_{SUSY}$, the parameters $\lambda_{1-7}$ are real and are expressed using the coupling constants $g_1$ and $g_2$ of electroweak group of the gauge symmetry $SU(2) \times U(1)$ as follows:

$$\lambda_1(M_{SUSY}) = \lambda_2(M_{SUSY}) = \frac{1}{4} \left( g_2^2(M_{SUSY}) + g_1^2(M_{SUSY}) \right),$$

$$\lambda_3(M_{SUSY}) = \frac{1}{4} \left( g_2^2(M_{SUSY}) - g_1^2(M_{SUSY}) \right),$$

$$\lambda_4(M_{SUSY}) = -\frac{1}{2} g_2^2(M_{SUSY}),$$

$$\lambda_5(M_{SUSY}) = \lambda_6(M_{SUSY}) = \lambda_7(M_{SUSY}) = 0.$$
Boundary conditions
On the scale of the superpartners $M_{SUSY}$

\[ m_t \quad \leftrightarrow \quad M_{SUSY} \]

The effective potential method,
the method of Feynman diagrams
& finite-temperature corrections

\[
\begin{align*}
\lambda_{SUSY}^1 &= \lambda_{SUSY}^2 = \frac{g_1^2 + g_2^2}{8}, \\
\lambda_{SUSY}^3 &= \frac{g_2^2 - g_1^2}{4}, \\
\lambda_{SUSY}^4 &= -\frac{g_2^2}{2}, \\
\lambda_{SUSY}^5 &= \lambda_{SUSY}^6 = \lambda_{SUSY}^7 = 0.
\end{align*}
\]

The deviation from the parameters

\[
\lambda_i = \lambda_{i}^{SUSY} - \Delta \lambda_i
\]


Extensions of Higgs sector
Threshold corrections (example for $\lambda_1$)

$$\lambda_i = \lambda_i^{SUSY} - \Delta \lambda_i^{th}$$

$$\Delta \lambda^{thr}_1 = 3h_t^4 |\mu|^4 I_2[m_Q, m_U] + 3h_b^4 |A_b|^4 I_2[m_Q, m_D] + h_t^2 |\mu|^2 (-\frac{g_1^2 - 3g_2^2}{2} I_1[m_Q, m_U]$$

$$+ 2g_1^2 I_1[m_U, m_Q]) + h_b^2 |A_b|^2 (\frac{12h_b^2 - g_1^2 - 3g_2^2}{2} I_1[m_Q, m_D] + (6h_b^2 - g_1) I_1[m_d, m_Q])$$

$$- \Delta \lambda_f^1 = (h_b^2 - \frac{g_1^2}{6})^2 (I(m_Q) + I(m_D)) + \frac{g_1^4}{9} I(m_U)$$

$$\Delta \lambda^{log}_1 = -\frac{1}{384\pi^2} (11g_1^4 - 36h_b^2 g_1^2 + 9(g_2^4 - 4h_b^2 g_2^2 + 16h_b^4)) ln \left( \frac{m_Q m_U}{m_t^2} \right)$$

where

$$I_0[m_1, m_2] = \sum_{n=-\infty}^{\infty} \sum_{n \neq 0} \frac{1}{4\pi (\sqrt{4\pi^2 n^2 T^2 + m_1^2} + \sqrt{4\pi^2 n^2 T^2 + m_2^2})}$$

$$I_1[M_1, M_2] = -\frac{1}{64\pi^4 T^2} \sum_{n=-\infty, n \neq 0} \sqrt{M_1^2 + n^2} \left( \frac{1}{\sqrt{M_1^2 + n^2 + \sqrt{M_2^2 + n^2}}} \right)$$

$$I_2[M_1, M_2] = \frac{1}{256\pi^5 T^4} \sum_{n=-\infty, n \neq 0} \frac{1}{\sqrt{(M_1^2 + n^2)(M_2^2 + n^2)(\sqrt{M_1^2 + n^2 + \sqrt{M_2^2 + n^2})^3}}}$$

