Relaxation to equilibrium at NICA: hydro-like behavior, EOS and shear viscosity-to-entropy ratio

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in collaboration with

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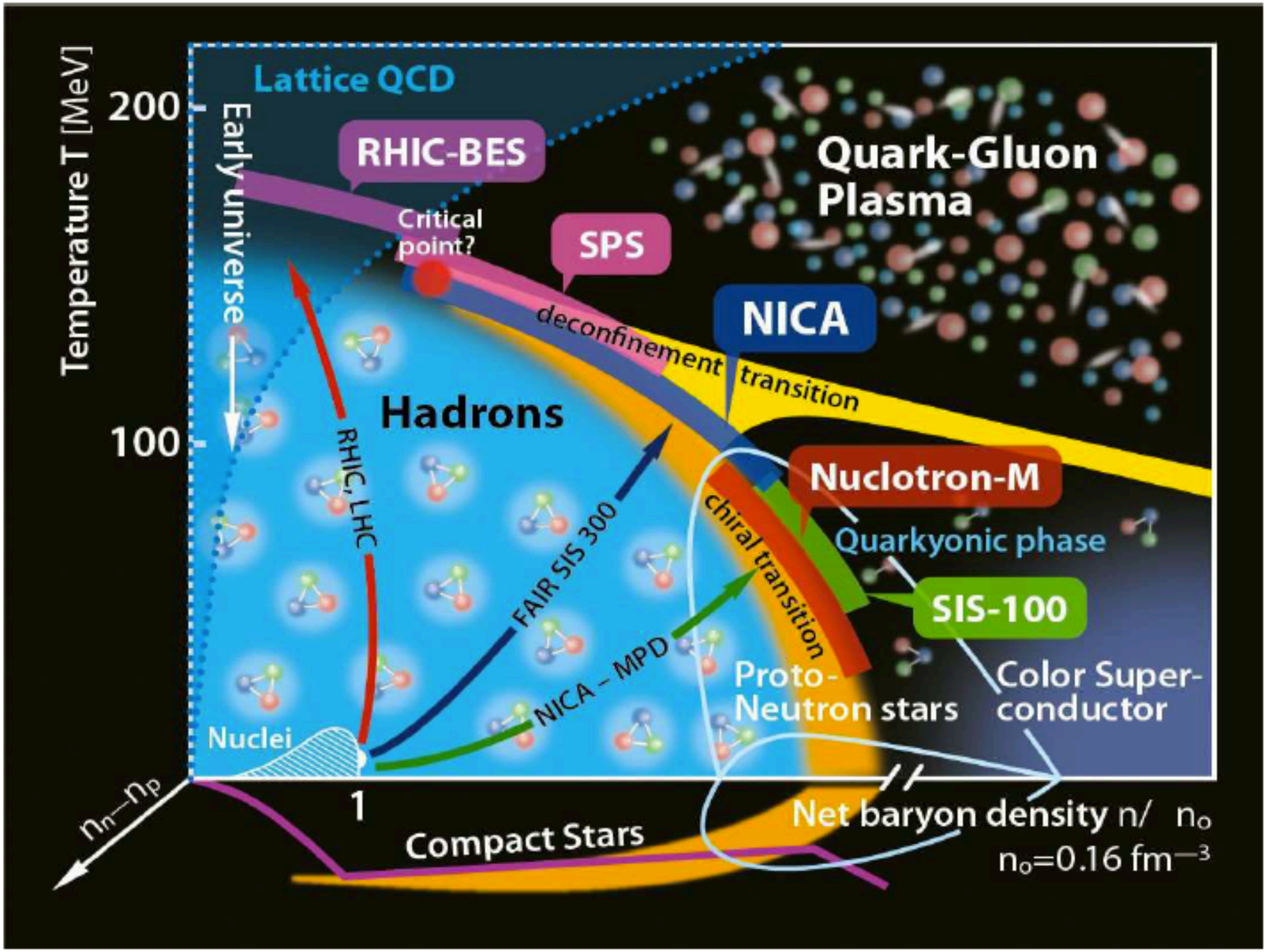
The XXIV International Workshop
High Energy Physics and Quantum Field Theory
Sochi, Russia, 22-29.09.2019

