Modified Cosmological Evolution in R^2-Gravity and Supersymmetric Dark Matter

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based on common works with A.D. Dolgov and R.S. Singh

The XXIV International Workshop

High Energy Physics and Quantum Field Theory

22 – 29 September 2019, Sochi, Russia

Outline

Universe evolution with an account of gravitational particle production in $R + R^2$ theory

- EA, A.D. Dolgov, R.S. Singh, "Distortion of the standard cosmology in $R + R^2$ theory", JCAP 1807 (2018) no.07, 019, arXiv:1803.01722

Kinetics of massive relics and supersymmetric dark matter in $R + R^2$ cosmology

- EA, A.D. Dolgov, R.S. Singh, "Dark matter in $R + R^2$ cosmology", JCAP 04 (2019) 014, arXiv:1811.05399

General Relativity (GR):

$$S_{EH} = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g}\, R$$

describes basic properties of the universe in very good agreement with observations.

- $m_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass

Beyond the frameworks of GR:

$$S_{tot} = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g}\, [R + F(R)] + S_m$$

$F(R) = -R^2/(6m^2)$:

- R^2-term was suggested by V.Ts. Gurovich and A.A. Starobinsky for elimination of cosmological singularity (JETP **50** (1979) 844).
- It was found that the the addition of the R^2-term leads to inflationary cosmology. (A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980))

Curvature $R(t)$ can be considered as an effective scalar field (scalaron) with the mass m and with the decay width Γ.

Action:

$$S_{tot} = -\frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}\left(R - \frac{R^2}{6m^2}\right) + S_m$$

- m is a constant parameter with dimension of mass

The modified Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{3m^2}\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} + g_{\mu\nu}D^2 - D_\mu D_\nu\right)R = \frac{8\pi}{m_{Pl}^2}T_{\mu\nu}$$

- $D^2 \equiv g^{\mu\nu}D_\mu D_\nu$ is the covariant D'Alembert operator.

The energy-momentum tensor of matter $T_{\mu\nu}$

$$T^\mu_\nu = diag(\varrho, -P, -P, -P)$$

where ϱ is the energy density, P is the pressure of matter.

The matter distribution is homogeneous and isotropic

$$P = w\varrho$$

- non-relativistic: $w = 0$, relativistic: $w = 1/3$, vacuum-like: $w = -1$

FRW: $ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta\, d\varphi^2\right]$, $H = \dot{a}/a$

The curvature scalar:

$$R = -6\dot{H} - 12H^2$$

The covariant conservation condition $D_\mu T^\mu_\nu = 0$:

$$\dot{\varrho} = -3H(\varrho + P) = -3H(1+w)\varrho$$

Trace equation:

$$D^2 R + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}\, T^\mu_\mu$$

For homogeneous field, $R = R(t)$, and with $P = w\varrho$:

$$\ddot{R} + 3H\dot{R} + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}(1-3w)\varrho$$

- This is the Klein-Gordon type equation for massive scalar field R (scalaron)
- It differs from KG Eq. in the flat space-time by the Hubble friction term.

Good approximation at the inflation, when part. production is practically absent.

At some stage R starts to oscillate efficiently producing particles

- It commemorates the end of inflation, the heating of the universe, and the transition from the accelerated expansion (inflation) to a de-accelerated one.
- The latter resembles the usual Friedmann matter dominated expansion regime but differs in many essential features.

Particle production for harmonic potential can be approximately described by the following production rate:

$$\Gamma = \frac{m^3}{48 m_{Pl}^2}$$

- Ya.B. Zeldovich, A. Starobinsky, JETP Lett. 26, 252 (1977); A. Vilenkin, Phys. Rev. **D32**, 2511 (1985); EA, A. Dolgov, L. Reverberi, JCAP **1202** (2012) 049.

