

Cornell potential for anisotropic QGP with non-zero chemical potential

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Motivation

Holography needs

- **Natural** theory with metric as solution of EOM.
- Description of anisotropic QGP.
- Non-zero chemical potential $\mu \neq 0$ (NICA).

I.Ya. Aref'eva “**Holography for HIC at LHC and NICA**”
EPJ Web Conf. **164** 01014 (2017)

Action in Einstein frame, metric ansatz

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \times$$
$$\times \left[R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$
$$A_\mu^{(1)} = A_t(z) \delta_\mu^0 \quad F^{(2)} = q dy^1 \wedge dy^2 \quad \phi = \phi(z)$$

$$ds^2 = \frac{b(z)}{z^2} \left[-g(z) dt^2 + dx^2 + z^{2-\frac{2}{\nu}} (dy_1^2 + dy_2^2) + \frac{dz^2}{g(z)} \right]$$

Boundary conditions:

$$g(0) = 1, \quad g(z_h) = 0 \quad A_t(0) = \mu, \quad A_t(z_h) = 0 \quad \phi(z_h) = 0$$

I.A., K.R. JHEP 1805 206 (2018)

Solution for anisotropic metric ansatz

$b(z) = \exp(cz^2/2) \Rightarrow$ heavy quarks

$$f_1 = z^{-2+\frac{2}{\nu}} \quad A_t(z) = \mu \frac{e^{-\frac{cz^2}{4}} - e^{-\frac{cz_h^2}{4}}}{1 - e^{-\frac{cz_h^2}{4}}}$$

$$g(z) = 1 - \frac{z^{2+\frac{2}{\nu}}}{z_h^{2+\frac{2}{\nu}}} \frac{\mathfrak{G}\left(\frac{3}{4}cz^2\right)}{\mathfrak{G}\left(\frac{3}{4}cz_h^2\right)} - \frac{\mu^2 c z^{2+\frac{2}{\nu}} e^{\frac{cz_h^2}{2}} \mathfrak{G}(cz^2)}{4 \left(1 - e^{-\frac{cz_h^2}{4}}\right)^2} \left(1 - \frac{\mathfrak{G}(cz_h^2)}{\mathfrak{G}(cz^2)} \frac{\mathfrak{G}\left(\frac{3}{4}cz^2\right)}{\mathfrak{G}\left(\frac{3}{4}cz_h^2\right)}\right),$$

$$\mathfrak{G}(x) = x^{-1-\frac{1}{\nu}} \gamma\left(1 + \frac{1}{\nu}, x\right) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!(1+n+\frac{1}{\nu})}$$

Solution for anisotropic metric ansatz

$$\begin{aligned}\phi = & \frac{1}{2\sqrt{2}\nu} \left\{ \sqrt{3c^2\nu^2 z^4 - 18c\nu^2 z^2 + 8(\nu - 1)} - \right. \\ & - \sqrt{3c^2\nu^2 z_h^4 - 18c\nu^2 z_h^2 + 8(\nu - 1)} + 2\sqrt{2(\nu - 1)} \ln \left(\frac{z^2}{z_h^2} \right) \\ & - 3\sqrt{3}\nu \ln \left(\frac{\sqrt{3c^2\nu^2 z^4 - 18c\nu^2 z^2 + 8(\nu - 1)} - \sqrt{3}\nu(3 - cz^2)}{\sqrt{3c^2\nu^2 z_h^4 - 18c\nu^2 z_h^2 + 8(\nu - 1)} - \sqrt{3}\nu(3 - cz_h^2)} \right) - \\ & - 2\sqrt{2(\nu - 1)} \times \\ & \left. \times \ln \left(\frac{9c\nu^2 z^2 - 8(\nu - 1) - \sqrt{2(\nu - 1)} \sqrt{3c^2\nu^2 z^4 - 18c\nu^2 z^2 + 8(\nu - 1)}}{9c\nu^2 z_h^2 - 8(\nu - 1) - \sqrt{2(\nu - 1)} \sqrt{3c^2\nu^2 z_h^4 - 18c\nu^2 z_h^2 + 8(\nu - 1)}} \right) \right\}\end{aligned}$$