Extensions of Higgs sector
The supersymmetric scalar potential of interaction of Higgs bosons with the third generation quark superpartners on the tree level has the form

$$V^0 = V_M + V_{\Gamma} + V_{\Lambda} + V_{\widetilde{Q}},$$

$$V_M = (-1)^{i+j} m_{ij}^2 \Phi_i^\dagger \Phi_j + M_\widetilde{Q}^2 (\widetilde{Q}^\dagger \widetilde{Q}) + M_D^2 \widetilde{U}^* \widetilde{U} + M_D^2 \widetilde{D}^* \widetilde{D},$$

$$V_{\Gamma} = \Gamma^D_i (\Phi_i^\dagger \widetilde{Q}) \widetilde{D} + \Gamma^U_i (i \Phi_i^T \sigma_2 \widetilde{Q}) \widetilde{U} + \Gamma^D_i (\widetilde{Q}^\dagger \Phi_i) \widetilde{D}^* - \Gamma^U_i (i \widetilde{Q}^\dagger \sigma_2 \Phi_i^*) \widetilde{U}^*,$$

$$V_{\Lambda} = \Lambda^{jl}_{ik} (\Phi_i^\dagger \Phi_j) (\Phi_k^\dagger \Phi_l) + (\Phi_i^\dagger \Phi_j) \left[ \Lambda^Q_{ij} (\widetilde{Q}^\dagger \widetilde{Q}) + \Lambda^U_{ij} \widetilde{U}^* \widetilde{U} + \Lambda^D_{ij} \widetilde{D}^* \widetilde{D} \right] +$$

$$+ \Lambda^Q_{ij} (\Phi_i^\dagger \widetilde{Q}) (\widetilde{Q}^\dagger \Phi_j) + \frac{1}{2} \left[ \Lambda_{ij} \left( i \Phi_i^T \sigma_2 \Phi_j \right) \widetilde{D}^* \widetilde{U} + \text{h.c.} \right], i, j, k, l = 1, 2,$$

$$V_{\widetilde{Q}}$$ denotes the terms of interaction of four scalar quarks.
Effective potential of NMSSM

The most general Hermitian form of the renormalized $SU(2) \times U(1)$ invariant potential for system of fields has the form:

$$U(\Phi_1, \Phi_2, S) = -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - \mu_3^2 S^* S - (\mu_{12}^2(\Phi_1^\dagger \Phi_2) + h.c.)$$

$$+ \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) +$$

$$+ \frac{\lambda_5}{2}(\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \frac{\lambda_5^*}{2}(\Phi_2^\dagger \Phi_1)(\Phi_2^\dagger \Phi_1) +$$

$$+ \lambda_6(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_7(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + h.c.$$}

$$+ k_1(\Phi_1^\dagger \Phi_1)S^* S + k_2(\Phi_2^\dagger \Phi_2)S^* S + (k_3(\Phi_1^\dagger \Phi_2)S^* S + h.c.)k_4(S^* S)^2 +$$

$$+ k_5(\Phi_1^\dagger \Phi_1)S + k_6(\Phi_2^\dagger \Phi_2)S + k_7(\Phi_1^\dagger \Phi_2)S^* + k_7^*(\Phi_2^\dagger \Phi_1)S^* + k_8 S^3.$$
Parameters of Effective Potential of NMSSM

