Palatini non-minimal gravity theories

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Modified gravity theories

Einstein gravity was positively tested in many aspects and situations, especially by recent LIGO-VIRGO probes. But several insolved questions:

- Inflation
- Dark energy
- Dark matter (may be)
- Quantum UV incompleteness
- Possibility of alternative interpretation of crucial tests

stimulate search of viable modifications of General Relativity (GR) not contradicting to existing tests. These vary from exotic (aether gravity, large extra dimensions) to more traditional, like f(R) gravity, higher curvature gravity, metric-affine gravity and scalar-tensor theories. The latter were extensively studied during the past decade and became the models of the first choice.

Choosing between alternative theories

- f(R) may solve inflation problem, but if $f'' \neq 0$, it reduces to a scalar-tensor model with a potential (Brans-Dicke with $\omega = 0$) which is simpler to manipulate
- Squares of Ricci, Riemann or Weyl tensors generate incurable ghosts
- Meanwhile, to date, several classes of scalar-tensor theories are known, which include phenomenologically attractive higher curvature terms and do not contain ghosts (Horndeski, DHOST).
- Unfortunately, a substantial part of them are refuted by the unprecedented accuracy of measuring the speed of gravitational waves in the recently detected double neutron star merging
- But the above results were found in the traditional metric formalism, assuming metric compatible Levi-Civita connection

Connection as independent variable

- A supergravity lesson teaches us that an *ad hoc* choice of the connection may lead to obstructions to get a consistent theory (Bouhdahl restriction on gravitino), while passing to the dynamical connection which, together with metric, must be obtained form a variational principle may solve the problem
- In the supergravity case, the dynamical (Palatini) derivation of the connection leads to (non-propagating) torsion which has to be added to Levi-Civita metric-compatible connection
- In purely bosonic scalar-tensor theories similar procedure is likely to generate (non-propagated) non-metricity, while torsion can be consistently set to zero (up to subtleties to be discussed later)
- Palatini variation of non-minimally coupled scalar-tensor models generically leads to different theories and generates new options for ghost-free models, while some of Horndeski class ghost-free models may contain ghosts in the Palatini version

When Palatini and metric verison are different theories

- It is known that Maxwell theory in the second and first order (independent A_μ and $F_{\mu\nu}$) formalisms lead to equivalent theories both at the classical and quantum levels
- Vacuum Einstein enjoys the same property, though with an additional requirement of vanishing torsion
- Palatini f(R) theory reduces to Brans-Dicke scalar-tensor theory with $\omega = -3/2$ contrary to $\omega = 0$ in the metric formalism
- Setting torsion zero in the Palatini version of vacuum Einstein gravity may be attributed to *projective invariance* of Einstein action under the transformation

$$\Gamma^\lambda_{\mu\nu} \to \Gamma^\lambda_{\mu\nu} + \delta^\lambda_\nu U_\mu$$

  so that this setting may be considered as the gauge choice
- Recently it was argued (Aoki and Shimada 2019) that projective invariance may also serve an indication of absence of the Ostrogradski ghosts (not proved generally)

Horndeski and viable Horndeski actions in Palatini formalism

- Horndeski class (H) significantly contracted by the gravity speed test to a subclass of phenomenologically viable models 'VH' (also known as 'KGB' – kinetical gravity brading)
- Variation of Horndeski actions via Palatini was recently explored by M. Volkov showing that (again with assumed zero torsion) the subclass VH remains the same as a whole, while in each particular theory Palatini variation leads to a different theory, though still within VH
- Palatini variation of Horndeski actions outside the VH class leads to theories of different type, namely, either again within H-class, or belonging to DHOST (containing higher order equations of degenerate type), or to theories with incurable ghosts
- Meanwhile some ghostly actions in the metric formalism can lead to ghost-free Palatini theories (Gal'tsov and Zhidkova, 2018), we will consider one such case in more detail below

Torsion and non-metricity

Vacuum General Relativity uses Levi-Civita connection (Christoffel symbols), rigidly related to the metric

$$\left\{ {\lambda \atop \mu\nu} \right\} = \frac{1}{2} g^{\lambda\tau}(g_{\nu\tau,\mu} + g_{\mu\tau,\nu} - g_{\mu\nu,\tau})$$