Equation for R acquires an additional friction term:

$$\ddot{R} + (3H + \Gamma)\dot{R} + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}(1 - 3w)\varrho$$

Particle production leads to an emergence of the source term in Eq. for ϱ:

$$\dot{\varrho} = -3H(1 + w)\varrho + \frac{m R_{ampl}^2}{1152\pi}$$

Dimensionless Equations

Dimensionless time variable and dimensionless functions

$$\tau = tm, \quad H = mh, \quad R = m^2 r, \quad \varrho = m^4 y, \quad \Gamma = m\gamma.$$

The system of dimensionless equations

$$h' + 2h^2 = -r/6$$
$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1 - 3w)y$$
$$y' + 3(1 + w)h\, y = S[r]$$

- prime denotes derivative over τ, $\mu = m/m_{Pl}$, $\gamma = \mu^2/48$

The source term is taken as:

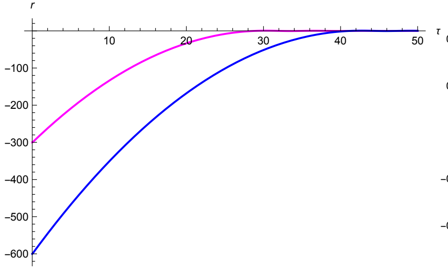
$$S[r] = \frac{\langle r^2 \rangle}{1152\pi}$$

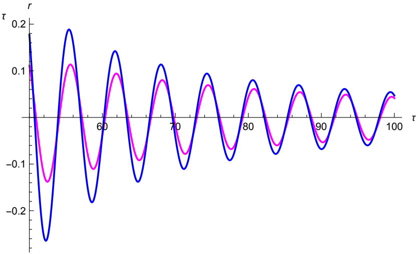
- We compare numerical solution of the system with the analytic asymptotic expansion at $\tau \gg 1$.
- Very good agreement allows to use asymptotic analytical solutions in cases when numerical calculations become unreliable.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Evolution of the dimensionless curvature scalar $\boldsymbol{r(\tau)}$ for $\boldsymbol{r_{in} = -300}$ (magenta) and $\boldsymbol{r_{in} = -600}$ (blue)





- Evolution during inflation.

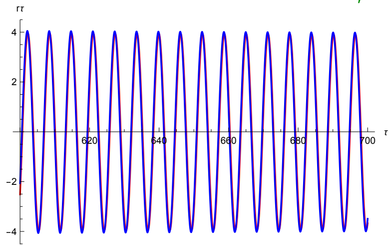
Evolution after the end of inflation, the curvature starts to oscillate.

The initial value of $\boldsymbol{R}$ should be quite large, $\boldsymbol{R \sim 300m^2}$, to ensure sufficiently long inflation ($N_e \sim 70$).

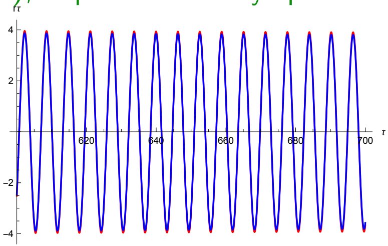
Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r] = (r')^2/1152\pi$

$$r'' + (3h+\gamma)r' + r = -8\pi\mu^2(1-3w)y, \;\; \mu = m/m_{Pl} = 0.1, \;\; \gamma = \mu^2/48$$

Evolution of the curvature scalar, $\boldsymbol{\tau r(\tau)}$, in post-infationary epoch.



$\boldsymbol{r_{in} = -300}$ (red), $\boldsymbol{r_{in} = -600}$ (blue)



$\boldsymbol{w = 1/3}$ (red) and $\boldsymbol{w = 0}$ (blue)

The amplitude $\boldsymbol{r_{ampl}\tau \to const}$. For large τ the result does not depend upon the initial value of r and very weakly depends on w.

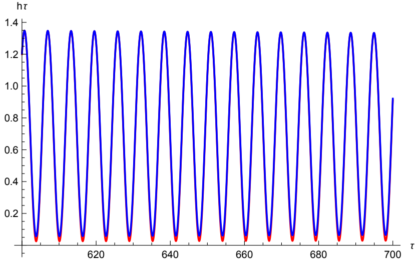
Analytical asymptotic solution:

$$\boldsymbol{r = -\frac{4\cos(\tau+\theta)}{\tau} - \frac{4}{\tau^2}}$$

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h+\gamma)r' + r = -8\pi\mu^2(1-3w)y, \quad \mu = m/m_{Pl} = 0.1$$

Evolution of the Hubble parameter, $\boldsymbol{h\tau}$, in post-inflationary epoch for $\boldsymbol{w = 1/3}$ (red) and $\boldsymbol{w = 0}$ (blue)



- The dependence on $\boldsymbol{w}$ is very weak, except for small values of $\boldsymbol{h}$.