Stable solution $\Rightarrow c < 0$

Confinement-deconfinement in hot anisotropic media

$$DW_x \equiv cz + \frac{1}{\nu z} \sqrt{\frac{2}{3}} \phi' + \frac{g'}{2g} - \frac{2}{z} \Big|_{z=z_{DW_x}} = 0$$

$$DW_y \equiv cz + \frac{1}{\nu z} \sqrt{\frac{2}{3}} \phi' + \frac{g'}{2g} - \frac{\nu + 1}{\nu z} \Big|_{z=z_{DW_y}} = 0$$

$$DW_\theta \equiv cz + \frac{1}{\nu z} \sqrt{\frac{2}{3}} \phi' + \frac{g'}{2g} - \frac{2}{z} + \left(1 - \frac{1}{\nu}\right) \frac{z^{1-\frac{2}{z}} \sin^2(\theta)}{\cos^2(\theta) + z^{2-\frac{2}{\nu}} \sin^2(\theta)} \Big|_{z=z_{DW_\theta}} = 0$$

I.A, K.R., P.S. PLB **792** 470 (2019)

Confinement/deconfinement phase transition for $\nu = 1$

$$b(z) = \exp(cz^2/2) \quad \mathbf{c = ?}$$

Heavy Quarks:

$\nu = 1$ 1-st order phase transition $T(\mu = 0) = 0.6 \text{ GeV}$

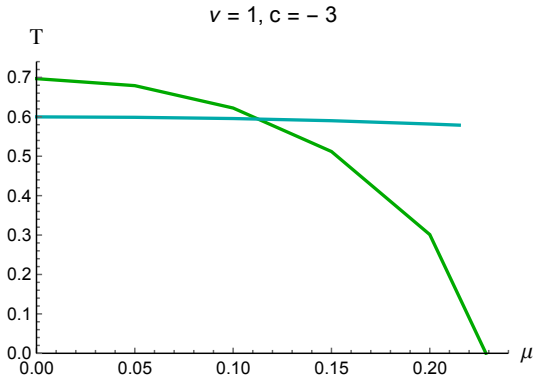
Y. Yang, P.-H. Yuan JHEP 1512 161 (2015)

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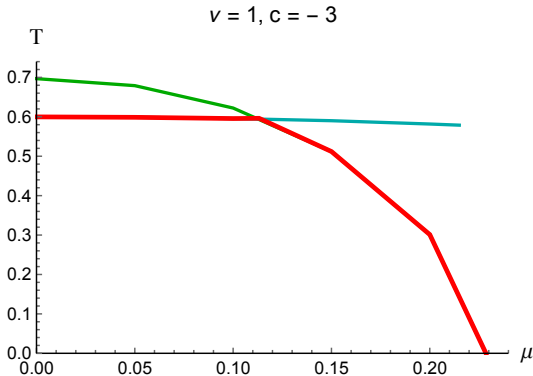


Confinement/deconfinement phase transition for $\nu = 1$

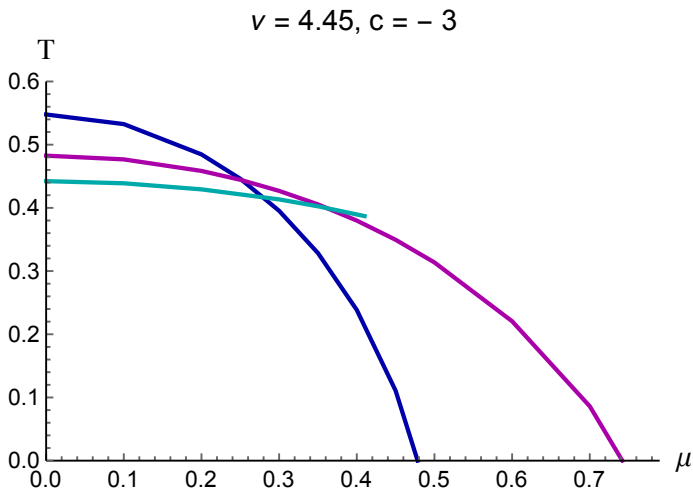
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Y. Yang, P.-H. Yuan JHEP 1512 161 (2015)

