Neutrino evolution in dense matter and electromagnetic field

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Lobanov A. E., Theoretical and Mathematical Physics, 2017, Vol. 192:1, 1000–1015
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- The fermions are combined in $SU(3)$-multiplets
- One-particle wave functions are elements of the representation space of the direct product of Poincaré group and $SU(3)$
- The procedure of quantization is well-defined
- The Fock space for the superposition of mass states can be constructed
- The probabilities of neutrino oscillations can be derived using standard S-matrix formalism
- The formulas for neutrino oscillations are in good agreement with those obtained in the phenomenological theory

An effective equation describing neutrino oscillations and its spin rotation in matter was obtained in *A. E. Lobanov*, Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, **59**, No. 11, 141, (2016).[Russ. Phys. J., **59**, No. 11, 1891, (2016)].

$$\left(i\gamma^{\mu}\partial_{\mu}\mathbb{I} - \mathbb{M} - \frac{1}{2}\gamma^{\alpha} f_{\alpha}^{(e)}(1+\gamma^5)\mathbb{P}^{(e)} - \frac{1}{2}\gamma_{\alpha} f^{\alpha(\mathrm{N})}(1+\gamma^5)\,\mathbb{I} \right) \Psi(x) = 0, \quad (2.1)$$

where $\mathbb{I}$ is the 3×3 identity matrix, $\mathbb{M}$ is the Hermitian mass matrix of the neutrino multiplet, $\mathbb{P}^{(e)}$ is the projector on the neutrino state with electron flavor. The potential

$$f^{\alpha(e)} = \sqrt{2} G_{\mathrm{F}} \left(j^{\alpha(e)} - \lambda^{\alpha(e)} \right) \quad (2.2)$$

determines the neutrino interaction with the electrons via charged currents, and the potential

$$f^{\alpha(\mathrm{N})} = \sqrt{2} G_{\mathrm{F}} \sum_{i=e,p,n} \left(j^{\alpha(i)} \left(T^{(i)} - 2Q^{(i)} \sin^2\theta_{\mathrm{W}} \right) - \lambda^{\alpha(i)} T^{(i)} \right) \quad (2.3)$$

determines the neutral current contribution.

Eq. (2.1) may be generalized in order to describe the neutrino direct interaction with the electromagnetic field

$$\Big(\mathrm{i}\gamma^{\mu}\partial_{\mu}\mathbb{I} - \mathbb{M} - \frac{1}{2}\gamma^{\alpha}f_{\alpha}^{(e)}(1+\gamma^{5})\mathbb{P}^{(e)} - \frac{1}{2}\gamma_{\alpha}f^{\alpha(\mathrm{N})}(1+\gamma^{5})\,\mathbb{I} - \\ - \frac{\mathrm{i}}{2}\mu_{0}F^{\mu\nu}\sigma_{\mu\nu}\mathbb{M} - \frac{\mathrm{i}}{2}F^{\mu\nu}\sigma_{\mu\nu}\mathbb{M}_{h} - \frac{\mathrm{i}}{2}{}^{\star}F^{\mu\nu}\sigma_{\mu\nu}\mathbb{M}_{ah}\Big)\Psi(x) = 0, \quad (3.1)$$

where $\mathbb{M}_h$, $\mathbb{M}_{ah}$ are the matrices of the transition magnetic and electric moments.

Chukhnova A. V., Lobanov A. E. Neutrino flavor oscillations and spin rotation in matter and electromagnetic field. arXiv:1906.09351

Quasi-classical evolution equation may be obtained (3.1), if we make the following substitution

$$\gamma^{\mu}\partial_{\mu} \Rightarrow \gamma^{\mu}\left(\frac{\partial\tau}{\partial x^{\mu}}\right)\frac{d}{d\tau} = \gamma^{\mu}u_{\mu}\frac{d}{d\tau}. \tag{4.1}$$

