The XXIV International Workshop
High Energy Physics
and
Quantum Field Theory

Sochi, 2019

Investigation of the properties of the Higgs fields basis

Mikhail Dolgopolov, V. Makeeva (Zagorovskaya),
E.N. Rykova, A. Gurskaya

SAMARA UNIVERSITY

Outline

Introduction

Outline

Introduction

Minimal supersymmetric standard model

- Effective potential of MSSM
- N-point loop integrals at vanishing external momenta

Outline

Introduction

Minimal supersymmetric standard model

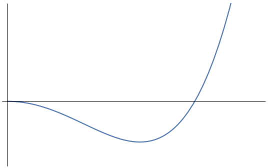
- Effective potential of MSSM
- N-point loop integrals at vanishing external momenta

Effective Potential parameters. One-loop temperature corrections

Introduction

In the simple isoscalar model the standard-like Higgs potential is

$$U(\varphi) = -\frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4.$$



The thermal Higgs boson mass:

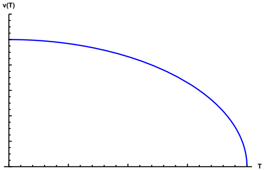
$$m_h^2 = -\mu^2 + \lambda\frac{T^2}{4}.$$

Two solutions

$$v(0) = 0 \quad \text{and} \quad v^2(T) = \frac{\mu^2}{\lambda} - \frac{T^2}{4},$$

demonstrate the second order phase transition at the critical temperature

$$T_c = \frac{2\mu}{\sqrt{\lambda}} = 2v(0),$$



Introduction

Temperature loop corrections from $\tilde{t}$ and other scalar states could be large and lead to the first order phase transition. The intensity depends on

$$\xi = \frac{v(T_c)}{T_c},$$

where $v(T_c) = \sqrt{v_1^2(T_c) + v_2^2(T_c)}$ is the vacuum expectation value at the critical temperature T_c.

The electroweak baryogenesis could be explained if

$$\frac{v(T_c)}{T_c} > 1$$

– the case of strong first order phase transition.

Introduction

In a number of analyzes the MSSM finite-temperature effective potential is taken in the representation

$$V_{eff}(v,T) = V_0(v_1, v_2, 0) + V_1(m(v), 0) + V_1(T) + V_{ring}(T), \qquad (1)$$

- V_0 **is the tree-level MSSM two-doublet potential at the SUSY scale**
- V_1 **is the (non-temperature) one-loop resumed Coleman-Weinberg term, dominated by** $\tilde{t}$ **and** $\tilde{b}$ **contributions**
- $V_1(T)$ **is the one-loop temperature term**
- V_{ring} **is the correction of re-summed leading infrared contribution from multi-loop ring (or daisy) diagrams**

Effective potential of MSSM

In two-doublet model there are two identical $SU(2)$ doublets of complex scalar fields Φ_1 and Φ_2

$$\Phi_1 = \left(\begin{array}{c} \phi_1^+(x) \\ \phi_1^0(x) \end{array} \right), \;\; \Phi_2 = \left(\begin{array}{c} \phi_2^+(x) \\ \phi_2^0(x) \end{array} \right)$$

with nonzero vacuum expectation values

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v_1 \end{array} \right), \qquad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v_2 \end{array} \right).$$

Neutral components of doublets

$$\phi_1^0(x) = \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1), \;\; \phi_2^0(x) = \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2).$$

Effective potential of MSSM

The most general renormalizable hermitian $SU(2) \times U(1)$ invariant potential: [Akhmetzyanova E.N., M.V.D, Dubinin M.N. Soft SUSY Breaking and Explicit CP Violation in the THDM // CALC 2003 & SQS0́3 Proc.]

$$U(\Phi_1, \Phi_2) = -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - \mu_{12}^2(\Phi_1^\dagger \Phi_2) - \overset{*}{\mu}{}_{12}^{2}(\Phi_2^\dagger \Phi_1) +$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) +$$

$$+\frac{\lambda_5}{2}(\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \frac{\overset{*}{\lambda}_5}{2}(\Phi_2^\dagger \Phi_1)(\Phi_2^\dagger \Phi_1) + \lambda_6(\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) +$$

$$+\overset{*}{\lambda}_6(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_1) + \lambda_7(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \overset{*}{\lambda}_7(\Phi_2^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

with effective real parameters μ_1^2, μ_2^2, λ_1,...,λ_4 and complex parameters μ_{12}^2, λ_5, λ_6, λ_7.