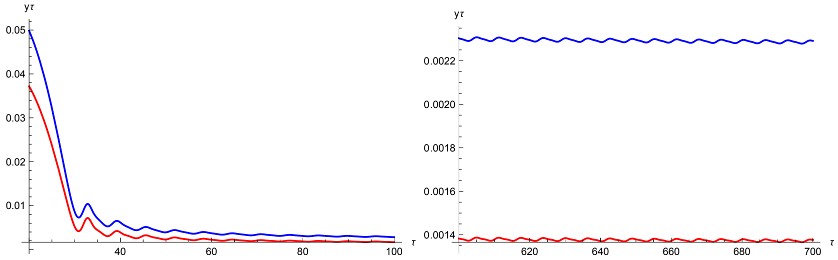
Analytical asymptotic solution:

$$\boldsymbol{h = \frac{2}{3\tau}\left[1 + \sin(\tau + \theta)\right]}$$

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Energy density of matter as a function of time in post-inflationary (scalaron dominated) epoch for $\boldsymbol{w = 1/3}$ (red) and $\boldsymbol{w = 0}$ (blue)



- Evolution of $\boldsymbol{y\tau}$ at small $\boldsymbol{\tau}$ (left) and at large $\boldsymbol{\tau}$ (right).

The product $\boldsymbol{y\tau \to const}$ with rising $\boldsymbol{\tau}$. It means that $\boldsymbol{\varrho \sim 1/t}$.
This behavior much differs from the standard matter density evolution $\boldsymbol{\varrho \sim 1/t^2}$.

Cosmological history in R^2-gravity: **4** distinct epoch

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2. **Scalaron dominated epoch:** R dropped down to 0 and started to oscillate around it. The curvature oscillations resulted in the onset of creation of usual matter, which remains subdominant.

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It opens the window for heavy LSPs to be the cosmological DM, modifies high temperature baryogenesis, leads to reconsideration of PBHs formation, *etc.*

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3. **Transition period to GR:** the oscillations of all relevant quantities damps down exponentially and the particle production by curvature becomes negligible.
   GR is recovered when the energy density of matter becomes larger than the energy density of the exponentially decaying scalaron.
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4. **After that we arrive to the conventional cosmology governed by GR.**

Scalaron dominated regime: SUSY Dark Matter (?)

- $R(t)$ approached zero and started to oscillate around it as

$$R = -\frac{4 m_R \cos(m_R t + \theta)}{t}, \quad m_R = 3 \times 10^{13} \ GeV$$

- The Hubble parameter:

$$H = \frac{2}{3t} \left[1 + \sin(m_R t + \theta)\right]$$

- Energy density of matter drops down as

$$\varrho_{R^2} = \frac{m_R^3}{120 \pi t} \quad \text{instead of} \quad \varrho_{GR} = \frac{3 H^2 m_{Pl}^2}{8\pi} = \frac{3 m_{Pl}^2}{32 \pi t^2}$$

Kinetics of massive species and the density of Dark Matter particles differ significantly from those in the conventional cosmology.

Dark Matter (DM)

First indications:

- J. C. Kapteyn, "First Attempt at a Theory of the Arrangement and Motion of the Sidereal System," Astrophys. J. **55** (1922) 302.
- J. H. Oort, "The force exerted by the stellar system in the direction perpendicular to the galactic plain and some related problems", Bull. Astron. Inst. Netherland **6** (1932) 249.
- F. Zwicky, "Die Rotverschiebung von extragalaktischen Nebeln," Helv. Phys. Acta **6** (1933) 110.

Later confirmation:

- V. C. Rubin and W. K. Ford, "Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions," Astrophys. J. **159** (1970) 379.
- Einasto, J., Kaasik, A., Saar, E., "Dynamic Evidence on Massive coronas of galaxies", Nature, **250** (1974) 309.
- J. P. Ostriker, P. J. E. Peebles and A. Yahil, "The size and mass of galaxies, and the mass of the universe," Astrophys. J. **193** (1974) L1.

Many theoretical models have been proposed to describe this elusive form of matter.