The supersymmetric scalar potential of interaction of Higgs bosons with the third generation quark superpartners on the tree level has the form

\[ V = |y_u (\tilde{Q} e H_u)|^2 + |y_d (\tilde{Q} e H_d)|^2 + |y_u \tilde{u}_R^* H^0_u - y_d \tilde{d}_R^* H^-_d|^2 + |y_d \tilde{d}_R^* H^0_d - y_d \tilde{u}_R^* H^+_u|^2 - \\ - y_u (\tilde{u}_R \tilde{u}_L^* \lambda S H^0_d + \tilde{u}_R \tilde{d}_L^* \lambda S H^-_d + c.c.) - y_d (\tilde{d}_R \tilde{d}_L^* \lambda S H^0_u + \tilde{d}_R \tilde{d}_L^* \lambda S H^+_u + c.c.) + \\ + \frac{g_2^2}{8} (4|H_d^\dagger \tilde{Q}|^2 - 2(H_d^\dagger H_d)(\tilde{Q}^\dagger \tilde{Q}) + 4|H_u^\dagger \tilde{Q}|^2 - 2(H_u^\dagger H_u)(\tilde{Q}^\dagger \tilde{Q})) + \\ + \frac{g_1^2}{2} \left( \frac{1}{6} (\tilde{Q}^\dagger \tilde{Q}) - \frac{2}{3} \tilde{u}_R^* \tilde{u}_R + \frac{1}{3} \tilde{d}_R^* \tilde{d}_R + \frac{1}{2} (H_u^\dagger H_u) - \frac{1}{2} (H_d^\dagger H_d) \right)^2 + \\ + (\tilde{u}_R^* y_u A_u (\tilde{Q}^T e H_u) - \tilde{d}_R y_d A_d (\tilde{Q}^T e H_d) + c.c.) \]
We study the evolution of Higgs potential shape in the framework of catastrophe theory for predicting conditions for the stable minimum existence, i.e. the true minimum, in which our Universe is expected now.

We take the effective 2HDM potential for MSSM and NMSSM with additional Higgs singlet, where the control parameters of Higgs potentials depend on the temperature.
In the early, high temperature stages of the Universe, the environment had a non-negligible matter and radiation density, making the hypotheses of conventional field theories impracticable. For that reason the methods of conventional field theories are no longer in use, and should be replaced by others, closer to thermodynamics, where the background state is a thermal bath. This field has been called field theory at finite temperature and it is extremely useful to study all phenomena which happened in the early Universe: phase transitions, inflationary cosmology, ... In the finite temperature field theory Feynman diagrams with boson propagators, containing Matsubara frequencies $\omega_n = 2\pi n T$ ($n = 0, \pm1, \pm2, \ldots$), lead to structures of the form

$$I[m_1, m_2, \ldots, m_b] = T \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^3} \prod_{i=1}^{b} \frac{(-1)^b}{(k^2 + \omega_n^2 + m_j^2)}; \quad (2)$$

$k$ is the three-dimensional momentum in a system with the temperature $T$. 
At \( n \neq 0 \) the result is

\[
I[m_1, m_2, \ldots, m_b] = 2T (2\pi T)^{3-2b} \frac{(-1)^b \pi^{3/2}}{(2\pi)^3} \frac{\Gamma(b - 3/2)}{\Gamma(b)} S(M, b - 3/2),
\]

where

\[
S(M, b - 3/2) = \int \{dx\} \sum_{n=1}^{\infty} \frac{1}{(n^2 + M^2)^{b-3/2}} , \quad M^2 \equiv \left( \frac{m}{2\pi T} \right)^2.
\]
N-point loop integrals at vanishing external momenta

We derive and collect the analytic forms of the relevant loop integrals for reconstruction of the effective Higgs potential parameters in extended models (MSSM, NMSSM etc.).

The smallness of external state masses in relation to the masses of loop particles allows us to take the limit where external particles are massless. These integrals are particularly easy to take using algebraic recursion relations or residue method. But it is necessary to keep the external momentum $p_\mu$ general (i.e. nonzero) until it can be converted into an external mass in the amplitude, after which point one may take accurate limit $p \to 0$, especially in case of equivalent internal masses in the loop.