Then the neutrino evolution is characterized by the neutrino proper time τ, which is connected to the neutrino path length L by the relation

$$\tau = L/|\mathbf{u}|. \tag{4.2}$$

The quasi-classical neutrino evolution equation is as follows

$$\left(\mathrm{i}\frac{d}{d\tau}\mathbb{I}-\mathcal{F}\right)\Psi(\tau)=0, \tag{4.3}$$

where

$$\mathcal{F} = \mathbb{M}+\frac{1}{2}(f^{(e)}u)\mathbb{P}^{(e)}+\frac{1}{2}(f^{(\mathrm{N})}u)+\frac{1}{2}R_e\mathbb{P}^{(e)}\gamma^5\gamma^\sigma s_\sigma^{(e)}\gamma^\mu u_\mu+\frac{1}{2}R_{\mathrm{N}}\gamma^5\gamma^\sigma s_\sigma^{(\mathrm{N})}\gamma^\mu u_\mu- \\ -\mu_0\mathbb{M}\gamma^5\gamma^\mu\, {}^\star F_{\mu\nu}u^\nu-\mathbb{M}_h\gamma^5\gamma^\mu\, {}^\star F_{\mu\nu}u^\nu+\mathbb{M}_{ah}\gamma^5\gamma^\mu F_{\mu\nu}u^\nu. \tag{4.4}$$

When the electromagnetic fields and the potentials of interaction with matter are constant, the solution of Eq. (4.4) can be written with the use of the resolvent $U(\tau)=e^{-\mathrm{i}\mathcal{F}\tau}$.

The density matrices are defined by the relation

$$\rho(\tau)=\frac{1}{4u^0}U(\tau)\left(\gamma^\mu u_\mu+1\right)\left(1-\gamma^5\gamma_\mu s_0^\mu\right)\mathbb{P}_0\bar{U}(\tau). \tag{4.5}$$

The probabilities of the transitions between the states with definite flavor and helicity

$$W_{\alpha\to\beta}=\mathrm{Sp}\,\rho_\alpha(\tau)\rho_\beta^\dagger(\tau=0). \tag{4.6}$$

The probabilities of the transitions between the states with definite flavor and helicity may be obtained with the help of the Backer–Campbell–Hausdorff formula as a series

$$W_{\alpha\to\beta} = \frac{1}{8u^0}\mathrm{Sp}\left\{e^{-\mathrm{i}\tau\mathcal{F}}\mathcal{P}_0^{(\alpha)}e^{\mathrm{i}\tau\mathcal{F}}\mathcal{P}_0^{(\beta)}(\gamma^\mu u_\mu + 1)\gamma^0\right\} =$$
$$= \frac{1}{8u^0}\sum_{n=0}^{\infty}\frac{(-\mathrm{i}\tau)^n}{n!}\mathrm{Sp}\left\{D_n\mathcal{P}_0^{(\beta)}(\gamma^\mu u_\mu + 1)\gamma^0\right\}, \quad (4.7)$$

where

$$D_0 = \mathcal{P}_0^{(\beta)},\ D_1 = [\mathcal{F}, \mathcal{P}_0^{(\beta)}],\ D_2 = [\mathcal{F}, [\mathcal{F}, \mathcal{P}_0^{(\beta)}]]... \quad (4.8)$$

Here

$$\mathcal{P}_0^{\alpha} = \left(1 - \gamma^5\gamma_\mu s_0^\mu\right)\mathbb{P}_0^{\alpha}. \quad (4.9)$$

There are several interesting cases, when an explicit analytical solution of the evolution equation may be obtained

- The neutrino propagates in dense moving unpolarized matter.
- The neutrino propagates in constant electromagnetic field.
- The neutrino propagates in matter composed of neutrons only in electromagnetic field.

The transition probabilities in the two-flavor model are as follows

$$W_{\alpha\to\beta} = \frac{1+\xi_0\xi_0'}{2}\frac{1+\zeta_0\zeta_0'}{2}W_1 + \frac{1+\xi_0\xi_0'}{2}\frac{1-\zeta_0\zeta_0'}{2}W_2 + \\ + \frac{1-\xi_0\xi_0'}{2}\frac{1+\zeta_0\zeta_0'}{2}W_3 + \frac{1-\xi_0\xi_0'}{2}\frac{1-\zeta_0\zeta_0'}{2}W_4. \quad (5.1)$$

Here for neutrino with initial electron flavor we choose $\xi_0 = 1$, otherwise $\xi_0 = -1$.
For neutrino with final electron flavor we choose $\xi_0' = 1$, otherwise $\xi_0' = -1$.
$\zeta_0 = -1$ corresponds to the left-handed neutrino in the initial state,
$\zeta_0 = +1$ corresponds to the right-handed neutrino in the initial state.
$\zeta_0' = -1$ corresponds to the left-handed neutrino in the final state,
$\zeta_0' = +1$ corresponds to the right-handed neutrino in the final state.