DM candidates

Dark matter:

- electrically neutral, since doesn't scatter light
- properties are practically unknown

Particles of many different types can be DM candidates.

Very popular candidate: Lightest Supersymmetric Particle (LSP)

- R. Catena and L. Covi, "SUSY dark matter(s)," arXiv:1310.4776 [hep-ph].
- G. B. Gelmini, "The Hunt for Dark Matter," arXiv:1502.01320 [hep-ph].
- T. R. Slatyer, "Indirect Detection of Dark Matter," arXiv:1710.05137 [hep-ph].

SUSY particles:

- Negative results at LHC $\Longrightarrow$ characteristic energy scale higher than 10 TeV.
- The cosmological energy density of LSPs $\boldsymbol{\varrho_{LSP} \sim m_{LSP}^2} \Longrightarrow$ for $m_{LSP} \sim 1$ Tev ϱ_{LSP} is of the order of the observed energy density of the universe.
- For larger masses LSPs would overclose the universe.

These unfortunate circumstances exclude LSPs as DM particles in the conventional cosmology.

Attempts to save SUSY Dark Matter

Modification of the cosmological scenarios of LSP production in such a way that the relic density of heavy LSPs would be significantly suppressed:

- Non-thermal production of heavy relics (G.L. Kane, P. Kumar, B.D. Nelson, B. Zheng, Phys.Rev. D93 (2016) no.6, 063527)
- Assumption: after the freezing of LSP the universe was matter dominated and this epoch transformed into the radiation dominated stage with low reheating temperature (M. Drees, F. Hajkarim, arXiv:1808.05706)
- Earlier: At some early stage the universe might be dominated by primordial black holes which created the necessary amount of entropy to dilute the heavy particle relics (A. D. Dolgov, P. D. Naselsky, I. D. Novikov, arXiv: astro-ph/0009407)

Generalization of R^2 inflation to supergravity:

- S. V. Ketov and A. A. Starobinsky, "Embedding $(R + R^2)$-Inflation into Supergravity," Phys. Rev. D **83** (2011) 063512 [arXiv:1011.0240 [hep-th]].
- S. Ketov and M. Khlopov, "Extending Starobinsky inflationary model in gravity and supergravity," arXiv:1809.09975 [hep-th].

Scenario with superheavy gravitino as a viable candidate for DM particle

- A. Addazi, S.V. Ketov, M.Yu. Khlopov "Gravitino and Polonyi production in supergravity", Eur.Phys.J. **C78** (2018) no.8, 642, [arXiv:1708.05393 [hep-ph]]

Our approach

In $(R + R^2)$-gravity the energy density of LSPs may be much lower than that in the GR cosmology $\Longrightarrow$ it reopens for them the chance to be the dark matter.

- EA, A. D. Dolgov and R. S. Singh, "Dark matter in $R + R^2$ cosmology," JCAP **04** (2019) 014, arXiv:1811.05399 [astro-ph.CO]

Outline:

- Evolution of matter density: $\boldsymbol{R^2}$-gravity versus GR
- LSP density for the scalaron decay into non-conformal scalars
- Decay into fermions or conformal scalars
- Anomalous decay into gauge bosons

EA, A. D. Dolgov and R. S. Singh, "Distortion of the standard cosmology in $R + R^2$ theory," JCAP **1807** (2018) no.07, 019 [arXiv:1803.01722 [gr-qc]]

EA, A. D. Dolgov and L. Reverberi, "Cosmological evolution in R^2 gravity," JCAP **1202** (2012) 049 [arXiv:1112.4995 [gr-qc]]

The energy density of the produced particles depends upon the form of their coupling to curvature, $R(t)$.

Parker theorem: gravitational production of massless particles in FLRW-metric is absent in conformally invariant theory.

Massless scalar field with minimal coupling to gravity is not conformally invariant $\Longrightarrow$ The width of the scalaron decay into 2 non-conformal scalars:

$$\Gamma_s = \frac{m_R^3}{48 m_{Pl}^2}, \qquad \varrho_s = \frac{m_R^3}{120\pi t}$$

The scalaron decay into pair of fermions (conformally invariant):

$$\Gamma_f = \frac{m_R m_f^2}{48 m_{Pl}^2}, \qquad \varrho_f = \frac{m_R m_f^2}{120\pi t}$$

Similar suppression factor appears for scalar particles with conformal coupling to gravity.