Effective potential of MSSM

Supersymmetry boundary conditions
In the tree approximation on the energy scale M_{SUSY}, the parameters λ_{1-7} are real and are expressed using the coupling constants g_1 and g_2 of electroweak group of the gauge symmetry $SU(2) \times U(1)$ as follows:

$$\lambda_1(M_{SUSY}) = \lambda_2(M_{SUSY}) = \frac{1}{4}\left(g_2^2(M_{SUSY}) + g_1^2(M_{SUSY})\right),$$

$$\lambda_3(M_{SUSY}) = \frac{1}{4}\left(g_2^2(M_{SUSY}) - g_1^2(M_{SUSY})\right),$$

$$\lambda_4(M_{SUSY}) = -\frac{1}{2}g_2^2(M_{SUSY}),$$

$$\lambda_5(M_{SUSY}) = \lambda_6(M_{SUSY}) = \lambda_7(M_{SUSY}) = 0.$$

Boundary conditions

On the scale of the superpartners M_{SUSY}

———————— m_t ——— $\Leftarrow$ —————————————— M_{SUSY} ——————————

The effective potential method,
the method of Feynman diagrams
& finite-temperature corrections

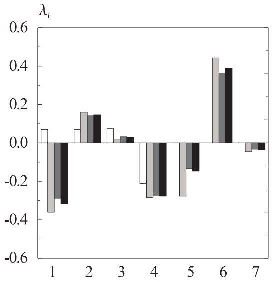
$$\lambda_1^{SUSY} = \lambda_2^{SUSY} = \frac{g_1^2 + g_2^2}{8}, \qquad \lambda_3^{SUSY} = \frac{g_2^2 - g_1^2}{4},$$

$$\lambda_4^{SUSY} = -\frac{g_2^2}{2}, \qquad \lambda_5^{SUSY} = \lambda_6^{SUSY} = \lambda_7^{SUSY} = 0.$$

The deviation from the parameters

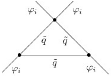
$$\lambda_i = \lambda_i^{SUSY} - \triangle\lambda_i$$

Dolgopolov M., Dubinin M., Rykova E. Threshold corrections to the MSSM finite-temperature Higgs potential. Journal of Modern Physics. 2011. PP.301-322



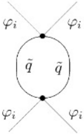
Threshold corrections (example for λ_1)

$$\lambda_i = \lambda_i^{SUSY} - \triangle\lambda_i^{th}$$



$$\triangle\lambda_1^{thr} = 3h_t^4|\mu|^4 I_2[m_Q, m_U] + 3h_b^4|A_b|^4 I_2[m_Q, m_D] + h_t^2|\mu|^2(-\frac{g_1^2 - 3g_2^2}{2} I_1[m_Q, m_U$$

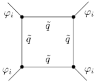
$$+2g_1^2 I_1[m_U, m_Q]) + h_b^2|A_b|^2(\frac{12h_b^2 - g_1^2 - 3g_2^2}{2} I_1[m_Q, m_D] + (6h_b^2 - g_1)I_1[m_d, m|_Q]$$



$$-\triangle\lambda_1^f = (h_b^2 - \frac{g_1^2}{6})^2(I(m_Q) + I(m_D)) + \frac{g_1^4}{9} I(m_U)$$

$$\triangle\lambda_1^{log} = -\frac{1}{384\pi^2}(11g_1^4 - 36h_b^2 g_1^2 + 9(g_2^4 - 4h_b^2 g_2^2 + 16h_b^4))ln(\frac{m_Q m_U}{m_t^2})$$

where



$$I_0[m_1, m_2] = \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{4\pi(\sqrt{4\pi^2 n^2 T^2 + m_1^2} + \sqrt{4\pi^2 n^2 T^2 + m_2^2})}$$

$$I_1[M_1, M_2] = -\frac{1}{64\pi^4 T^2} \sum_{n=-\infty, \neq 0}^{\infty} \frac{1}{\sqrt{M_1^2 + n^2}(\sqrt{M_1^2+n^2} + \sqrt{M_2^2+n^2})^2}$$

$$I_2[M_1, M_2] = \frac{1}{256\pi^5 T^4} \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{\sqrt{(M_1^2+n^2)(M_2^2+n^2)}(\sqrt{M_1^2+n^2} + \sqrt{M_2^2+n^2})^3}$$