Motivation



Temperature T [MeV]
200
100
Lattice QCD
Early universe
RHIC-BES
Critical point?
SPS
NICA
deconfinement transition
Quark-Gluon Plasma
Hadrons
RHIC, LHC
FAIR SIS 300
NICA – MPD
chiral transition
Nuclotron-M
Quarkyonic phase
SIS-100
Proto-Neutron stars
Color Super-conductor
Nuclei
1
Compact Stars
Net baryon density n/ n₀
n₀=0.16 fm⁻³
nn–np

Relativistic Hydrodynamics

Basic Equations

Energy-momentum tensor

$$\mathbf{T}^{\mu\nu} = \underbrace{(\varepsilon + \mathbf{P})\mathbf{u}^{\mu}\mathbf{u}^{\nu} - \mathbf{P}\mathbf{g}^{\mu\nu}}_{\text{inertial}} + \underbrace{\eta^{\mu\nu}}_{\text{dissipative}}$$

The space-time evolution of relativistic fluid is described by the set of differential equations

$$\partial_{\mu}\mathbf{N}^{\mu}(\mathbf{x}) = \mathbf{0}$$

$$\partial_{\mu}\mathbf{T}^{\mu\nu} = \mathbf{0};\ \ \mu,\nu = \mathbf{0},\mathbf{1},\mathbf{2},\mathbf{3}$$

For perfect fluid (i.e. $\eta^{\mu\nu} = 0$) these equations take the familiar form

$$\begin{aligned}(\partial_t + \vec{\mathbf{v}}\cdot grad)\mathcal{N} &= -\mathcal{N}\,div\vec{\mathbf{v}} & \mathcal{N} &\equiv \gamma\,\mathbf{N}^{\mu}\mathbf{u}_{\mu}\\ (\partial_t + \vec{\mathbf{v}}\cdot grad)\vec{\mathcal{M}} &= -\vec{\mathcal{M}}\cdot div\vec{\mathbf{v}} - grad\,\mathbf{P} & \vec{\mathcal{M}} &\equiv \mathbf{T}^{\mathbf{0i}} = (\varepsilon + \mathbf{P})\gamma^2\vec{\mathbf{v}}\\ (\partial_t + \vec{\mathbf{v}}\cdot grad)\mathcal{E} &= -\mathcal{E}div\vec{\mathbf{v}} - div(\mathbf{P}\vec{\mathbf{v}}) & \mathcal{E} &\equiv \mathbf{T}^{\mathbf{00}} = (\varepsilon + \mathbf{P}\vec{\mathbf{v}}^{\mathbf{2}})\gamma^2\end{aligned}$$

$$\partial_\mu N^\mu(x) = 0$$

$$\partial_\mu T^{\mu\nu} = 0; \quad \mu,\nu = 0,1,2,3$$

$$T^{\mu\nu} = \underbrace{(\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu}}$$

Number of variables – 6

Number of equations – 4

Missing equations:

(1) EOS, that links energy density and pressure

- **Four-velocity**

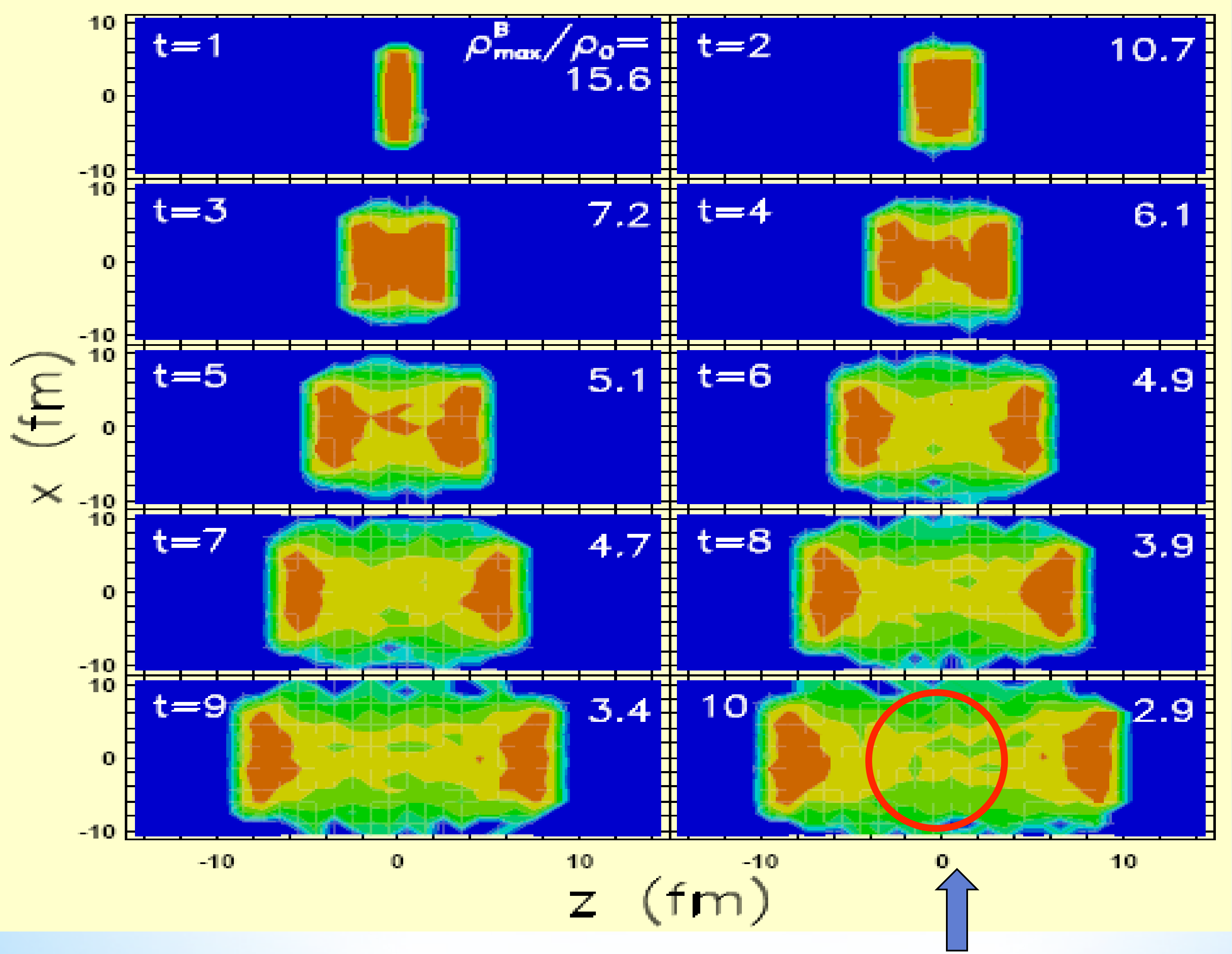
$$u^\mu = (\gamma, \gamma\vec{v}); \quad \vec{v} \equiv \frac{\vec{p}}{p^0}; \quad \gamma = \frac{1}{\sqrt{1-(\vec{v})^2}}$$

thus

(2) $$u^\mu u_\mu = 1$$

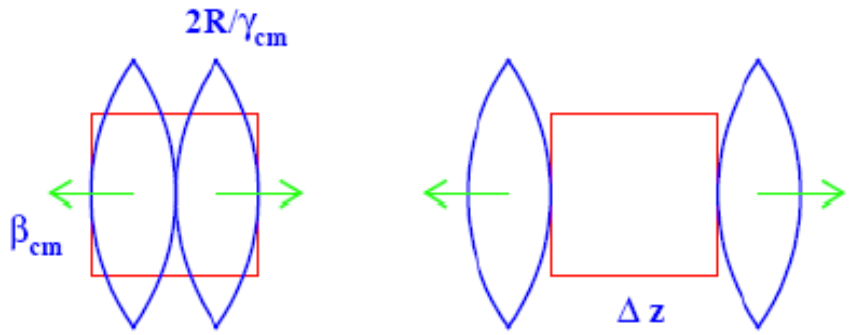
Pre-equilibrium: Homogeneity of baryon matter