The N-point loop integrals at vanishing external momenta are defined as:

$$\frac{1}{(4\pi)^2}N_{2z}(m_1^2, m_2^2, ..., m_N^2) = \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{i k^{2z}}{\prod_i^N (k^2 - m_i^2)},$$

Numerical check of the zero temperature limiting case $T \to 0$ in (2) demonstrates that the results obtained using non-temperature field theory are successfully reproduced. In the high-temperature limit zero mode gives dominant contribution in agreement with known suppression of quantum effects at increasing temperatures.
Finite temperature corrections of squarks

We calculate the integral

\[ J_0[a_1, a_2] = \int \frac{dk}{(2\pi)^3} \frac{1}{(k^2 + a_1^2)(k^2 + a_2^2)} = \frac{1}{4\pi(a_1 + a_2)}, \]

taking a residue in the spherical coordinate system. 
\( a_{1,2}^2 \) are the sums of squared frequency and squared mass. Derivatives of \( J_0 \) with respect to \( a_1 \) and \( a_2 \) can be used for calculation of integrals

\[ J_1[a_1, a_2] = \int \frac{dk}{(2\pi)^3} \frac{1}{(k^2 + a_1^2)^2(k^2 + a_2^2)} = \frac{1}{8\pi a_1(a_1 + a_2)^2}, \]

\[ J_2[a_1, a_2] = \int \frac{dk}{(2\pi)^3} \frac{1}{(k^2 + a_1^2)^2(k^2 + a_2^2)^2} = \frac{1}{8\pi a_1 a_2(a_1 + a_2)^3}. \]
Parameters of Effective Potential of MSSM

\[ \Delta \lambda_1 = C_{31}^4 I_2[m_Q, m_U] + C_{32}^4 I_2[m_Q, m_D] + \\
+ C_{31}^2 (C_{41} I_1[m_Q, m_U] + C_{43} I_1[m_U, m_Q]) + \\
+ C_{32}^2 (C_{42} I_1[m_Q, m_D] + C_{44} I_1[m_D, m_Q]). \]

\[ \Delta \lambda_2 = C_{33}^4 I_2[m_Q, m_U] + C_{34}^4 I_2[m_Q, m_D] + \\
+ C_{33}^2 (C_{45} I_1[m_Q, m_U] + C_{47} I_1[m_U, m_Q]) + \\
+ C_{34}^2 (C_{46} I_1[m_Q, m_D] + C_{48} I_1[m_D, m_Q]). \]

\[ \Delta(\lambda_3 + \lambda_4) = C_{31}^2 C_{33}^2 I_2[m_Q, m_U] + C_{32}^2 C_{34}^2 I_2[m_Q, m_D] + \\
+ (C_{31}^2 C_{45} + C_{33}^2 C_{41}) I_1[m_Q, m_U] + (C_{31}^2 C_{47} + C_{33}^2 C_{43}) I_1[m_U, m_Q] + \\
+ (C_{32}^2 C_{46} + C_{34}^2 C_{42}) I_1[m_Q, m_D] + (C_{32}^2 C_{48} + C_{34}^2 C_{44}) I_1[m_D, m_Q]. \]

\[ \Delta \lambda_5 = C_{31}^2 C_{33}^2 I_2[m_Q, m_U] + C_{32}^2 C_{34}^2 I_2[m_Q, m_D]. \]
Parameters of Effective Potential of MSSM