Effective potential of MSSM

The supersymmetric scalar potential of interaction of Higgs bosons with the third generation quark superpartners on the tree level has the form

$$\mathcal{V}^{\,0} = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\widetilde{Q}}\,,$$

$$\mathcal{V}_M = (-1)^{i+j} m^2_{ij} \Phi^\dagger_i \Phi_j + M^{\,2}_{\widetilde{Q}} \left(\widetilde{Q}^{\,\dagger} \widetilde{Q}\right) + M^{\,2}_{\widetilde{U}} \widetilde{U}^* \widetilde{U} + M^{\,2}_{\widetilde{D}} \widetilde{D}^* \widetilde{D}\,,$$

$$\mathcal{V}_\Gamma = \Gamma^D_i \left(\Phi^\dagger_i \widetilde{Q}\right) \widetilde{D} + \Gamma^U_i \left(i\Phi^T_i \sigma_2 \widetilde{Q}\right) \widetilde{U} + \overset{*}{\Gamma}{}^D_i \left(\widetilde{Q}^{\,\dagger} \Phi_i\right) \widetilde{D}^* - \overset{*}{\Gamma}{}^U_i \left(i\widetilde{Q}^{\,\dagger} \sigma_2 \Phi^*_i\right) \widetilde{U}^*\,,$$

$$\mathcal{V}_\Lambda = \Lambda^{jl}_{ik} \left(\Phi^\dagger_i \Phi_j\right) \left(\Phi^\dagger_k \Phi_l\right) + \left(\Phi^\dagger_i \Phi_j\right) \left[\Lambda^Q_{ij} \left(\widetilde{Q}^{\,\dagger} \widetilde{Q}\right) + \Lambda^U_{ij} \widetilde{U}^* \widetilde{U} + \Lambda^D_{ij} \widetilde{D}^* \widetilde{D}\right] +$$

$$+ \overline{\Lambda}{}^{\,Q}_{ij} \left(\Phi^\dagger_i \widetilde{Q}\right) \left(\widetilde{Q}^{\,\dagger} \Phi_j\right) + \frac{1}{2} \left[\Lambda \epsilon_{ij} \left(i\Phi^T_i \sigma_2 \Phi_j\right) \widetilde{D}^* \widetilde{U} + \text{h.c.}\right]\,, i, j,\, k, l = 1, 2\,,$$

$\mathcal{V}_{\widetilde{Q}}$ denotes the terms of interaction of four scalar quarks.

Effective potential of NMSSM

The most general Hermitian form of the renormalized $SU(2)\times U(1)$ invariant potential for system of fields has the form:

$$U(\Phi_1,\Phi_2,S)=-\mu_1^2(\Phi_1^\dagger\Phi_1)-\ \mu_2^2(\Phi_2^\dagger\Phi_2)-\ \mu_3^2S^*S-\ (\mu_{12}^2(\Phi_1^\dagger\Phi_2)+h.c.)$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2+\frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)+$$

$$+\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2)+\frac{\overset{*}{\lambda}_5}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1)+$$

$$+\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)+\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2)+h.c.$$

$$+k_1(\Phi_1^\dagger\Phi_1)S^*S+k_2(\Phi_2^\dagger\Phi_2)S^*S+(k_3(\Phi_1^\dagger\Phi_2)S^*S+h.c.)k_4(S^*S)^2+$$

$$+k_5(\Phi_1^\dagger\Phi_1)S+k_6(\Phi_2^\dagger\Phi_2)S+k_7(\Phi_1^\dagger\Phi_2)S+\ \overset{*}{k}_7\ (\Phi_2^\dagger\Phi_1)S^*+k_8S^3.$$

Parameters of Effective Potential of NMSSM

The supersymmetric scalar potential of interaction of Higgs bosons with the third generation quark superpartners on the tree level has the form $V =$

$$|y_u(\widetilde{Q}\epsilon H_u)|^2 + |y_d(\widetilde{Q}\epsilon H_d)|^2 + |y_u\widetilde{u}^*_R H^0_u - y_d\widetilde{d}^*_R H^-_d|^2 + |y_d\widetilde{d}^*_R H^0_d - y_d\widetilde{u}^*_R H^+_u|^2 -$$

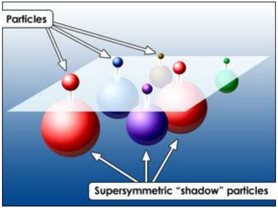
$$-y_u(\widetilde{u}_R\widetilde{u}^*_L\lambda S H^0_d + \widetilde{u}_R\widetilde{d}^*_L\lambda S H^-_d + c.c.) - y_d(\widetilde{d}_R\widetilde{d}^*_L\lambda S H^0_u + \widetilde{d}_R\widetilde{d}^*_L\lambda S H^+_u + c.c.) +$$