Much slower decrease of the energy density of matter than normally for relativistic matter is ensured by the influx of energy from the scalaron decay.

- Without energy influx: $\varrho \sim 1/a^4(t) \sim 1/t^{8/3}$, since $a(t) \sim t^{(2/3)}$ at SD.

LSP density for the scalaron decay into massless scalars

The freezing of massive species $X \Longrightarrow$ Lee-Weinberg equation (1977) (Zeldovich, 1965):

$$\dot{n}_X + 3Hn_X = -\langle \sigma_{ann} v \rangle \left(n_X^2 - n_{eq}^2 \right),$$

- n_X is a number density of particles X, $\boldsymbol{v}$ is the center-of-mass velocity
- σ_{ann} is the annihilation cross-section, g_s is the number of spin states.

For annihilation of the non-relativistic particles:

$$\langle \sigma_{ann} v \rangle = \sigma_{ann} v = \frac{\alpha^2 \beta_{ann}}{M_X^2},$$

- M_X is a mass of X-particle, α is a coupling constant, in SUSY theories $\boldsymbol{\alpha \sim 0.01}$
- $\boldsymbol{\beta_{ann}}$ is a numerical parameter $\sim$ the number of annihilation channels, $\boldsymbol{\beta \sim 10}$.

The equilibrium number density of X-particles:

$$n_{eq} = g_s \left(\frac{M_X T}{2\pi} \right)^{3/2} e^{-M_X/T}$$

Some comments

An additional term describing X-particle production by $R(t)$ should be included. However, we assume that this channel is suppressed in comparison with inverse annihilation of light particles into $X\bar{X}$-pair.

We do not specify which precisely supersymmetric particle is the lightest (it can be, e.g., sneutrino, neutralino or gravitino). We only need the value of its mass, M_X, and the magnitude of the annihilation cross-section.

We assume, that the plasma is thermalised $\Longrightarrow$ the temperature satisfies

$$\left(\alpha^2 \beta_{scat} T t\right)_s = \frac{\alpha^2 \beta_{scat}}{4\pi^3 g_*} \left(\frac{m_R}{T}\right)^3 \approx 8 \cdot 10^{-7} \left(\frac{m_R}{T}\right)^3 > 1$$

The energy density of relativistic matter in thermal equilibrium:

$$\varrho_{therm} = \frac{\pi^2 g_*}{30} T^4$$

- g_* is the number of relativistic species in the plasma, $g_* \sim 100$.

GR cosmology $\Longleftrightarrow$ R^2 cosmology (decay into scalars)

Equating the energy densities ϱ_{GR} and ϱ_s to the energy density of relativistic plasma in thermal equilibrium $\varrho_{therm} = \pi^2 g_* T^4/30$, we obtain:

$$\varrho_{GR} = \frac{3m_{Pl}^2}{32\pi t^2} = \frac{\pi^2 g_* T^4}{30} \qquad \varrho_s = \frac{m_R^3}{120\pi t} = \frac{\pi^2 g_* T^4}{30}$$

- g_* is the number of relativistic species in the plasma, $g_* \sim 100$.

Connection of the temperature with time:

$$(tT^2)_{GR} = \left(\frac{90}{32\pi^3 g_*}\right)^{1/2} m_{Pl} = const \qquad (tT^4)_s = \frac{m_R^3}{4\pi^3 g_*} = const$$

Correspondingly

$$\left(\frac{\dot{T}}{T}\right)_{GR} = -\frac{1}{2t} \qquad \left(\frac{\dot{T}}{T}\right)_s = -\frac{1}{4t}$$

Kinetic equation

New function $\boldsymbol{f}$:

$$n_X = n_{in}\left(\frac{a_{in}}{a}\right)^3 \boldsymbol{f}, \quad n_{in} = 0.12 g_s T_{in}^3 = 0.12 g_s M_X^3$$

NB: The final result does not depend upon $\boldsymbol{n_{in}}$ and $\boldsymbol{T_{in}}$.