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi_1^0 \tilde{u}_L \tilde{u}_R$</td>
<td>$-h_u v_3 \lambda$</td>
<td>$C_{31}$</td>
</tr>
<tr>
<td>$\varphi_1^{0*} \tilde{d}_L \tilde{d}_R$</td>
<td>$h_d A_d$</td>
<td>$C_{32}$</td>
</tr>
<tr>
<td>$\varphi_2^0 \tilde{u}_L \tilde{u}_R$</td>
<td>$h_u A_u$</td>
<td>$C_{33}$</td>
</tr>
<tr>
<td>$\varphi_2^{0*} \tilde{d}_L \tilde{d}_R$</td>
<td>$-h_d v_3 \lambda$</td>
<td>$C_{34}$</td>
</tr>
<tr>
<td>$\varphi_1^{0*} \varphi_1^0 \tilde{u}_L \tilde{u}_L$</td>
<td>$g_2^2/4 - g_1^2/12$</td>
<td>$C_{41}$</td>
</tr>
<tr>
<td>$\varphi_1^{0*} \varphi_1^0 \tilde{d}_L \tilde{d}_L$</td>
<td>$-g_1^2/12 - g_2^2/4 + h_d^2$</td>
<td>$C_{42}$</td>
</tr>
<tr>
<td>$\varphi_1^{0*} \varphi_1^0 \tilde{u}_R \tilde{u}_R$</td>
<td>$g_1^2/3$</td>
<td>$C_{43}$</td>
</tr>
<tr>
<td>$\varphi_1^{0*} \varphi_1^0 \tilde{d}_R \tilde{d}_R$</td>
<td>$h_d^2 - g_1^2/6$</td>
<td>$C_{44}$</td>
</tr>
<tr>
<td>$\varphi_2^{0*} \varphi_2^0 \tilde{u}_L \tilde{u}_L$</td>
<td>$g_1^2/12 - g_2^2/4 + h_u^2$</td>
<td>$C_{45}$</td>
</tr>
<tr>
<td>$\varphi_2^{0*} \varphi_2^0 \tilde{d}_L \tilde{d}_L$</td>
<td>$g_1^2/12 + g_2^2/4$</td>
<td>$C_{46}$</td>
</tr>
<tr>
<td>$\varphi_2^{0*} \varphi_2^0 \tilde{u}_R \tilde{u}_R$</td>
<td>$h_u^2 - g_1^2/3$</td>
<td>$C_{47}$</td>
</tr>
<tr>
<td>$\varphi_2^{0*} \varphi_2^0 \tilde{d}_R \tilde{d}_R$</td>
<td>$g_1^2/6$</td>
<td>$C_{48}$</td>
</tr>
</tbody>
</table>
Parameters of Effective Potential of MSSM

The one loop corrections to the parameters of effective potential

\[ \Delta \lambda_1 = h_u^4 \lambda^4 v_3^4 I_2[m_Q, m_U] + h_d^4 A_d^4 I_2[m_Q, m_D] + \\
+ h_u^2 \lambda^2 v_3^2 \left( \left( \frac{g_2^2}{4} - \frac{g_1^2}{12} \right) I_1[m_Q, m_U] + \frac{1}{3} g_1^2 I_1[m_U, m_Q] \right) + \\
+ h_d^2 A_d^2 \left( \left( h_d^2 - \frac{g_1^2}{12} - \frac{g_2^2}{4} \right) I_1[m_Q, m_D] + \left( h_d^2 - \frac{g_1^2}{6} \right) I_1[m_D, m_Q] \right) \]

\[ \Delta \lambda_2 = h_u^4 A_u^4 I_2[m_Q, m_U] + h_d^4 \lambda^4 v_3^4 I_2[m_Q, m_D] + \\
+ h_u^2 A_u^2 \left( \left( \frac{g_1^2}{12} - \frac{g_2^2}{4} \right) I_1[m_Q, m_U] + \left( h_u^2 - \frac{1}{3} g_1^2 I_1[m_U, m_Q] \right) + \\
+ h_d^2 \lambda^2 v_3^2 \left( \left( \frac{g_1^2}{12} + \frac{g_2^2}{4} \right) I_1[m_Q, m_D] + \frac{g_1^2}{6} I_1[m_D, m_Q] \right) \right) \]
Restrictions on the parameters of the MSSM