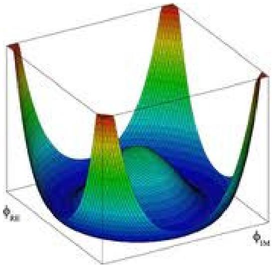
$$+\frac{g_2^2}{8}(4|H^\dagger_d\widetilde{Q}|^2 - 2(H^\dagger_d H_d)(\widetilde{Q}^\dagger\widetilde{Q}) + 4|H^\dagger_u\widetilde{Q}|^2 - 2(H^\dagger_u H_u)(\widetilde{Q}^\dagger\widetilde{Q})) +$$

$$+\frac{g_1^2}{2}(\frac{1}{6}(\widetilde{Q}^\dagger\widetilde{Q}) - \frac{2}{3}\widetilde{u}^*_R\widetilde{u}_R + \frac{1}{3}\widetilde{d}^*_R\widetilde{d}_R + \frac{1}{2}(H^\dagger_u H_u) - \frac{1}{2}(H^\dagger_d H_d))^2 +$$

$$+(\widetilde{u}^*_R y_u A_u(\widetilde{Q}^T\epsilon H_u) - \widetilde{d}_R y_d A_d(\widetilde{Q}^T\epsilon H_d) + c.c.)$$

We study the evolution of Higgs potential shape in the framework of catastrophe theory for predicting conditions for the stable minimum existence, i.e. the true minimum, in which our Universe is expected now





We take the effective 2HDM potential for MSSM and NMSSM with additional Higgs singlet, where the control parameters of Higgs potentials depend on the temperature.

Finite temperature corrections of squarks

In the early, high temperature stages of the Universe, the environment had a non-negligible matter and radiation density, making the hypotheses of conventional field theories impracticable. For that reason the methods of conventional field theories are no longer in use, and should be replaced by others, closer to thermodynamics, where the background state is a thermal bath. This field has been called field theory at finite temperature and it is extremely useful to study all phenomena which happened in the early Universe: phase transitions, inflationary cosmology, ...
In the finite temperature field theory Feynman diagrams with boson propagators, containing Matsubara frequencies $\omega_n = 2\pi n T$ $(n = 0, \pm 1, \pm 2, ...)$, lead to structures of the form

$$I[m_1, m_2, ..., m_b] = T \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^3} \prod_{i=1}^{b} \frac{(-1)^b}{(\mathbf{k}^2 + \omega_n^2 + m_j^2)}, \qquad (2)$$

$\mathbf{k}$ is the three-dimensional momentum in a system with the temperature T.

At $n \neq 0$ the result is

$$I[m_1, m_2, ..., m_b] = 2T\,(2\pi T)^{3-2b}\ \frac{(-1)^b \pi^{3/2}}{(2\pi)^3}\ \frac{\Gamma(b-3/2)}{\Gamma(b)}\ S(M, b-3/2), \quad (3)$$

where

$$S(M, b-3/2) = \int \{dx\} \sum_{n=1}^{\infty} \frac{1}{(n^2+M^2)^{b-3/2}}, \qquad M^2 \equiv \left(\frac{m}{2\pi T}\right)^2.$$

N-point loop integrals at vanishing external momenta

We derive and collect the analytic forms of the relevant loop integrals for reconstruction of the effective Higgs potential parameters in extended models (MSSM, NMSSM etc.).
The smallness of external state masses in relation to the masses of loop particles allows us to take the limit where external particles are massless. These integrals are particularly easy to take using algebraic recursion relations or residue method. But it is necessary to keep the external momentum p_μ general (i. e. nonzero) until it can be converted into an external mass in the amplitude, after which point one may take accurate limit $p \to 0$, especially in case of equivalent internal masses in the loop.
The N-point loop integrals at vanishing external momenta are defined as:

$$\frac{1}{(4\pi)^2} N_{2z}(m_1^2, m_2^2, ..., m_N^2) = \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{i k^{2z}}{\prod_i^N (k^2 - m_i^2)}, \qquad (4)$$

Numerical check of the zero temperature limiting case $T \to 0$ in (2) demonstrates that the results obtained using non-temperature field theory are successfully reproduced. In the high-temperature limit zero mode gives dominant contribution in agreement with known suppression of quantum effects at increasing temperatures.

Finite temperature corrections of squarks

We calculate the integral

$$J_0[a_1, a_2] = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)(\mathbf{k}^2 + a_2^2)} = \frac{1}{4\pi(a_1 + a_2)},$$

taking a residue in the spherical coordinate system.
$a_{1;2}^2$ are the sums of squared frequency and squared mass. Derivatives of J_0 with respect to a_1 and a_2 can be used for calculation of integrals

$$J_1[a_1, a_2] = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)} = \frac{1}{8\pi a_1(a_1 + a_2)^2},$$

$$J_2[a_1, a_2] = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)^2} = \frac{1}{8\pi a_1 a_2(a_1 + a_2)^3}.$$