$$W_1 = \frac{1}{2}\Bigg(\frac{1}{2}(1-\zeta_0(\bar{s}s_{sp}))^2(1-S_{+1}^2X_{+1}^2)+\frac{1}{2}(1+\zeta_0(\bar{s}s_{sp}))^2(1-S_{-1}^2X_{-1}^2)+ \\ +(1-(\bar{s}s_{sp})^2)(C_{+1}C_{-1}+S_{+1}S_{-1}Y_{+1}Y_{-1})\cos(\omega\tau)+ \\ +\xi_0(1-(\bar{s}s_{sp})^2)(S_{-1}Y_{-1}C_{+1}-C_{-1}S_{+1}Y_{+1})\sin(\omega\tau)\Bigg),$$

$$W_2 = \frac{1}{2}\Bigg(\frac{1}{2}(1-(\bar{s}s_{sp})^2)(2-S_{+1}^2X_{+1}^2-S_{-1}^2X_{-1}^2)- \\ -(1-(\bar{s}s_{sp})^2)(C_{+1}C_{-1}+S_{+1}S_{-1}Y_{+1}Y_{-1})\cos(\omega\tau)- \\ -\xi_0(1-(\bar{s}s_{sp})^2)(S_{-1}Y_{-1}C_{+1}-C_{-1}S_{+1}Y_{+})\sin(\omega\tau)\Bigg),$$

$$W_3 = \frac{1}{2}\Bigg(\frac{1}{2}(1-\zeta_0(\bar{s}s_{sp}))^2S_{+1}^2X_{+1}^2+\frac{1}{2}(1+\zeta_0(\bar{s}s_{sp}))^2S_{-1}^2X_{-1}^2+ \\ +(1-(\bar{s}s_{sp})^2)S_{+1}S_{-1}X_{+1}X_{-1}\cos(\omega\tau)\Bigg),$$

$$W_4 = \frac{1}{2}\Bigg(\frac{1}{2}(1-(\bar{s}s_{sp})^2)(S_{+1}^2X_{+1}^2+S_{-1}^2X_{-1}^2)- \\ -(1-(\bar{s}s_{sp})^2)S_{+1}S_{-1}X_{+1}X_{-1}\cos(\omega\tau)\Bigg).$$

Lobanov A.E., Chukhnova A.E. Vestn. MGU. Fiz. Astron. **58**, No. 5, 22–26 (2017) [Mosc. Univ. Phys. Bull. **72**, No. 5, 454–459 (2017)]

$$C_{\pm 1} = \cos\big(\tau Z_{\pm 1}/2\big), \qquad S_{\pm 1} = \sin\big(\tau Z_{\pm 1}/2\big),$$

For moving unpolarized matter

$$R = \sqrt{(fu)^2 - f^2}, \quad s^\mu = \frac{u^\mu (fu) - f^\mu}{\sqrt{(fu)^2 - f^2}}. \tag{5.3}$$

$$\begin{aligned} Y_\zeta &= \frac{1}{Z_\zeta}\Big((m_2 - m_1)\cos 2\theta - \big((fu) - \zeta R\big)/2\Big), \\ X_\zeta &= \frac{1}{Z_\zeta}\Big((m_2 - m_1)\sin 2\theta\Big), \\ Z_\zeta &= \sqrt{\Big(\big((fu) - \zeta R\big)/2 - (m_2 - m_1)\cos 2\theta\Big)^2 + \Big((m_2 - m_1)\sin 2\theta\Big)^2}. \end{aligned} \tag{5.4}$$

$$X_\zeta = \sin 2\theta_\zeta, \qquad Y_\zeta = \cos 2\theta_\zeta \tag{5.5}$$

If we assume that $u^0 \approx |\mathbf{u}|$, then for the medium at rest we have

$$\begin{aligned} Y_+ &= \cos 2\theta, & X_+ &= \sin 2\theta, \\ Y_- &= \cos 2\theta_{eff}\,, & X_- &= \sin 2\theta_{eff}\,, \end{aligned} \tag{5.6}$$

where θ_{eff} is the effective mixing angle in the medium.
While the medium density varies, $\cos\theta_{eff}$ changes its sign. It means that the Mikheev–Smirnov–Wolfenstein resonance occurs
S. P. Mikheev and A. Yu. Smirnov., Yad. Phys. 42, 1441–1448 (1985) [Sov. J. Nucl. Phys. 42, 913 (1985)]