With new variable $\boldsymbol{x = M_X/T}$ we arrive to the equations:

$$\mathbf{GR}: \qquad \frac{df}{dx} = -\sigma v\, m_{Pl} M_X \left(\frac{45}{4\pi^3 g_*}\right)^{1/2} \left(\frac{a_{in}}{a}\right)^3 \frac{n_{in}}{T^3}\, \frac{(f^2 - f_{eq}^2)}{x^2}$$

$$\boldsymbol{R^2}: \qquad \frac{df}{dx} = -\sigma v\, \frac{m_R^3}{\pi^3 g_* M_X} \left(\frac{a_{in}}{a}\right)^3 \frac{n_{in}}{T^3} \left(f^2 - f_{eq}^2\right)$$

GR-regime: $\boldsymbol{aT \approx const}$

$$\boldsymbol{\frac{a_{in}^3 n_{in}}{a^3 T^3} \approx 1}$$

R^2 theory: $\boldsymbol{T \sim t^{-1/4}}$, $\boldsymbol{a \sim t^{2/3}}$

$$\boldsymbol{\frac{a_{in}^3\, n_{in}}{a^3\, T^3} = \frac{0.12\, g_s}{x^5}}$$

Evolution of X-particles in the scalaron dominated regime

$$\frac{df}{dx} = -\frac{0.12 g_s \alpha^2 \beta_{ann}}{\pi^3 g_*} \left(\frac{m_R}{M_X}\right)^3 \frac{f^2 - f_{eq}^2}{x^5} \equiv -Q_s \frac{f^2 - f_{eq}^2}{x^5}$$

The coefficient Q_s is normally huge $\Longrightarrow$ initially the solution is close to the equilibrium one:

$$f = f_{eq}(1+\delta) \quad with \quad \delta = -\frac{x^5}{2Q_s f_{eq}^2}\frac{df_{eq}}{dx} \approx \frac{x^5}{2Q_s f_{eq}}$$

The equilibrium solution:

$$f_{eq} = \frac{1}{0.12}\left(\frac{x}{2\pi}\right)^{3/2} e^{-x}x^5, \qquad \delta = \frac{x^5}{2Q_s f_{eq}} = \frac{0.06}{Q_s}\left(\frac{x}{2\pi}\right)^{-3/2} e^{x}$$

This solution is valid till $\boldsymbol{\delta}$ remains small, $\delta \leq 1$.

The freezing temperature: termination of annihilation

The deviation from equilibrium becomes of order of unity, or $\delta = 1$, at the so-called freezing temperature T_{fr} or at x_{fr}:

$$x_{fr} \approx \ln Q_s + \frac{3}{2}\ln(\ln Q_s) - \frac{3}{2}\ln(2\pi) + \ln 0.06 \approx \ln Q_s + \frac{3}{2}\ln(\ln Q_s) - 5.7$$

Since $Q_s \gg 1$, then x_{fr} is also large, typically $x_{fr} \sim (10 - 100)$ depending upon the interaction strength.

After $\boldsymbol{x}$ becomes larger than $\boldsymbol{x_{fr}}$ $\Longrightarrow$ the kinetic equation with the initial condition $f = f_{fr}$ at $x = x_{fr}$ is simply integrated $\Longrightarrow$ the asymptotic result at $\boldsymbol{x} \rightarrow \infty$:

$$f(x) = \frac{f_{fr}}{1 + \frac{Q_s f_{fr}}{4}\left(\frac{1}{x_{fr}^4} - \frac{1}{x^4}\right)} \rightarrow \frac{4x_{fr}^4}{Q_s} = f_{fin}$$

Thus $\boldsymbol{f}$ tends to a constant value $\boldsymbol{f_{fin}}$, when $\boldsymbol{x} \gg \boldsymbol{x_{fr}}$ and $Q_s f_{fr}/(4x_{fr}^4) > 1$

Energy density of the X-particles

For $g_* = 100$, $\alpha = 0,01$, $\beta_{ann} = 10$, $m_R = 3 \times 10^{13}$ GeV, and $n_\gamma = 412/\text{cm}^3$

The present day energy density of the X-particles:

$$\varrho_X = M_X n_\gamma f_{fin} \approx 1.7 \times 10^8 \left(\frac{10^{10}\text{Gev}}{M_X} \right) \text{keV}/\text{cm}^3$$

The observed energy density of dark matter: $\varrho_{DM} \approx 1$ keV/cm^3.