Parameters of Effective Potential of MSSM

$$\Delta\lambda_1 = C_{31}^4 I_2[m_Q, m_U] + C_{32}^4 I_2[m_Q, m_D] +$$
$$+ C_{31}^2 (C_{41} I_1[m_Q, m_U] + C_{43} I_1[m_U, m_Q]) +$$
$$+ C_{32}^2 (C_{42} I_1[m_Q, m_D] + C_{44} I_1[m_D, m_Q]).$$

$$\Delta\lambda_2 = C_{33}^4 I_2[m_Q, m_U] + C_{34}^4 I_2[m_Q, m_D] +$$
$$+ C_{33}^2 (C_{45} I_1[m_Q, m_U] + C_{47} I_1[m_U, m_Q]) +$$
$$+ C_{34}^2 (C_{46} I_1[m_Q, m_D] + C_{48} I_1[m_D, m_Q]).$$

$$\Delta(\lambda_3 + \lambda_4) = C_{31}^2 C_{33}^2 I_2[m_Q, m_U] + C_{32}^2 C_{34}^2 I_2[m_Q, m_D] +$$
$$+ (C_{31}^2 C_{45} + C_{33}^2 C_{41}) I_1[m_Q, m_U] + (C_{31}^2 C_{47} + C_{33}^2 C_{43}) I_1[m_U, m_Q] +$$
$$+ (C_{32}^2 C_{46} + C_{34}^2 C_{42}) I_1[m_Q, m_D] + (C_{32}^2 C_{48} + C_{34}^2 C_{44}) I_1[m_D, m_Q].$$

$$\Delta\lambda_5 = C_{31}^2 C_{33}^2 I_2[m_Q, m_U] + C_{32}^2 C_{34}^2 I_2[m_Q, m_D].$$

Parameters of Effective Potential of MSSM

$\varphi_1^0\tilde{u}_L\tilde{u}_R$	$-h_u v_3\lambda$	C_{31}
$\varphi_1^{0*}\tilde{d}_L\tilde{d}_R$	$h_d A_d$	C_{32}
$\varphi_2^{0*}\tilde{u}_L\tilde{u}_R$	$h_u A_u$	C_{33}
$\varphi_2^{0*}\tilde{d}_L\tilde{d}_R$	$-h_d v_3\lambda$	C_{34}
$\varphi_1^{0*}\varphi_1^0\tilde{u}_L^*\tilde{u}_L$	$\frac{{g_2}^2}{4} - \frac{{g_1}^2}{12}$	C_{41}
$\varphi_1^{0*}\varphi_1^0\tilde{d}_L^*\tilde{d}_L$	$-\frac{{g_1}^2}{12} - \frac{{g_2}^2}{4} + {h_d}^2$	C_{42}
$\varphi_1^{0*}\varphi_1^0\tilde{u}_R^*\tilde{u}_R$	$\frac{{g_1}^2}{3}$	C_{43}
$\varphi_1^{0*}\varphi_1^0\tilde{d}_R^*\tilde{d}_R$	${h_d}^2 - \frac{{g_1}^2}{6}$	C_{44}
$\varphi_2^{0*}\varphi_2^0\tilde{u}_L^*\tilde{u}_L$	$\frac{{g_1}^2}{12} - \frac{{g_2}^2}{4} + {h_u}^2$	C_{45}
$\varphi_2^{0*}\varphi_2^0\tilde{d}_L^*\tilde{d}_L$	$\frac{{g_1}^2}{12} + \frac{{g_2}^2}{4}$	C_{46}
$\varphi_2^{0*}\varphi_2^0\tilde{u}_R^*\tilde{u}_R$	${h_u}^2 - \frac{{g_1}^2}{3}$	C_{47}
$\varphi_2^{0*}\varphi_2^0\tilde{d}_R^*\tilde{d}_R$	$\frac{{g_1}^2}{6}$	C_{48}

Parameters of Effective Potential of MSSM

The one loop corrections to the parameters of effective potential

$$\Delta\lambda_1 = h_u^4\lambda^4 v_3^4 I_2[m_Q, m_U] + h_d^4 A_d^4 I_2[m_Q, m_D] +$$

$$+h_u^2\lambda^2 v_3^2 \left(\left(\frac{g_2^2}{4} - \frac{g_1^2}{12} \right) I_1[m_Q, m_U] + \frac{1}{3} g_1^2 I_1[m_U, m_Q] \right) +$$

$$+h_d^2 A_d^2 \left(\left(h_d^2 - \frac{g_1^2}{12} - \frac{g_2^2}{4} \right) I_1[m_Q, m_D] + \left(h_d^2 - \frac{g_1^2}{6} \right) I_1[m_D, m_Q] \right)$$

$$\Delta\lambda_2 = h_u^4 A_u^4 I_2[m_Q, m_U] + h_d^4\lambda^4 v_3^4 I_2[m_Q, m_D] +$$