In electromagnetic field

$$\omega = \mu_0(m_2 + m_1)N,$$
$$\bar{s}^{\mu} = -{}^{\star}F^{\mu\nu}u_{\nu}/N, \qquad N = \sqrt{u_{\mu}\,{}^{\star}F^{\mu\alpha}\,{}^{\star}F_{\alpha\nu}u^{\nu}}. \tag{6.1}$$

$$\begin{aligned}
Y_{\zeta} &= \frac{1}{Z_{\zeta}}\Big((m_2 - m_1)\big(1 - \zeta\mu_0 N\big)\cos 2\theta + \zeta\mu_1 N\big(m_2 + m_1\big)\sin 2\theta\Big),\\
X_{\zeta} &= \frac{1}{Z_{\zeta}}\Big((m_2 - m_1)\big(1 - \zeta\mu_0 N\big)\sin 2\theta - \zeta\mu_1 N\big(m_2 + m_1\big)\cos 2\theta\Big),\\
Z_{\zeta} &= \Big\{\Big((m_2 - m_1)\big(1 - \zeta\mu_0 N\big)\Big)^2 + \Big((m_2 + m_1)\mu_1 N\Big)^2\Big\}^{1/2}
\end{aligned} \tag{6.2}$$

$$X_{\zeta} = \sin 2\theta_{\zeta}, \qquad Y_{\zeta} = \cos 2\theta_{\zeta} \tag{6.3}$$

A resonance behavior of the spin-flavor transition probabilities in electromagnetic field is due to the neutrino transition magnetic moments! The resonance condition is $u_0\mu_0 B \approx 1$.

Even in the two flavor model a detailed analysis of the results is rather complicated. For clarity, we consider the spin-flip probability $W = W_2 + W_4$. Because of the correlations with the flavor transitions, this probability is defined by the expression

$$W = \frac{\mathcal{A}}{2}\ \left(A_1(1-\cos\omega_1\tau)+A_2(1-\cos\omega_2\tau)+A_3(1-\cos\omega_3\tau)+A_4(1-\cos\omega_4\tau)\right), \tag{7.1}$$

where the total amplitude of the spin-flip transitions is as follows

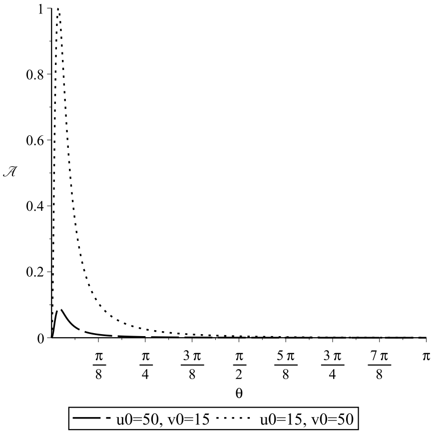
$$\mathcal{A} = 1-(ss_{sp})^2. \tag{7.2}$$

For neutrino in matter

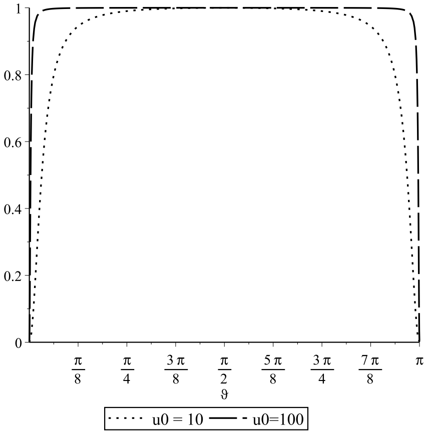
$$\mathcal{A} = \frac{(v_0^2-1)\sin^2\vartheta}{(v_0 u_0 - \sqrt{u_0^2-1}\sqrt{v_0^2-1}\cos\vartheta)^2-1}, \tag{7.3}$$