Meanwhile, differential geometry offers more general connection including torsion and non-metricity:

$$\Gamma_{\mu\nu}{}^{\lambda} = \left\{ {\lambda \atop \mu\nu} \right\} + L_{\mu\nu}{}^{\lambda} + K_{\mu\nu}{}^{\lambda}$$

If one impose by hand metricity $\nabla_\lambda g_{\mu\nu} = 0$ and no-torsion $\Gamma_{\mu\nu}{}^{\lambda} = \Gamma_{\nu\mu}{}^{\lambda}$ conditions, one can meet contradictions with matter equations, the well-known example being the Bouchdal constraint for spin $s = 3/2$. In this case the tetrad and the spin connection must be used. Consistent coupling can be found considering connection and the tetrad as independent variables and using Palatini variation in the action $S_{EH}(\omega, e) + S_{3/2}(\psi, \omega, e)$

Supergravity hint

The resulting theory is supergravity, in which the spin connection depends on gravitino $\omega = \omega(e, \psi)$ and contains torsion, while non-metricity remains zero. At the same time, the scalar, Dirac and vector fields can be consistently coupled to Einstein gravity without torsion and non-metricity.

The lesson from this is that one should use dynamics of a theory to learn which connection is more adequate, rather than impose it by hand. If one explores vacuum Einstein gravity in the Palatini formalism one finds that torsion, though is not forbidden, can be consistently set zero. Similarly for minimally coupled scalar and vector theories. But various theories with non-minimal coupling to curvature no more share this property. General features are the following

1) In absence of fermions, torsion can be consistently set to zero
2) Non-metricity is often required, once non-minimal coupling of scalar and vector fields is introduced.

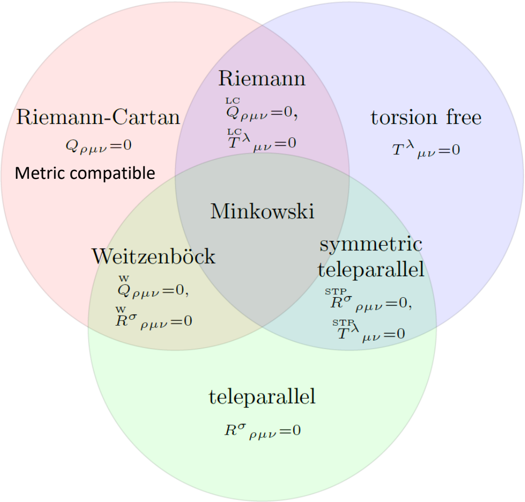

Рис.: Various combination of curvature, torsion and non-metricity

Kinetically coupled scalar

The problem of Ostrogradski ghosts is typical for scalar-tensor gravity with non-minimal kinetic coupling to curvature via the derivatives $\phi_\mu = \partial_\mu \phi$. The simplest coupling of this kind is (Amendola:1993)

$$R_{\mu\nu}\phi^\mu\phi^\nu$$

This term, in metric approach generates ghosts (though $G_{\mu\nu}\phi^\mu\phi^\nu$ with Einstein tensor does not). But as we will see, $R_{\mu\nu}\phi^\mu\phi^\nu$ in the Palatini formalism leads to consistent theory. Even simpler Platatini theory contains this term together with

$$R\phi_\mu\phi^\mu$$

but in different combination than $G_{\mu\nu}$. Torsion can still be zero, but non-metricity is present (non-propagating).
1) It is disformmaly dual to minimally coupled theory
2) Disformal duality relates singular metrics to non-singular ones

Disformal transformations

In many theories involving scalar fields two metric naturally coexist related by conformal transformations, a typical example being the string frame and the Einstein frame in string theory. In theories with non-minimal coupling of scalar field to curvature (such as Horndeski models) two frames usually are related by the *disformal* transformations

$$\hat{g}_{\mu\nu} = \alpha(\phi, \psi) g_{\mu\nu} + \beta(\phi, \psi) \phi_{,\mu} \phi_{,\nu}, \tag{1}$$

where $\psi = \phi_{,\alpha}\phi^{,\alpha}$. These were introduced by Bekenstein in 1992 on the basis of the Finsler geometry, but further reappeared in various contexts. In contrast to conformal, the disformal transformations may change the causal structure of spacetime, so additional conditions must be imposed on the functions involved. Applications include inflation, cosmology with varying speed of light, dark energy, screening mechanism, dark matter and other problems.