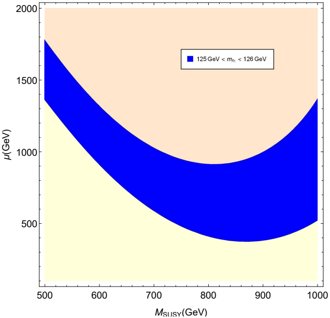
$$+h_u^2 A_u^2 \left(\left(\frac{g_1^2}{12} - \frac{g_2^2}{4} \right) I_1[m_Q, m_U] + (h_u^2 - \frac{1}{3} g_1^2 I_1[m_U, m_Q] \right) +$$

$$+h_d^2\lambda^2 v_3^2 \left(\left(\frac{g_1^2}{12} + \frac{g_2^2}{4} \right) I_1[m_Q, m_D] + \frac{g_1^2}{6} I_1[m_D, m_Q] \right)$$

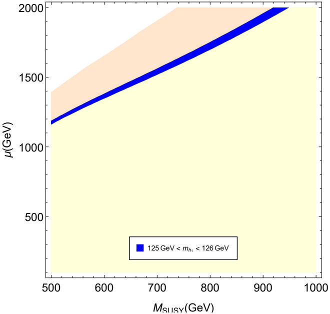
Restrictions on the parameters of the MSSM

Parameters	Values
μ	$100 \div 2000(\mathrm{GeV})$
$A_{t,b}$	$-1000 \div 1000(\mathrm{GeV})$
M_{SUSY}	$500 \div 1000(\mathrm{GeV})$
m_A	$100 \div 500(\mathrm{GeV})$
$\tan\beta$	$3 \div 50$
φ	$0 \div 2\pi$
T	$10 \div 500\ (\mathrm{GeV})$

Restrictions on the parameters of the MSSM



a) $\tan\beta = 5$



b) $\tan\beta = 50$

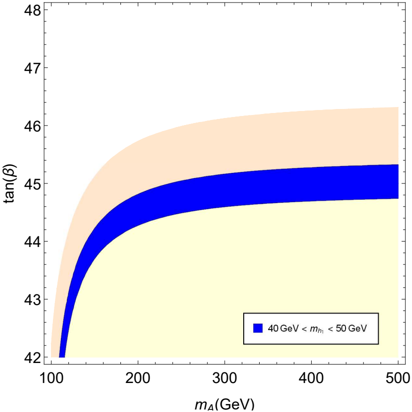
Contour plot in μ – M_{SUSY} plane.

Selected region: 125 GeV $< m_{h_1} <$ 126 GeV (blue region).

Fixed parameters:

$A_{t,b} = 1000$ GeV, $m_{H^{\pm}} = 300$ GeV, $\varphi = \dfrac{\pi}{3}$.

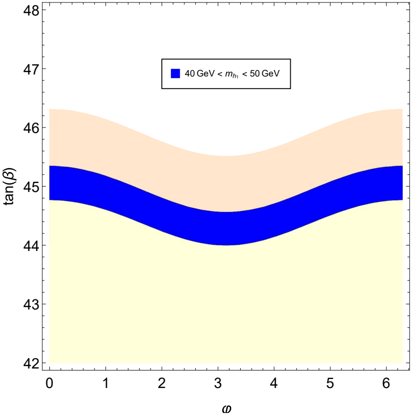
Restrictions on the parameters of the MSSM



Contour plot in $m_A - tan\beta$ plane.
Selected region: 40 GeV $< m_{h_1} <$ 50 GeV (blue region).
Fixed parameters:

$\mu = 2000$ GeV, $A_{t,b} = 1000$ GeV, $M_{SUSY} = 500$ GeV, $\varphi = \frac{\pi}{3}$.

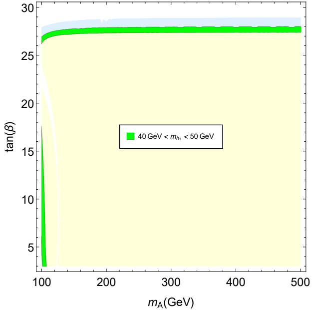
Restrictions on the parameters of the MSSM



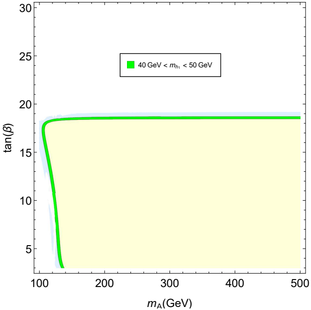
Contour plot in $\varphi - tan\beta$ plane.
Selected region: 40 GeV $< m_{h_1} <$ 50 GeV (blue region).
Fixed parameters:

$\mu = 2000$ GeV, $A_{t,b} = 1000$ GeV, $M_{SUSY} = 500$ GeV, $m_{H^{\pm}} = 300$ GeV.