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$100 \div 2000$ (GeV)</td>
</tr>
<tr>
<td>$A_{t,b}$</td>
<td>$-1000 \div 1000$ (GeV)</td>
</tr>
<tr>
<td>$M_{SUSY}$</td>
<td>$500 \div 1000$ (GeV)</td>
</tr>
<tr>
<td>$m_A$</td>
<td>$100 \div 500$ (GeV)</td>
</tr>
<tr>
<td>$\tan \beta$</td>
<td>$3 \div 50$</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>$0 \div 2\pi$</td>
</tr>
<tr>
<td>$T$</td>
<td>$10 \div 500$ (GeV)</td>
</tr>
</tbody>
</table>
Restrictions on the parameters of the MSSM

a) $\tan \beta = 5$

Contour plot in $\mu - M_{SUSY}$ plane.
Selected region: 125 GeV < $m_{h_1}$ < 126 GeV (blue region).
Fixed parameters:
$A_t,b = 1000$ GeV, $m_{H^\pm} = 300$ GeV, $\varphi = \frac{\pi}{3}$.

b) $\tan \beta = 50$
Restrictions on the parameters of the MSSM

Contour plot in $m_A - \tan\beta$ plane.  
Selected region: $40 \text{ GeV} < m_{h_1} < 50 \text{ GeV}$ (blue region).  
Fixed parameters:  
$\mu = 2000 \text{ GeV}$, $A_{t,b} = 1000 \text{ GeV}$, $M_{SUSY} = 500 \text{ GeV}$, $\varphi = \frac{\pi}{3}$.  

Restrictions on the parameters of the MSSM

Contour plot in $\varphi - \tan \beta$ plane.
Selected region: $40 \text{ GeV} < m_{h_1} < 50 \text{ GeV}$ (blue region).
Fixed parameters:
$\mu = 2000 \text{ GeV}$, $A_{t,b} = 1000 \text{ GeV}$, $M_{SUSY} = 500 \text{ GeV}$, $m_{H^\pm} = 300 \text{ GeV}$. 
Restrictions on the parameters of the MSSM

\[ 40 \text{ GeV} < m_{h_1} < 50 \text{ GeV} \]

\[
\begin{align*}
\text{a)} &\ T = 10 \text{GeV} \\
\text{Contour plot in } \varphi - \tan \beta \text{ plane.} \\
\text{Selected region: } 40 \text{ GeV} < m_{h_1} < 50 \text{ GeV} \text{ (green region).} \\
\text{Fixed parameters:} \\
\mu &= 2000 \text{ GeV}, \ A_{t,b} = 1000 \text{ GeV}, m_Q = 500 \text{ GeV}, \ m_t = 800 \text{ GeV}, \ m_b = 200 \text{ GeV}, \ m_{H^\pm} = 300 \text{ GeV}. 
\end{align*}
\]

\[
\begin{align*}
\text{b)} &\ T = 500 \text{GeV} \\
\text{Contour plot in } \varphi - \tan \beta \text{ plane.} \\
\end{align*}
\]
Restrictions on the parameters of the MSSM

a) $T = 10 \text{GeV}$
Contour plot in $m_A - \tan \beta$ plane.
Selected region: $40 \text{ GeV} < m_{h_1} < 50 \text{ GeV}$ (green region).
Fixed parameters:
$\mu = 2000 \text{ GeV}, A_{t,b} = 1000 \text{ GeV}, m_Q = 500 \text{ GeV}, m_t = 800 \text{ GeV}, m_b = 200 \text{ GeV}, \varphi = \frac{\pi}{3}$.

b) $T = 500 \text{GeV}$
Nonlinear transformations (Peccei–Quinn symmetry)

in first coordinate system

\[
\lambda_1 v_1^3 + \frac{\lambda_{345}}{2} v_1 v_2^2 - \mu_1^2 v_1 - \mu_{12}^2 v_2 = 0
\]

\[
\lambda_2 v_2^3 + \frac{\lambda_{345}}{2} v_1^2 v_2 - \mu_2^2 v_2 - \mu_{12}^2 v_1 = 0
\]