L.Bravina et al., PRC 60 (1999) 024904



The local equilibrium in the central zone is quite possible

Equilibration in the Central Cell



$t^{cross} = 2R/(\gamma_{cm}\,\beta_{cm})$

$t^{eq} \geq t^{cross} + \Delta z/(2\beta_{cm})$

Kinetic equilibrium:

Isotropy of velocity distributions

Isotropy of pressure

Thermal equilibrium:

Energy spectra of particles are described by Boltzmann distribution

$$\frac{dN_i}{4\pi p E dE} = \frac{V g_i}{(2\pi\hbar)^3} \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

L.Bravina et al., PLB 434 (1998) 379; JPG 25 (1999) 351

Chemical equlibrium:

Particle yields are reproduced by SM with the same values of $(T,\ \mu_B,\ \mu_S)$:

$$N_i = \frac{V g_i}{2\pi^2\hbar^3} \int_0^\infty p^2 dp \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Statistical model of ideal hadron gas

input values — output values

$$\varepsilon^{\rm mic} = \frac{1}{V}\sum_i E_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm B}^{\rm mic} = \frac{1}{V}\sum_i B_i \cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm S}^{\rm mic} = \frac{1}{V}\sum_i S_i \cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}).$$

Multiplicity

$$N_i^{\rm SM} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 f(p,m_i)dp,$$

Energy

$$E_i^{\rm SM} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 \sqrt{p^2+m_i^2}\, f(p,m_i)dp$$