Restrictions on the parameters of the MSSM



a)$T = 10GeV$

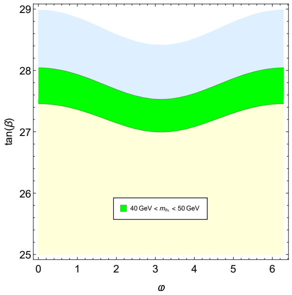


b)$T = 500GeV$

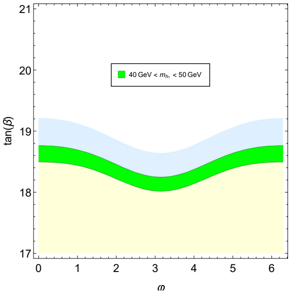
Contour plot in $\varphi - \tan\beta$ plane.
Selected region: 40 GeV $<m_{h_1}<$50 GeV (green region).
Fixed parameters:

$\mu = 2000$ GeV, $A_{t,b} = 1000$ GeV,m_Q=500 GeV, m_t=800 GeV, m_b=200 GeV,$m_{H^\pm} = 300$ GeV.

Restrictions on the parameters of the MSSM



a)$T = 10GeV$



b)$T = 500GeV$

Contour plot in $m_A - \tan\beta$ plane.
Selected region: 40 GeV $< m_{h_1} <$ 50 GeV (green region).
Fixed parameters:

$\mu = 2000$ GeV, $A_{t,b} = 1000$ GeV,m_Q=500 GeV, m_t=800 GeV, m_b=200 GeV, $\varphi = \frac{\pi}{3}$.

Nonlinear transformations (Peccei–Quinn symmetry)

in first coordinate system

$$\lambda_1 v_1^3 + \frac{\lambda_{345}}{2} v_1 v_2^2 - \mu_1^2 v_1 - \mu_{12}^2 v_2 = 0$$

$$\lambda_2 v_2^3 + \frac{\lambda_{345}}{2} v_1^2 v_2 - \mu_2^2 v_2 - \mu_{12}^2 v_1 = 0$$

in new coordinate system

1. $U_{\overline{v}_1,\overline{v}_2} = \overline{\mu}_1^2 \overline{v}_1^2 + \overline{\mu}_1^2 \overline{v}_2^2$ (Morse lemma)

$$\overline{v}_1 (\lambda_1 \overline{v}_1^2 + \frac{\lambda_{345}}{2} \overline{v}_2^2 - \overline{\mu}_1^2) = 0$$

$$\overline{v}_2 (\lambda_2 \overline{v}_2^2 + \frac{\lambda_{345}}{2} \overline{v}_1^2 - \overline{\mu}_2^2) = 0$$

$$\overline{\mu}_{1,2}^2 = \frac{1}{2} \left(\mu_1^2 + \mu_2^2 \pm \sqrt{(\mu_1^2 - \mu_2^2)^2 + 4 Re \mu_{12}^4} \right), \qquad \cos^2\theta = \frac{1}{2} - \frac{1}{2} \frac{|\mu_1^2 - \mu_2^2|}{\sqrt{(\mu_1^2 - \mu_2^2)^2 + 4 Re \mu_{12}^4}}$$

2. $U = U_{NM}(\overline{v}_1, \overline{v}_2) + (\overline{\mu}_1^2 \overline{v}_1^2 + \overline{\mu}_2^2 \overline{v}_2^2)$ (Thom theorem)