The disformal transformations are *non-local* but still can be invertible, in which case the respective theories are expected to be classically equivalent. A restricted class of disformal transformations leaves invariant the Horndeski set of non-minimal scalar-tensor theories in the second-order formalism. Another case is manifestly *non-invertible* disformal trabsformation in certain theories, such as mimetic gravity by Chamseddine and Mukhanov or more general DHOST theories (Langlois 2018). (For reviews of scalar-tensor theories see Capozziello and Faraoni (2010), Nojiri, Odintsov and Oikonomou (2017), Harko and Lobo (2019)).

Disformal duality may serve as a tool opening the way to move metric singularities of one theory to the pure scalar sector of the partner theory ensuring the *metric* of the latter non-singular (Gal'tsov and Zhidkova 2018)

Derivatively coupled scalar: metric formalism

Consider first the non-minimal action with two independent couplings of the derivatives of the scalar field $\phi_\mu \equiv \phi_{,\mu}$ to the Ricci tensor and the Ricci scalar which has the long history (Amendola:1993, Capozziello:1999, Sushkov:2009, Granda:2010,...)

$$S = \int d^4x\sqrt{-g}\,[R - (g_{\mu\nu} + \kappa_1 g_{\mu\nu} R + \kappa_2 R_{\mu\nu})\,\phi^\mu\phi^\nu]\,, \qquad (2)$$

where $\phi^\mu = \phi_\nu g^{\mu\nu}$. Einstein tensor $G_{\mu\nu}$ can be represented as $G_{\mu\nu} = \Theta_{\mu\nu}$, where the right hand side contains the third derivative terms

$$\Theta^{(3)}_{\mu\nu} = (\kappa_2 + 2\kappa_1)\,(g_{\mu\nu}\phi^\alpha\nabla_\alpha\Box\phi - \phi^\alpha\phi_{\alpha\mu\nu})\,, \qquad (3)$$

where ∇_λ is covariant derivative with respect to the Levi-Civita connection of $g_{\mu\nu}$,

$$\Box = \nabla_\lambda\nabla^\lambda, \quad \phi_{\mu\nu} = \nabla_\mu\phi_\nu, \quad \phi_{\alpha\mu\nu} = \nabla_\alpha\phi_{\mu\nu}$$

.

Similarly, the scalar equation

$$g^{\mu\nu}\phi_{\mu\nu} + \nabla_\mu \left[\phi_\nu(\kappa_1 g^{\mu\nu} R + \kappa_2 R^{\mu\nu})\right] = 0, \tag{4}$$

in the general case contains the third derivatives of the metric. But for $-2\kappa_1 = \kappa_2$ the Ricci-terms are combined into the Einstein tensor satisfying $\nabla_\mu G^{\mu\nu} = 0$. Then the scalar equations becomes the second order $(g^{\mu\nu} + \kappa G^{\mu\nu})\,\phi_{\mu\nu} = 0$, while (3) disappears.

This theory, in the metric version, was found capable to provide inflationary mechanism without a potential Sushkov:2009, Unfortunately, this metric model is likely to have problems when confronted with the speed of gravity waves measurement in the recent observation of the binary neutron stars merging.

Palatini version

The Palatini version (hereinafter abbreviated PDST) has the same action $S = \int d^4x\sqrt{-g}\,L_P$, where in the Lagrangian

$$L = (\hat{R}_{\mu\nu} - \phi_\mu\phi_\nu)g^{\mu\nu} - \hat{R}_{\alpha\beta}\phi_\mu\phi_\nu(\kappa_1 g^{\alpha\beta}g^{\mu\nu} + \kappa_2 g^{\alpha\mu}g^{\beta\nu}) \quad (5)$$

the Ricci tensor $\hat{R}_{\mu\nu}$ is a function of the independent connection $\hat{\Gamma}^\lambda_{\mu\nu}$, while the Ricci scalar $\hat{R} = R_{\mu\nu}g^{\mu\nu}$ depends on the metric and on the connection. In the absence of fermions, when the Ricci tensor is contracted with symmetric tensors, the torsion can be consistently set equal to zero, so the variation of $\hat{R}_{\mu\nu}$ is

$$\delta\hat{R}_{\mu\nu} = \hat{\nabla}_\lambda\delta\hat{\Gamma}^\lambda_{\mu\nu} - \hat{\nabla}_\nu\delta\hat{\Gamma}^\lambda_{\mu\lambda},$$

where $\hat{\nabla}_\lambda \equiv \hat{\nabla}_\lambda(\hat{\Gamma})$ stands for covariant derivative with respect to the Palatini connection.