- X-particles must have huge mass $M_X \gg m_R$ to make reasonable DM density.
- However, if $M_X > m_R$, then classical scalaron field can still create X-particles, but the probability of their production would be strongly suppressed $\Longrightarrow$ such LSP with the mass somewhat larger than m_R could successfully make the cosmological dark matter.

Scalaron decay into fermions or conformal scalars

If the bosons are coupled to curvature as $\xi R \phi^2$ with $\xi = 1/6 \Longrightarrow$
they are conformally invariant $\Longrightarrow$ are not produced if their mass is zero.

- The probability of production of both bosons and fermions $\sim \boldsymbol{m^2_{particle}}$
- In what follows we confine ourselves to consideration of fermions only.

The width of the scalaron decay into a pair of fermions:

$$\Gamma_f = \frac{m_R m_f^2}{48 m_{Pl}^2}$$

We assume:

- The mass of the LSP is considerably smaller than the masses of the other decay products, $\boldsymbol{m_X < m_f}$, at least as $\boldsymbol{m_X \lesssim 0.1 m_f}$.
- the direct production of X-particles by $\boldsymbol{R(t)}$ can be neglected.

In such a case LSPs are dominantly produced by the secondary reactions in the plasma, which was created by the scalaron production of heavier particles.

Kinetic equation for freezing of fermionic species

$$\frac{df}{dx} = -\frac{\alpha^2 \beta_{ann}}{\pi^3 g_*} \frac{n_{in}\, m_R m_f^2}{m_X^6} \frac{f^2 - f_{eq}^2}{x^5} = -Q_f \frac{f^2 - f_{eq}^2}{x^5}$$

$n_{in} = 0.09 g_s m_X^3$ is the initial number density of X-particles at $T \sim m_X$.

The asymptotic solution of this equation for large $x \gg x_{fr}$ with the initial condition $f(x_{fr}) = f_{fr}$ (in complete analogy with scalars):

$$f(x) = \frac{f_{fr}}{1 + \frac{Q_f f_{fr}}{4}\left(\frac{1}{x_{fr}^4} - \frac{1}{x^4}\right)} \to \frac{4x_{fr}^4}{Q_f} = f_{fin}$$

Here the freezing temperature is defined by

$$x_{fr} \approx \ln Q_f + \frac{3}{2}\ln(\ln Q_f) - \frac{3}{2}\ln(2\pi) + \ln 0.045 \approx \ln Q_f + \frac{3}{2}\ln(\ln Q_f) - 5.86$$

and the so called frozen value of $\boldsymbol{f}$ is equal to

$$f_{fr} = x_{fr}^5 / Q_f$$

The contemporary energy density of X-particle

$$\varrho_X = m_X n_\gamma \left(\frac{n_X}{n_{rel}}\right)_{now} = 7 \cdot 10^{-9} \frac{m_f^3}{m_X m_R} \mathrm{cm}^{-3}$$

- $n_\gamma \approx 412/\mathrm{cm}^3$, $n_{rel} \approx \varrho^{rel}/3T$ is the number density of relativistic particles
- $\alpha = 0.01$, $\beta_{ann} = 10$, $g_* = 100$, $Q_f \approx 1.7 \cdot 10^4$ and $\ln Q_f \approx 10$.
- If we take $m_f = 10^5$ GeV and $m_X = 10^4$ GeV, then $\varrho_X \ll \varrho_{DM}$.

This energy density should be close to the energy density of the cosmological dark matter, $\varrho_{DM} \approx 1$ keV/cm^3.