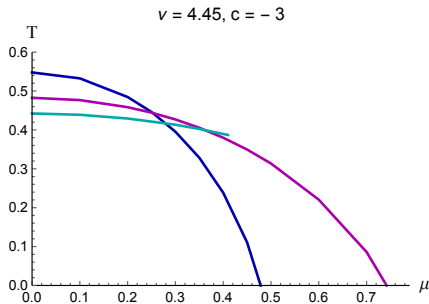
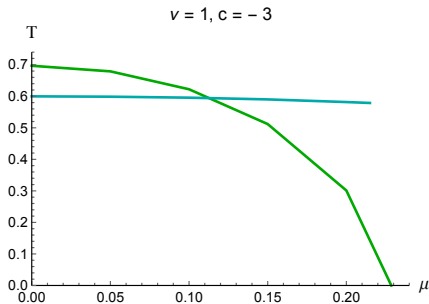


Confinement/deconfinement phase transition for $\nu = 4.45$



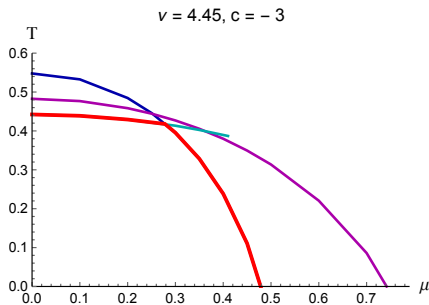
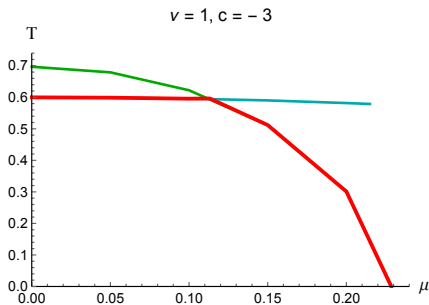
Confinement/deconfinement phase transition for $\nu = 4.45$

Anisotropy: $T \downarrow$, $\mu_{max} \uparrow$



Confinement/deconfinement phase transition for $\nu = 4.45$

Anisotropy: $T \downarrow$, $\mu_{max} \uparrow$



String tension

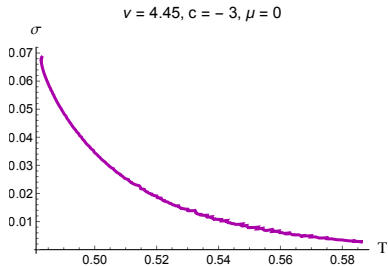
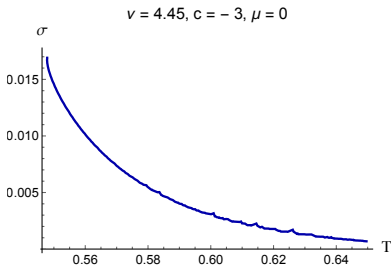
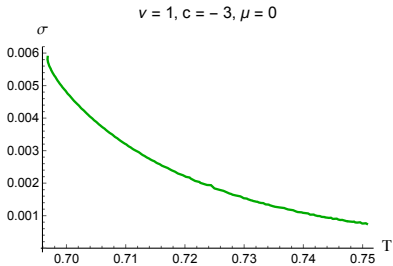
$$\sigma_{DW} = z^{-2} e^{\frac{cz^2}{2} + \sqrt{\frac{2}{3}}\phi(z_{DW})} \sqrt{g(z_{DW}) \left(\cos^2(\theta) + z^{2+\frac{2}{\nu}} \sin^2(\theta) \right)}$$

$$\sigma_{DW_x} = z^{-2} e^{\frac{cz^2}{2} + \sqrt{\frac{2}{3}}\phi(z_{DW_x})} \sqrt{g(z_{DW_x})}$$

$$\sigma_{DW_y} = z^{-1-\frac{1}{\nu}} e^{\frac{cz^2}{2} + \sqrt{\frac{2}{3}}\phi(z_{DW_y})} \sqrt{g(z_{DW_y})}$$

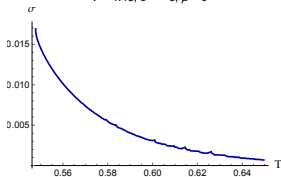
$$z_{DW} = Z_{min} : \sigma'|_{z=z_{min}} = 0$$