Integrating by parts, we arrive at the Euler-Lagrange equation:

$$\hat{\nabla}_\lambda \left(\sqrt{-g} W^{\mu\nu}\right) = 0, \quad W^{\mu\nu} = \lambda g^{\mu\nu} - \kappa_2 \phi^\mu \phi^\nu,$$

$$\lambda = (1 - \kappa_1 \psi), \quad \psi = \phi_\alpha \phi^\alpha.$$

Variation of the action with respect to the metric leads to the Einstein-Palatini equation

$$\lambda \hat{R}_{\mu\nu} - \phi_\mu \phi_\nu (1 + \kappa_1 \hat{R}) - 2\kappa_2 \hat{R}_{\alpha(\mu} \phi_{\nu)} \phi^\alpha - g_{\mu\nu} L/2 = 0. \tag{6}$$

Finally, a variation over ϕ gives rise to a scalar equation

$$\partial_\mu \left[\sqrt{-g} \left(\phi^\mu + \kappa_1 \hat{R} \phi^\mu + \kappa_2 \hat{R}_{\alpha\beta} g^{\beta\mu} \phi^\alpha \right) \right] = 0,$$

which, in principle, can contain third derivatives of the metric and the fourth derivatives of the scalar field.

Second metric

The standard way to find the connection $\hat{\Gamma}$ is to transform the Palatini connection equation into an equation

$$\hat{\nabla}_\lambda \hat{g}_{\mu\nu} = 0$$

for some second metric $\hat{g}_{\mu\nu}$, in which case $\hat{\Gamma}$ can be identified with the Levi-Civita connection of the latter. For this, it is sufficient to ensure the following relation between the matrix $W^{\mu\nu}$ and the inverse metric $\hat{g}^{\mu\nu}$:

$$\sqrt{-g}\,W^{\mu\nu} = \sqrt{-\hat{g}}\,\hat{g}^{\mu\nu}, \quad \hat{g} = \det(\hat{g}_{\mu\nu}).$$

Then we get the equation in terms of the inverse new metric which is equivalent to what we are looking for.

The inverse matrix $W_{\mu\nu}$ reads:

$$W_{\mu\nu} = \lambda^{-1}\left(g_{\mu\nu} + \kappa_2 \Lambda^{-1}\phi_\mu\phi_\nu\right), \quad \Lambda = 1 - (\kappa_1 + \kappa_2)\psi.$$

Calculating the determimants one can find :

$$\hat{g} = g\,\Lambda\lambda^3.$$

Using this, the second metric can be represented as

$$\hat{g}_{\mu\nu} = \sqrt{\Lambda\lambda}\left(g_{\mu\nu} + \kappa_2\Lambda^{-1}\phi_\mu\phi_\nu\right),$$

and the Palatini connection, will read:

$$\hat{\Gamma}^{\lambda}_{\mu\nu} = \frac{1}{2}\hat{g}^{\lambda\tau}\left(\partial_\mu\hat{g}_{\lambda\nu} + \partial_\mu\hat{g}_{\mu\lambda} - \partial_\lambda\hat{g}_{\mu\nu}\right).$$

Disformal transformation

The connection generating metric $\hat{g}_{\mu\nu}$ (is related to the physical metric $g_{\mu\nu}$, to which couples the matter, by disfomal transformation. Not only the Palatini connection is expressed in terms of the dual metric $\hat{g}_{\mu\nu}$, but the full PDST theory will have a simpler form in the dual variables. Note that disformal dualities in non-minimal theories were extensively discussed recently (Bettoni:2013, Sakstein:2015, Domenech:2015, deRham:2016, Takahashi:2017).

The crucial question is whether the two theories related by non-pointlike transformations of variables are classically equivalent, with no extra degrees of freedom. A number of investigations Exirifard:2007,Deruelle:2014,Tsujikawa:2014,Domenech:2015,Arroja:2015, deRham:2016,Takahashi:2017) suggest that it is indeed the case, provided the disformal transformations are *reversible* .