It can be easily achieved with $m_X \sim 10^6$ GeV and $m_f \sim 10^7$ GeV:

$$\varrho_X = 0.23 \left(\frac{m_f}{10^7\,\mathrm{GeV}}\right)^3 \left(\frac{10^6\,\mathrm{GeV}}{m_X}\right) \frac{\mathrm{keV}}{\mathrm{cm}^3}$$

Anomalous decay into gauge bosons

The coupling of the massless gauge bosons to gravity is determined by the anomaly in the trace of energy-momentum tensor of the gauge fields:

$$T^{\mu}_{\mu} = \frac{\beta^2 \alpha}{8\pi} G_{\mu\nu} G^{\mu\nu}$$

- α is the fine structure constant, $G^{\mu\nu}$ is the gauge field strength
- β is the first coefficient in perturbative expansion of the beta-function

Due to this anomaly massless gauge bosons are efficiently produced in cosmology:

- A. D. Dolgov: "Massless Particle Production By Conformal Plane Gravitation Field" (in Russian), Pisma Zh. Eksp. Teor. Fiz. **32** (1980) 673; "Conformal Anomaly and the Production of Massless Particles by a Conformally Flat Metric," Sov. Phys. JETP **54** (1981) 223; "Breaking of conformal invariance and electromagnetic field generation in the universe," Phys. Rev. D **48** (1993) 2499

In particular, this coupling leads to the decay of the curvature $R(t)$ into gauge bosons.

Particle production due to conformal anomaly

The decay width of particle production by curvature due to conformal anomaly

- D. Gorbunov and A. Tokareva, JCAP **1312** (2013) 021 [arXiv:1212.4466]

is to be compared with the decay widths into

minimally coupled massless scalars and fermions:

$$\Gamma_{anom} = \frac{\beta\alpha^2 N}{96\pi^2}\frac{m_R^3}{m_{Pl}^2} \iff \Gamma_s = \frac{m_R^3}{48 m_{Pl}^2} \iff \Gamma_f = \frac{m_R m_f^2}{48 m_{Pl}^2}$$

Γ_{anom} is suppressed by factor $(\beta\alpha^2 N)/(2\pi^2)$ in comparison with Γ_s, but still is much larger than the decay width into fermions Γ_f with e.g. mass $m_f \sim 10^5$GeV.

Another route of escape: $N = 4$ supersymmetry for which beta-function vanishes and the conformal anomaly is absent.

- M. F. Sohnius, "Introducing Supersymmetry," Phys. Rept. **128** (1985) 39.

$N = 4$ SUSY

$N = 4$ super Yang-Mills theories:

- are believed to be unrealistic because they do not allow to introduce chiral fermions, even if the symmetry is broken spontaneously.

Though spontaneous symmetry breaking is the most appealing way to deal with the theories with broken symmetries, it is not obligatory!

- The symmetry can be broken explicitly.
- It is possible to break the symmetry "by hand" introducing different masses to particles in the same multiplet.

This would allow to construct a phenomenologically acceptable model.

- Since the symmetry is broken by mass, the theory would remain renormalizable.

At higher energies, much larger than the particle masses, it would behave as $\boldsymbol{N = 4}$ super Yang-Mills theory and at this energy scale the trace anomaly would vanish.

$N = 1$ and $N = 2$ SUSY

$N = 1$ and $N = 2$ supersymmetric theories:

- phenomenologically acceptable and possess the so called conformal window $\Longrightarrow$ trace anomaly vanishes with a certain set of the multiplets.

Review:

- M. Chaichian, W. F. Chen and C. Montonen, "New superconformal field theories in four-dimensions and N=1 duality," Phys. Rept. **346** (2001) 89 [hep-th/0007240].

The conformal coupling of scalar fields to gravity postulated above indeed breaks supersymmetry but supersymmetry is broken anyhow and this kind of breaking does not lead to revival of the conformal anomaly.

Gravitational corrections to the trace anomaly $\Longrightarrow$
contribution $\sim R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$, $R_{\mu\nu}R^{\mu\nu}$, ...
This contribution does not lead to production of gauge bosons.

Higher loop gravitational corrections, even if result in gauge boson production, are strongly suppressed.

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- The canonical relation between temperature of matter and the cosmological time becomes $T^4 t = C_{R^2}$, while usually $T^2 t = C_{GR}$. Here $C_{GR} \sim m_{Pl}$ is a universal constant, while in R^2-cosmology C_{R^2} depends on the model and may be strongly different for the scalaron decay into bosons or fermions.

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- In the case of the scalaron decay into fermions or conformally coupled scalars the mass of LSP may be at the level of 10^3 TeV to form DM.
- The search for such dark matter particles in low background experiments looks presently feasible. If they are discovered, it would be an interesting confirmation of R^2 inflationary model.

The END

Thank You for Your Attention