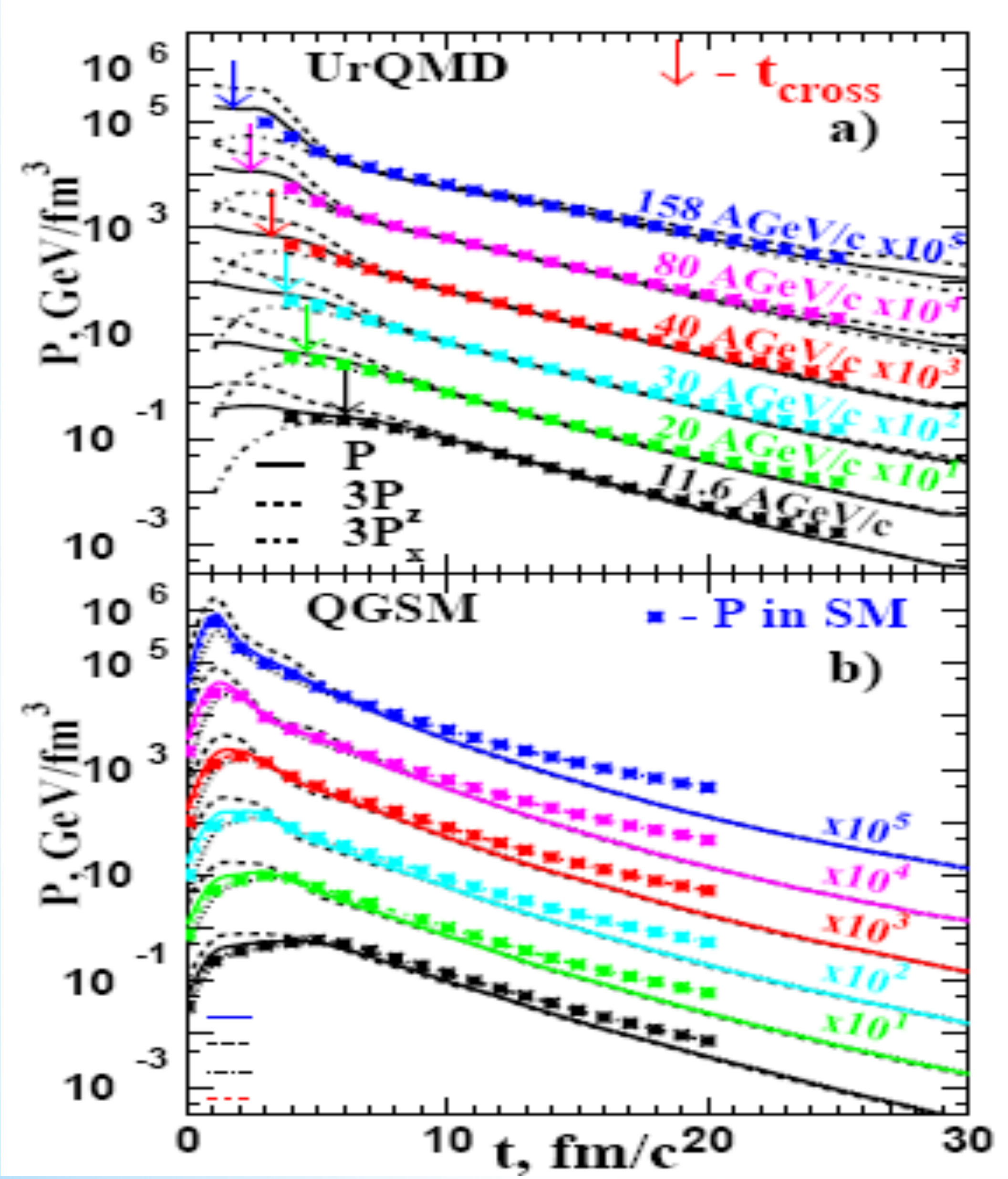
Pressure

$$P^{\rm SM} = \sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 \frac{p^2}{3(p^2+m_i^2)^{1/2}}\, f(p,m_i)dp$$

Entropy density

$$s^{\rm SM} = -\sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty f(p,m_i)\,[\ln f(p,m_i) - 1]\, p^2 dp$$