in new coordinate system

1. \( U_{\bar{v}_1, \bar{v}_2} = \bar{\mu}_1^2 \bar{v}_1^2 + \bar{\mu}_1^2 \bar{v}_2^2 \) (Morse lemma)

\[
\bar{v}_1 (\lambda_1 \bar{v}_1^2 + \frac{\lambda_{345}}{2} \bar{v}_2^2 - \bar{\mu}_1^2) = 0
\]

\[
\bar{v}_2 (\lambda_2 \bar{v}_2^2 + \frac{\lambda_{345}}{2} \bar{v}_1^2 - \bar{\mu}_2^2) = 0
\]

\[
\bar{\mu}_{1,2}^2 = \frac{1}{2} \left( \mu_1^2 + \mu_2^2 \pm \sqrt{(\mu_1^2 - \mu_2^2)^2 + 4Re\mu_{12}^4} \right), \quad \cos^2 \theta = \frac{1}{2} - \frac{1}{2} \frac{|\mu_1^2 - \mu_2^2|}{\sqrt{(\mu_1^2 - \mu_2^2)^2 + 4Re\mu_{12}^4}}
\]

2. \( U = U_{NM}(\bar{v}_1, \bar{v}_2) + (\bar{\mu}_1^2 \bar{v}_1^2 + \bar{\mu}_2^2 \bar{v}_2^2) \) (Thom theorem)

\( U_{NM} \) – simple sprout of catastrophe \( A_4 \) or \( A_6 \)
## 1. Bifurcation sets

<table>
<thead>
<tr>
<th>N</th>
<th>Solutions</th>
<th>Hessian $H(\bar{v}_1, \bar{v}_2) =$</th>
<th>local minimum conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\bar{v}_1 = 0, \quad \bar{v}_2 = 0$</td>
<td>$- \begin{pmatrix} \bar{\mu}_1^2 &amp; 0 \ 0 &amp; \bar{\mu}_2^2 \end{pmatrix}$</td>
<td>$\bar{\mu}_1^2 + \bar{\mu}_2^2 &lt; 0, \quad \bar{\mu}_1^2 \cdot \bar{\mu}_2^2 \geq 0$</td>
</tr>
<tr>
<td>2</td>
<td>$\bar{v}_1 = 0, \quad \lambda_2 \bar{v}_2^2 - \bar{\mu}_2^2 = 0$</td>
<td>$-\begin{pmatrix} \lambda_345 \bar{v}_2^2 &amp; 0 \ 0 &amp; 2\lambda_2 \bar{v}_2^2 \end{pmatrix}$</td>
<td>$-\bar{\mu}_1^2 + \bar{\mu}_2^2 (2\lambda_2 + \frac{1}{2} \lambda_345) &gt; 0$ $(-\bar{\mu}_1^2 + \frac{1}{2} \lambda_345 \bar{v}_2^2) \lambda_2 \bar{v}_2^2 \geq 0$</td>
</tr>
<tr>
<td>3</td>
<td>$\bar{v}_2 = 0, \quad \lambda_1 \bar{v}_1^2 - \bar{\mu}_1^2 = 0$</td>
<td>$\begin{pmatrix} 2\lambda_1 \bar{v}_1^2 &amp; 0 \ 0 &amp; -\bar{\mu}_2^2 + \frac{\lambda_345 \bar{v}_2^2}{2} \end{pmatrix}$</td>
<td>$-\bar{\mu}_1^2 + \bar{\mu}_2^2 (2\lambda_1 + \frac{1}{2} \lambda_345) &gt; 0$ $(-\bar{\mu}_2^2 + \frac{1}{2} \lambda_345 \bar{v}_1^2) \lambda_1 \bar{v}_1^2 \geq 0$</td>
</tr>
<tr>
<td>4</td>
<td>$\lambda_1 \bar{v}<em>1^2 + \frac{\lambda</em>{345} \bar{v}_2^2}{2} - \bar{\mu}_1^2 = 0, \quad \lambda_2 \bar{v}<em>2^2 + \frac{\lambda</em>{345} \bar{v}_1^2}{2} - \bar{\mu}_2^2 = 0$</td>
<td>$\begin{pmatrix} 2\lambda_1 \bar{v}<em>1^2 &amp; \lambda</em>{345} \bar{v}_1 \bar{v}<em>2 \ \lambda</em>{345} \bar{v}_1 \bar{v}_2 &amp; 2\lambda_2 \bar{v}_2^2 \end{pmatrix}$</td>
<td>$\lambda_1 \bar{v}_1^2 + \lambda_2 \bar{v}_2^2 &gt; 0$ $\bar{v}_1^2 \bar{v}<em>2^2 (4\lambda_1 \lambda_2 - \lambda</em>{345}^2) \geq 0$</td>
</tr>
</tbody>
</table>
**The NMSSM case** \((v_1 \neq 0, v_2 \neq 0, v_3 \neq 0)\):