The dual theory

Expressing the action in terms of the new metric we find for generic κ_1, κ_2 the Einstein-Hilbert gravity non-minimally coupled to ϕ:

$$S = \int \sqrt{-\hat{g}} \left[R_{\mu\nu}(\hat{g}) - \phi_\mu \phi_\nu \hat{\Lambda}^{-1} \right] \hat{g}^{\mu\nu} d^4x,$$

where we denoted by $\hat{\Lambda}$ the scale factor Λ with $\psi = \phi_\mu \phi_\nu g^{\mu\nu}$ expressed through $\hat{\psi} = \phi_\mu \phi_\nu \hat{g}^{\mu\nu}$.
These two are related by the equation

$$\hat{\psi} = \psi(1 - \kappa_1 \psi)^{1/2}([1 - (\kappa_1 + \kappa_2)\psi]^{-3/2},$$

This relation becomes singular if one of the scale factors reaches zero, when the determinant ratio degenerates. We, therefore, demand $\lambda > 0, \Lambda > 0$ in the physical region of spacetime.

Reversibility

By the inverse function theorem, the solution $\psi(\hat{\psi})$ exists at any point where the derivative $d\hat{\psi}/d\psi \neq 0$. In our case this derivative is zero at $\psi = \psi_{\rm cr} = 2/(2\kappa_1 + 3\kappa_2)$. But there one easily finds

$$\hat{\psi}(\psi_{\rm cr}) = 2/(\sqrt{3}\kappa_2).$$

To the right and to the left of this point, the function $\hat{\psi}(\psi)$ is monotonous (see Left panel in Fig.1). I general, the cubic equation has three solutions $\psi(\hat{\psi})$, from which one is real in the physical domain, and we naturally choose it. Thus, it is found that the disformal transformation is one-to-one and reversible indeed within the physical region. However, the existence of a physical region restricts the range of possible parameters (a more detailed discussion will be given elsewhere).

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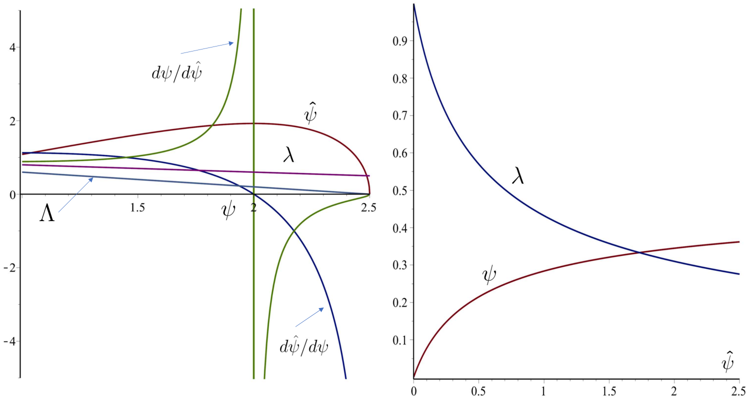

Рис.: Left panel: dependence $\hat{\psi}(\psi)$, derivatives $d\hat{\psi}/d\psi$, $d\psi/d\hat{\psi}$ and scale factors $\lambda(\psi)$, $\Lambda(\psi)$ for generic theory ($\kappa_1 = \kappa_2 = 0.2$. Right panel: solution $\psi(\hat{\psi})$ and scale factor λ for exceptional theory ($\kappa_1 = -\kappa_2 = 1$).

No Ostrogradski ghosts

Note that we are still working in the Palatini formalism, so the Ricci tensor in be considered as a functional of the connection. But for such actions, both the metric and the Palatini formalism give the same equations, so, with some abuse on notation, we denote the Ricci tensor as metric one Obviously, the Einstein-scalar theory does

not suffer from Ostrogradski instabilities, so we conclude that the PDST theory with two generic coupling constants has no ghosts.

Exceptional theory

The case $\kappa_2 = -\kappa_1 \equiv \kappa$ is exceptional. Then $\Lambda = 1$, and the above theory reduces to the Einstein theory, minimally coupled to a massless scalar:

$$S = \int \sqrt{-\hat{g}}\,[R_{\mu\nu}(\hat{g}) - \phi_\mu \phi_\nu]\,\hat{g}^{\mu\nu} d^4x.$$

The Einstein equation reads

$$R_{\mu\nu} = \phi_\mu \phi_\nu,$$

and the scalar obeys the covariant d'Alembert equation

$$\hat{\Box}\phi = 0,$$

On shell the following conditions are satisfied: $L = 0$, $\hat{R} = \psi$, implying that the Palatini Einstein equation for $g_{\mu\nu}$ is valid.