For neutrino in electromagnetic field

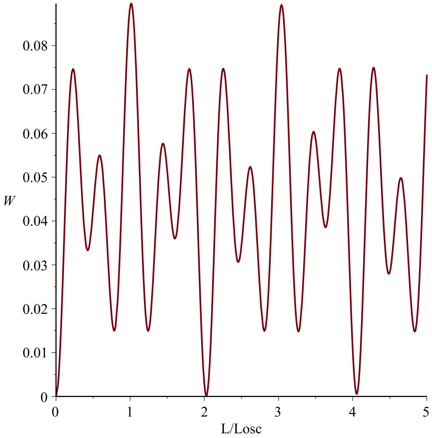
$$\mathcal{A} = \frac{u_0^2\sin^2\vartheta_M}{u_0^2-|\mathbf{u}|^2\cos^2\vartheta_M}, \tag{7.4}$$



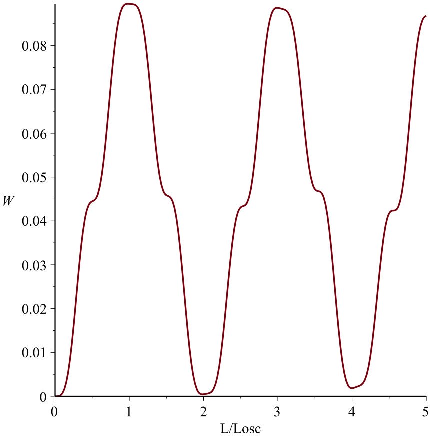
Total amplitude of spin-flip transitions in matter



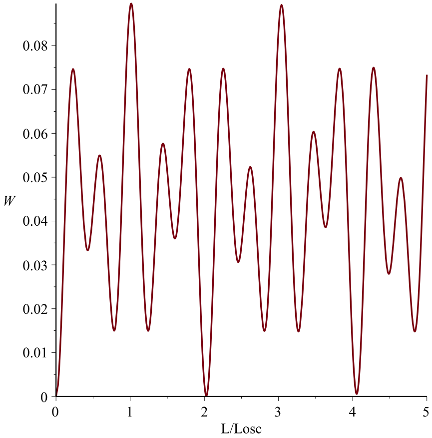
Total amplitude of spin-flip transitions in magnetic field



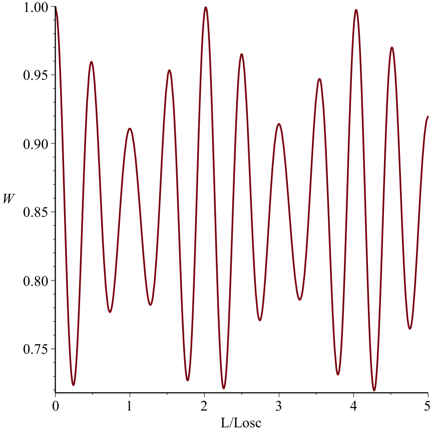
Spin-flip probability for $\xi_0 = 1$,
$u_0 = 50$, $v_0 = 15$, $k = 10$,
$\cos\vartheta = \cos\vartheta_{max}$



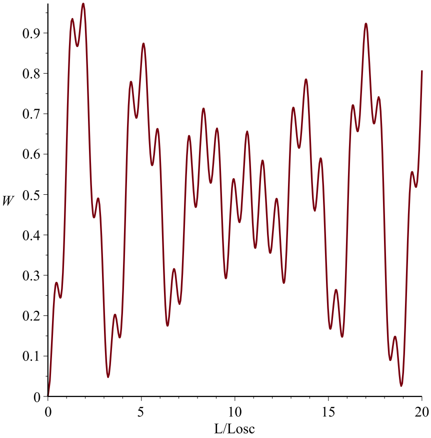
Spin-flip probability for $\xi_0 = -1$,
$u_0 = 50$, $v_0 = 15$, $k = 10$,
$\cos\vartheta = \cos\vartheta_{max}$