String tension

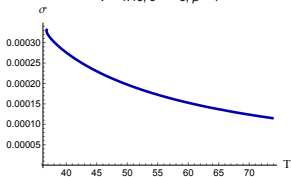


String tension

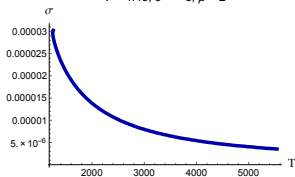
$v = 4.45, c = -3, \mu = 0$



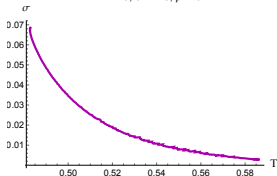
$v = 4.45, c = -3, \mu = 1$



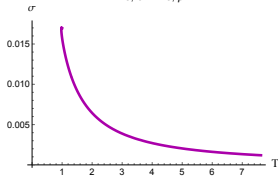
$v = 4.45, c = -3, \mu = 2$



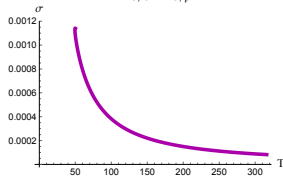
$v = 4.45, c = -3, \mu = 0$



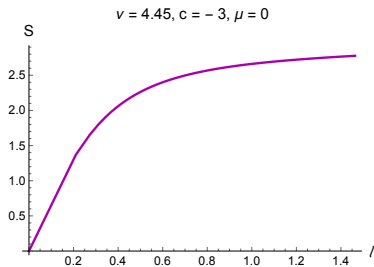
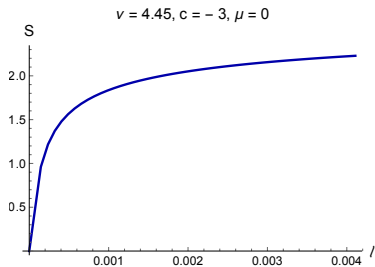
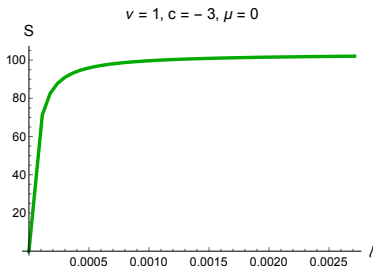
$v = 4.45, c = -3, \mu = 1$



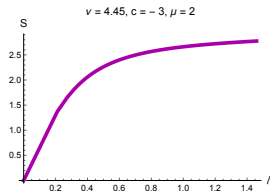
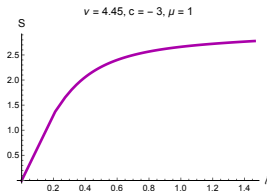
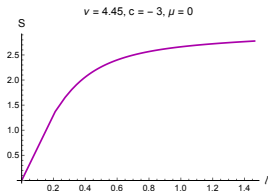
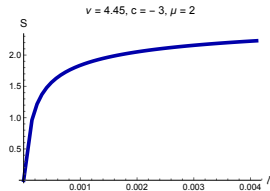
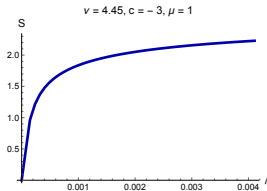
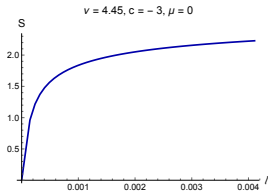
$v = 4.45, c = -3, \mu = 2$



Cornell potential



Cornell potential



Conclusions

Anisotropy changes

- phase diagram for confinement/deconfinement phase transition,
- string tension,
- Cornell potential.

Quark mass influences on

- phase diagram for confinement/deconfinement phase transition,
- string tension,
- Cornell potential.

Chemical potential influences on

- string tension,
- coefficients of the Cornell potential for heavy quarks (weakly).

To do

- Light quarks $b(z) = \exp(-a \ln(cz^2 + 1))$.
- Anisotropy for strong magnetic field.
- Shock waves.
- RG-flow.

Thank you
for your attention

Parameters

Holographic model \leftrightarrow $\beta(E)$ -function (HRG)

L — scale

ν — anisotropy

c — quark mass

μ — chemical potential