Non-singular cosmology

Assume that $\kappa_2 = -\kappa_1 \equiv \kappa > 0$. Using disformal duality, we can construct an exact cosmological solutions of our theory starting with the spatially flat FRW cosmology in the Einstein's frame theory

$$d\hat{s}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu = dt^2 - \hat{a}^2 \delta_{ij} dx^i dx^j .$$

The relevant Einstein and scalar field equations

$$R_{tt} = -\frac{3\ddot{\hat{a}}}{\hat{a}} = \dot{\phi}^2, \quad \ddot{\phi} + 3\frac{\dot{\hat{a}}}{\hat{a}}\dot{\phi} = 0$$

give a solution for "stiff-matter" cosmology proposed by Zel'dovich in 1972

$$\hat{a} = a_0 t^{1/3}, \quad \phi = \sqrt{2} \ln t/\sqrt{3}.$$

Obviously, it is singular at $t = 0$ and describes the decelerating expansion.

Now we derive a solution to our theory. Since the metric is diagonal and the scalar field depends only on t, we obtain a cubic algebraic equation for g_{tt}:

$$\left(|g_{tt}| - 2x/(3\sqrt{3})\right)^3 = |g_{tt}|, \quad x = \kappa\sqrt{3}/t^2.$$

Its solution is smooth, although in terms of real functions it looks piecewise:

$$|g_{tt}| = \frac{2x}{3\sqrt{3}} + \frac{1}{\sqrt{3}} \begin{cases} 2\cos\left(\frac{1}{3}\arccos(x)\right), & x < 1, \\ A^{1/3} + A^{-1/3}, & x > 1, \end{cases}$$

where $A = \left(x + \sqrt{x^2-1}\right)^{1/3}$. For large x (small t) one has:

$$|g_{tt}| = 2x/3\sqrt{3} + (2x)^{\frac{1}{3}}/\sqrt{3} + (4/x)^{\frac{1}{3}}/(2\sqrt{3}) + \dots,$$

for small x (large t),

$$|g_{tt}| = 1 + x/\sqrt{3} - x^2/18 + \dots .$$

For g_{ij} one obtains :

$$g_{ij} = \delta_{ij} a^2, \quad a^2 = \hat{a}^2 |g_{tt}|^{1/3}.$$

Since $|g_{tt}| = 1$ only at large t, we need to go to the synchronous time $t \to \tau(t)$, so that $g_{\tau\tau} \equiv 1$, solving the equation

$$dt/d\tau = |g_{tt}|^{-1/2}.$$

For small t, keeping the leading term one finds:

$$dt/d\tau = 1/(H_0\, t) \implies t = \mathrm{e}^{H_0 \tau},\ H_0 = \sqrt{3/(2\kappa)}.$$

For the spatial compoments for small t, one can find $a^2 \sim a_0^2 H_0^{-2/3}$, with no dependence on time at all. Finally, after scaling the spatial coordinates, we get the Minkowski metric for $t \to 0$. Theerefore, the space-time $g_{\mu\nu}$ starts at $\tau = -\infty$ as Minkowski space. Computing the second derivative of the scale factor $\ddot{a}$ one finds that the universe starts in accelerating phase which, however, ends soon with an insignificant gain of the cosmological radius (Fig. 2). After a short period of acceleration, the expansion decelerates and at large t the Zel'dovich's power law $a \sim t^{1/3}$ is restored, since the scale factor $\lambda \to 1$ (and the comoving time coincides with the Einstein frame comoving time).

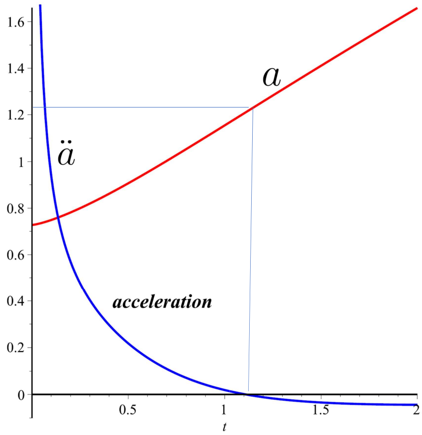

Рис.: Non-singular PDST cosmology. Expansion starts with finite scale factor with positive acceleration.