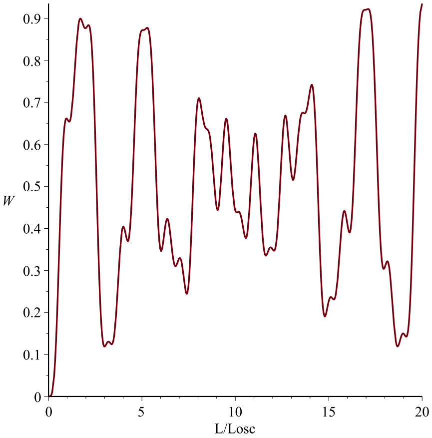
Spin-flip probability for $\xi_0 = 1$, $u_0 = 50$, $v_0 = 15$, $k = 10$, $\cos\vartheta = \cos\vartheta_{max}$



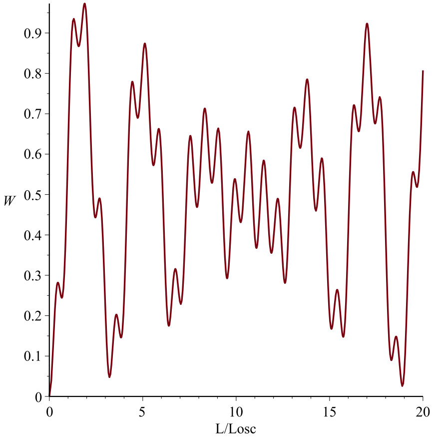
$\nu_{e,L} - \nu_{e,L}$ transition probability for $u_0 = 50$, $v_0 = 15$, $k = 10$, $\cos\vartheta = \cos\vartheta_{max}$



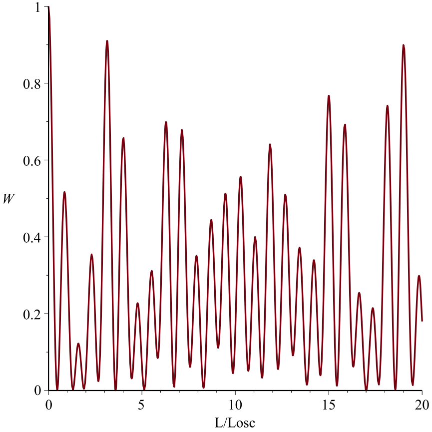
Spin-flip probability for $\xi_0 = 1$, $u_0 = 15$, $v_0 = 50$, $k = 10$, $\cos\vartheta = \cos\vartheta_{max}$



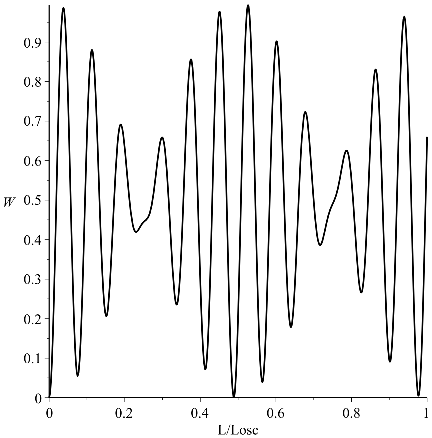
Spin-flip probability for $\xi_0 = -1$, $u_0 = 15$, $v_0 = 50$, $k = 10$, $\cos\vartheta = \cos\vartheta_{max}$



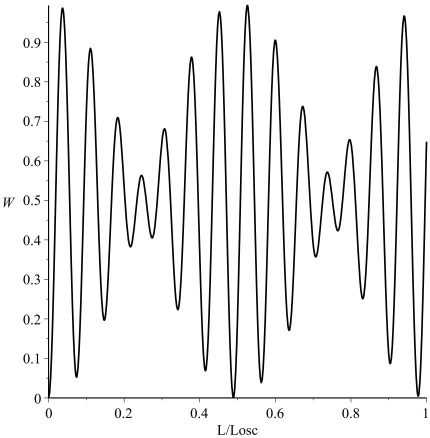
Spin-flip probability for $\xi_0 = 1$, $u_0 = 15$, $v_0 = 50$, $k = 10$, $\cos\vartheta = \cos\vartheta_{max}$



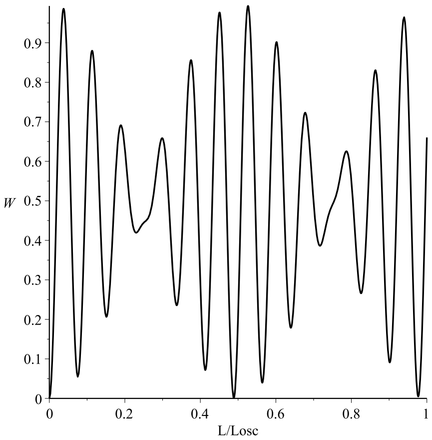
$\nu_{e,L} - \nu_{e,L}$ transition probability for $u_0 = 15$, $v_0 = 50$, $k = 10$, $\cos\vartheta = \cos\vartheta_{max}$