\[
- \frac{k_5 v_1 v_2}{v_3} + 8k_4 v_3^2 + 6k_6 v_3 - v_3(k_3 v_3 + k_5) \frac{v_1^2 + v_2^2}{v_1 v_2} + \lambda_1 v_1^2 + \lambda_2 v_2^2 > 0,
\]

\[
\frac{1}{v_1 v_2 v_3} \cdot (v_3(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3)) (v_1 v_2(k_5 v_1 + 2k_3 v_1 v_3 + 2k_2 v_2 v_3) \times
\]

\[
\left( k_3 v_3^2 + k_5 v_3 + v_1 v_2(\lambda_3 + \lambda_4) \right) - v_1(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3) \left( -k_3 v_1 v_3 - k_5 v_1 v_3 + \lambda_2 v_2^3 \right) - v_3(k_5 v_1 + 2k_3 v_1 v_3 + 2k_2 v_2 v_3)(v_2(k_5 v_1 + 2k_3 v_1 v_3 + 2k_2 v_2 v_3) \times
\]

\[
\left( k_3 v_3^2 + k_5 v_3 + v_1 v_2(\lambda_3 + \lambda_4) \right) + \left( 8k_4 v_3^2 + 6k_6 v_3^2 - k_5 v_1 v_2 \right) \left( -k_3 v_2 v_3 - k_5 v_2 v_3 + \lambda_1 v_1^3 \right) - v_1 v_2(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3) \times
\]

\[
\left( k_3 v_3^2 + k_5 v_3 + v_1 v_2(\lambda_3 + \lambda_4) \right) + \left( 8k_4 v_3^2 + 6k_6 v_3^2 - k_5 v_1 v_2 \right) \left( -k_3 v_2 v_3 - k_5 v_2 v_3 + \lambda_1 v_1^3 \right) - v_1 v_2(k_3 v_3^2 + k_5 v_3 + v_1 v_2(\lambda_3 + \lambda_4))^2 \right) > 0.
\]
Conclusion

- Our analysis of the effective MSSM and NMSSM finite-temperature potentials is based on the diagram calculation of various one-loop temperature corrections for the case of nonzero trilinear parameters $A_t$, $A_b$ and Higgs superfield parameter $\mu$.

- Quantum corrections are incorporated in control parameters $\lambda_1, \ldots, 7(T)$ of the effective two-doublet (+singlet) potential, which is then explicitly rewritten in terms of Higgs boson mass eigenstates.

- Types bifurcation sets for the two-Higgs-doublet(+singlet) potential $U_{\text{eff}}(v_1, v_2)$ are determined.

- Bifurcation sets for Higgs potential in the case of Peccei–Quinn symmetry are obtained. These sets always describe the system at the local minimum (critical morse point).

- Constrains on MSSM and NMSSM allowed parameter space are evaluated at the presence of the effective potential local minimum.

- Higgs prepotential as canonical morse form and non-morse term (catastrophe function at critical temperature) are reconstructed.