It is worth noting that although the PDST metric is nonsingular at the initial stage of expansion, the scalar field diverges. In the Palatini theories it is assumed that matter couples to the metric $g_{\mu\nu}$, so geodesics, defined as curves of minimal length, do not stop at $t \to 0$, and the space is geodesically complete. At the same time, the auto-parallel curves, defined in terms of the Palatini connection, which coincides with the Levi-Civita connection of the singular metric will meet the singularity at $t = 0$. But, if matter couples to the metric and not to the connection, the scalar acts as Deus Ex Machina realizing the cherished dream of General Relativity - removing of space-time singularities.
To repeat, in the Einstein frame, the universe begins at a curvature singularity, which is also a singularity of the scalar field, and expands with negative acceleration. In the PDST frame, the initial space-time is Minkowski. As far as the scalar field decays, the universe enters into an expanding phase with positive acceleration for some finite time, then slows down and asymtotically joins the regime of the Einstein frame.

Static Geon

In the static case, interesting solutions arise for $\kappa_1 = -\kappa_2 > 0$, so here we denote $\kappa = \kappa_1$. Minimal scalar gravity has a well-known FJNW solution

$$\hat{g}_{tt} = -\hat{g}_{rr}^{-1} = -\left(1 - b/r\right)^{\gamma}, \; \hat{g}_{\theta\theta} = r^2 \left(1 - b/r\right)^{1-\gamma},$$

$$\phi = q(b)^{-1} \ln\left(1 - b/r\right),$$

where q is the scalar charge and $0 < \gamma < 1$, $\gamma = \left(1 - 4q^2/b^2\right)^{1/2}$. It is asymptotically flat and has a singular horizon at $r = b$. We want to find a PDST counterpart for this solution. The disformal transformation generates somewhat more complicated cubic equation for g_{rr}:

$$[g_{rr} - 2x/(3\sqrt{3})]^3 = w^2 g_{rr}, \quad w = \hat{g}_{rr} = (1 - b/r)^{-\gamma},$$

$$x = 3\sqrt{3}\kappa q^2/[2r^2(r-b)^2]$$

smooth real solution is:

$$g_{rr} = \frac{2x}{3\sqrt{3}} + \frac{1}{\sqrt{3}} \begin{cases} 2w\cos[\frac{1}{3}\arccos(x/w)], & x < w, \\ w^{2/3}B + w^{4/3}B^{-1}, & x > w, \end{cases}$$

where $B = \left(x + \sqrt{x^2 - w^2}\right)^{1/3}$. The remaining metric components then are:

$$g_{tt} = \hat{g}_{tt}/\lambda^{1/2},\ g_{\theta\theta} = \hat{g}_{\theta\theta}/\lambda^{1/2},\ \lambda = (g_{rr}/w)^{-2/3}.$$

At infinity $r \to \infty$, the variables $x \to 0,\ w \to 1$, while $g_{tt} \sim 1 + x/\sqrt{3}$, so $\lambda = 1 + O(r^{-4})$ and the solution remains asymptotically flat:

$$g_{tt} \sim -1 + \gamma b/r, \quad g_{rr} \sim 1 - \gamma b/r, \quad g_{\theta\theta} \sim r^2.$$

In the singularity $r = b$, one has $w \sim \xi^{-\gamma}, x \sim \xi^{-2}$, where $\xi = r/b - 1 \to 0$ leading to:

$$\lambda \sim \mu^{-2} \xi^{2(2-\gamma)/3}, \;\; g_{tt} \sim \mu \xi^{2(2\gamma-1)/3}, \;\; g_{rr} \sim \mu \xi^{-2},$$

$$g_{\theta\theta} \sim \mu b^2 \xi^{(1-2\gamma)/3}, \;\; \mu = (\kappa q^2/b^4)^{1/3}$$

For $\gamma = 1/2$, the interval in the neighborhood of $r = b$ reads:

$$\mu^{-1} ds^2 = -dt^2 + \xi^{-2} dr^2 + b^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

Passing to the new radial coordinate $\rho = b \ln \xi$ extending the domain $r \in (b, \infty)$ to a complete real line, one can find that the manifold is isomorphfic to the product $M_{1,1} \times S^2$ of the two-dimensional Minkowski space and a sphere of radius b. This manifold is geodesically complete. Therefore, our solution is a regular geon of the PDST theory. Its striking feature, however, that it is supported by a singular scalar.