U_{NM} – simple sprout of catastrophe A_4 or A_6

1. Bifurcation sets

N	Solutions	Hessian $H(\overline{v}_1, \overline{v}_2) =$	local minimum conditions
1	$\overline{v}_1 = 0, \qquad \overline{v}_2 = 0$	$-\begin{pmatrix} \overline{\mu}_1^2 & 0 \\ 0 & \overline{\mu}_2^2 \end{pmatrix}$	$\overline{\mu}_1^2 + \overline{\mu}_2^2 < 0, \qquad \overline{\mu}_1^2 \cdot \overline{\mu}_2^2 \geq 0$
2	$\overline{v}_1 = 0, \qquad \lambda_2 \overline{v}_2^2 - \overline{\mu}_2^2 = 0$	$\begin{pmatrix} -\overline{\mu}_1^2 + \frac{\lambda_{345}}{2} \overline{v}_2^2 & 0 \\ 0 & 2\lambda_2 \overline{v}_2^2 \end{pmatrix}$	$-\overline{\mu}_1^2 + \overline{v}_2^2 (2\lambda_2 + \frac{1}{2}\lambda_{345}) > 0$ $(-\overline{\mu}_1^2 + \frac{1}{2}\lambda_{345}\overline{v}_2^2)\lambda_2 \overline{v}_2^2 \geq 0$
3	$\overline{v}_2 = 0, \qquad \lambda_1 \overline{v}_2^2 - \overline{\mu}_1^2 = 0$	$\begin{pmatrix} 2\lambda_1 \overline{v}_1^2 & 0 \\ 0 & -\overline{\mu}_2^2 + \frac{\lambda_{345}}{2} \overline{v}_1^2 \end{pmatrix}$	$-\overline{\mu}_2^2 + \overline{v}_1^2 (2\lambda_1 + \frac{1}{2}\lambda_{345}) > 0$ $(-\overline{\mu}_2^2 + \frac{1}{2}\lambda_{345}\overline{v}_1^2)\lambda_1 \overline{v}_1^2 \geq 0$
4	$\lambda_1 \overline{v}_1^2 + \frac{\lambda_{435}}{2} \overline{v}_2^2 - \overline{\mu}_1^2 = 0,$ $\lambda_2 \overline{v}_2^2 + \frac{\lambda_{435}}{2} \overline{v}_1^2 - \overline{\mu}_2^2 = 0$	$\begin{pmatrix} 2\lambda_1 \overline{v}_1^2 & \lambda_{345}\overline{v}_1\overline{v}_2 \\ \lambda_{345}\overline{v}_1\overline{v}_2 & 2\lambda_2 \overline{v}_2^2 \end{pmatrix}$	$\lambda_1 \overline{v}_1^2 + \lambda_2 \overline{v}_2^2 > 0$ $\overline{v}_1^2 \overline{v}_2^2 (4\lambda_1\lambda_2 - \lambda_{345}^2) \geq 0$

$***$ The NMSSM case ($v_1 \neq 0$, $v_2 \neq 0$, $v_3 \neq 0$):

$$-\frac{k_5 v_1 v_2}{v_3} + 8k_4 v_3^2 + 6k_6 v_3 - v_3(k_3 v_3 + k_5)\frac{v_1^2 + v_2^2}{v_1 v_2} + \lambda_1 v_1^2 + \lambda_2 v_2^2 > 0,$$

$$\frac{1}{v_1 v_2 v_3} \cdot \Bigg(v_3 \Big(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3 \Big) \Big(v_1 v_2 \Big(k_5 v_1 + 2k_3 v_1 v_3 + 2k_2 v_2 v_3 \Big) \times$$

$$\times \Big(k_3 v_3^2 + k_5 v_3 + v_1 v_2 (\lambda_3 + \lambda_4) \Big) - v_1 \Big(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3 \Big) \Big(-k_3 v_1 v_3^2 -$$

$$-k_5 v_1 v_3 + \lambda_2 v_2^3 \Big) \Big) - v_3 \Big(k_5 v_1 + 2k_3 v_1 v_3 + 2k_2 v_2 v_3 \Big) \Big(v_2 \Big(k_5 v_1 + 2k_3 v_1 v_3 +$$

$$+2k_2 v_2 v_3 \Big) \Big(-k_3 v_2 v_3^2 - k_5 v_2 v_3 + \lambda_1 v_1^3 \Big) - v_1 v_2 \Big(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3 \Big) \times$$

$$\times \Big(k_3 v_3^2 + k_5 v_3 + v_1 v_2 (\lambda_3 + \lambda_4) \Big) \Big) + \Big(8k_4 v_3^3 + 6k_6 v_3^2 - k_5 v_1 v_2 \Big) \Big(\Big(-k_3 v_2 v_3^2 -$$

$$-k_5 v_2 v_3 + \lambda_1 v_1^3 \Big) \Big(-k_3 v_1 v_3^2 - k_5 v_1 v_3 + \lambda_2 v_2^3 \Big) - v_1 v_2 \Big(k_3 v_3^2 + k_5 v_3 +$$

$$+v_1 v_2 (\lambda_3 + \lambda_4) \Big)^2 \Big) \Bigg) > 0.$$

Conclusion

- Our analysis of the effective MSSM and NMSSM finite-temperature potentials is based on the diagram calculation of various one-loop temperature corrections for the case of nonzero trilinear parameters A_t, A_b and Higgs superfield parameter μ.
- Quantum corrections are incorporated in control parameters $\lambda_{1,\ldots 7}...(T)$ of the effective two-doublet (+singlet) potential, which is then explicitly rewritten in terms of Higgs boson mass eigenstates.
- Types bifurcation sets for the two-Higgs-doublet(+singlet) potential $U_{eff}(v_1, v_2)$ are determined.
- Bifurcation sets for Higgs potential in the case of Peccei–Quinn symmetry are obtained. These sets always describe the system at the local minimum (critical morse point).

———————————————————————————————————————

- Constrains on MSSM and NMSSM allowed parameter space are evaluated
  at the presence of the effective potential local minimum.

———————————————————————————————————————

- Higgs prepotential as canonical morse form and non-morse term (catastrophe function at critical temperature) are reconstructed.