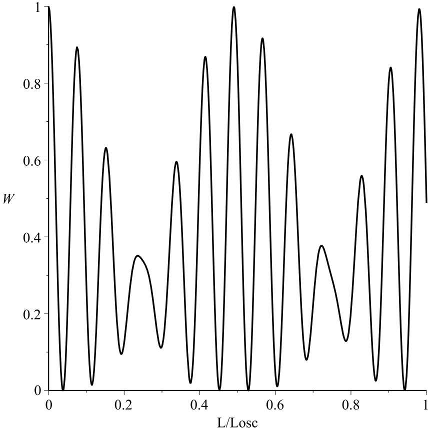
Spin-flip probability in magnetic field for $\xi_0 = 1,\ \mu_0 u_0 B = 1,\ k_m = 13.3$



Spin-flip probability in magnetic field for $\xi_0 = -1,\ \mu_0 u_0 B = 1,\ k_m = 13.3$



Spin-flip probability in magnetic field for $\xi_0 = 1$, $\mu_0 u_0 B = 1$, $k_m = 13.3$



$\nu_{e,L} - \nu_{e,L}$ transition probability in magnetic field for $\mu_0 u_0 B = 1$, $k_m = 13.3$

In the cases considered before, it is possible to write the explicit form of the solutions, as we are able to find spin integrals of motion for the neutrino evolution equation. This is exactly the reason, why the solutions take similar form. However, there is one more physically interesting case, when the explicit form of the solutions may be written.
Consider neutrino moving in dense matter composed of neutrons only. If the magnetic field is present, but its value is not enough for the MSW-like resonance to occur, we can neglect the effect of the transition neutrino moments. Then the evolution equation takes the form

$$\left(\mathrm{i}\gamma^{\mu}\partial_{\mu}\mathbb{I} - \mathbb{M} - \frac{1}{2}\gamma_{\alpha}f^{\alpha(\mathrm{N})}(1+\gamma^{5})\,\mathbb{I} - \frac{\mathrm{i}}{2}\mu_{0}F^{\mu\nu}\sigma_{\mu\nu}\mathbb{M}\right)\Psi(x) = 0, \quad (8.1)$$

We can find some flavor integrals of motion. Namely, neutrino propagates in its mass eigenstates. Like in the previous case, we can obtain the explicit form of the neutrino wave function in the quasi-classical approximation. Then we also obtain the probabilities of the spin-flavor transitions in the two flavor model.

- We obtain the equation for neutrino evolution taking into account neutrino interaction with matter and with external electromagnetic field.
- This equation has no purely spin or purely flavor integrals of motion. Thus, we introduce the concept of spin-flavor states of the neutrino, which can be described by eigenvectors of a spin-flavor integral of motion.
- For ultra-relativistic particles the quasi-classical approximation of this evolution equation is valid.
- In the case, when the external conditions do not depend on the coordinates of the event space, we obtain a formal solution of this equation.
- Using the Backer–Campbell–Hausdorff formula, we develop the general method of calculating the probabilities of the transitions between arbitrary neutrino spin-flavor states.
- In particular cases we find analytical solutions of the quasi-classical evolution equation and the explicit expressions for the spin-flavor transition probabilities of the neutrino.

- We study in detail the neutrino spin rotation in moving dense matter and in the electromagnetic field. In the latter case we take into account the transition magnetic moments.
- We show that the wave functions of neutrino in external conditions cannot in general case be considered as superposition of the wave functions of the mass eigenstates.
- We discover the resonance behaviour of the probabilities for neutrino in magnetic field due to the transition magnetic moment.
- We study the neutrino propagation in neutron medium in the presence of electromagnetic field.
- For neutrino in neutron medium in electromagnetic field we discover no resonance behavior.
- We discover that the resonance behavior of the transition probabilities is possible only when the neutrino state cannot be described as a superposition of the mass eigenstates.

Thank you for your attention!