PP-wave

Wave space-times (and more general classes of Kundt metrics) in Einstein theory coupled to the minimal scalar were constructed recently (Tahamtan:2015). Here we consider the simplest pp-wave spacetime

$$d\hat{s}^2 = F(u,x,y)\,du^2 - 2dudv + dx^2 + dy^2,$$

whose tensor Ricci is

$$R_{\mu\nu} = -\delta^u_\mu \delta^u_\nu \,\Delta\, F/2, \quad \Delta = \partial_x^2 + \partial_y^2.$$

Assuming that the scalar field depends only on u, it is found that the d'Alembert equation is satisfied: $\hat{\Box}\phi(u) = 0$, and the Einstein equation $R_{\mu\nu} = \phi_\mu \phi_\nu$ reduces to an equation for F:

$$\Delta\, F = -2\phi'^2.$$

Now construct the corresponding PDST solution. In this case, the disformal transformation is light-like . Assuming that the metric has nonzero only g_{uu}, g_{uv}, g_{ij}, we find only non-zero inverse metric components g^{vv}, g^{uv}, g^{ij}, therefore $\psi = \phi_\mu \phi_\nu g^{\mu\nu} = 0$, and the scale factor $\lambda = 1$. Then we easily find the pp-wave solution of the PDST theory:

$$d\hat{s}^2 = \left(F(u,x,y) - \kappa\phi'^2\right) du^2 - 2dudv + dx^2 + dy^2.$$

It is clear that it propagates at the speed of light.

Matter fields interacting only with metric

Matter fields can be added to PDST frame theory so that the interaction term does not dpend on the Palatini connection. In particular, point-like particle action depends on the metric only. Therefore parfticle's world-line will be a minimal curve, which can be thought of as geodesic with respect to the Levi-Civita connection of the non-singular metric. It therefore will not see singularities hidden in the scalar sector.

At the same time, the Palatini connection is singular together with the scalar field, so the entire theory remains to be singular. The singularities, however, are not seen by observers interacting only with the metric. It should be emphasized that the particle world-line are no more the autoparallel curves in terms of Palatini connection, which plays the role of auxiliary field of the theory.

Our theory has solutions which have non-singular metric, but singular scalar field and Palatini connection. So as a whole the PDST theory contains singularities. But for observers related to matter whose lagrangian depends on the metric only these singularities will be unseen. The class of such lagrangians includes those for point particles and other branes, the scalar fields, the Maxwell field. In the latter case the field strength has to be defined in terms of partial and not covariant derivatives. In all these theories the Levi-Civita connection of the regular metric can be introduced, defining the non-singular curvature. Thus we deal with the theory with two curvatures, from which the physical one (i.e. that with which interacts the matter) is non-singular. This provides a novel view on the problem of singularities.

Conclusion

- Scalar-tensor theories with non-minimal coupling to curvature in Palatini formalism is viable approach to get ghost-free theories with phenomenologically attractive features
- Such theories are simpler once matter is coupled to metric but not to connection. Then they contain non-propagated non-metricity which can be computed algebraically
- In some cases connection can be a Levi-Civita connections from an auxiliary metric, related to the physical metric by disformal transformation which can be solved algebraically. In the case of kinetical coupling of scalar to Ricci tensor and Ricci scalar, there is a particular ratio of couplings when the initial non-minimal theory is dual to Einsten gravity with minimally coupled scalar

- The PDST counterpart to Zeldochich cosmology describes the universe which starts in Minkowski state, then the scale factor experiences an accelerating growth of about 1.7 times, changing to decelerating phase afterwards.
- Singular Fisher solution of the minimal Einstein-scalar theory in PDST has non-singular metric and singular scalar field
- In a similar way we constructed the non-singular PDST pp-waves showing that they propagate exactly at the speed of light.
- Here we concentrated on some exact solutions of PDST with remarkable features. The cosmological one apparently can not describe a real word. But numerical analysis show the existence of other cosmological solutions with more appealing features, once the scalar potential is added

Thank You for Attention!