Kinetic Equilibrium



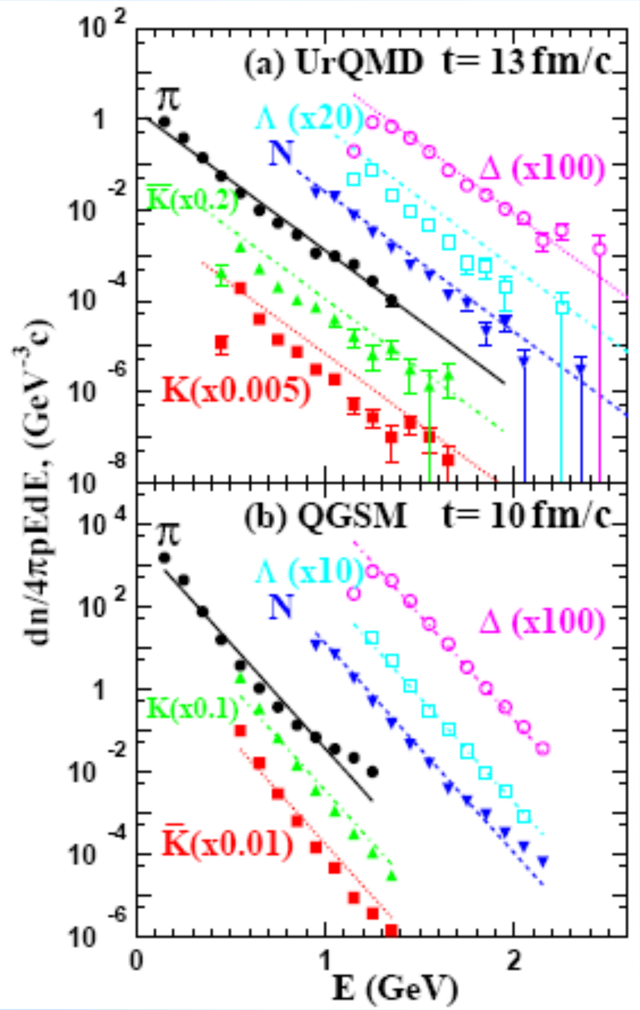
Isotropy of pressure

L.Bravina et al., PRC 78 (2008) 014907

Pressure becomes isotropic for all energies from 11.6 AGeV to 158 AGeV

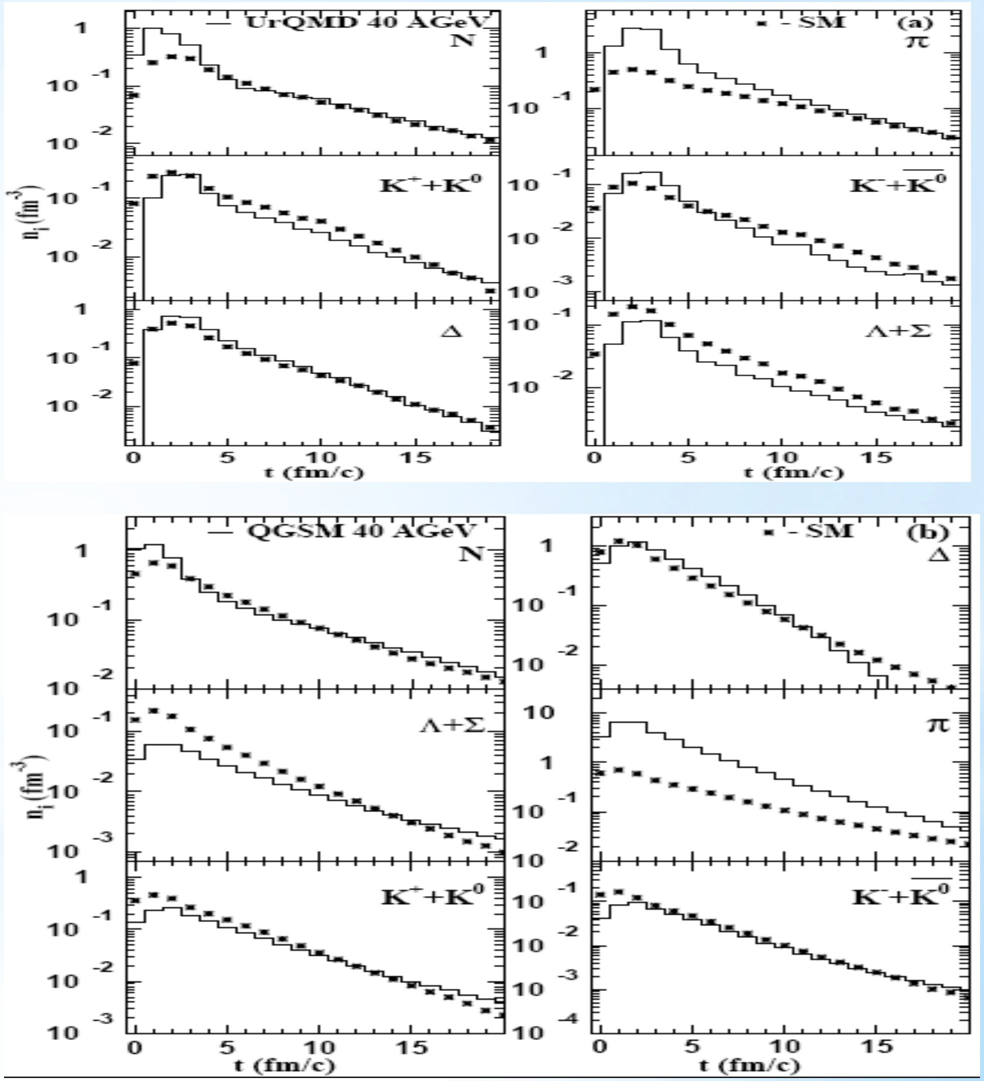
Thermal and Chemical Equilibrium

Boltzmann fit to the energy spectra



PRC 78 (2008) 014907

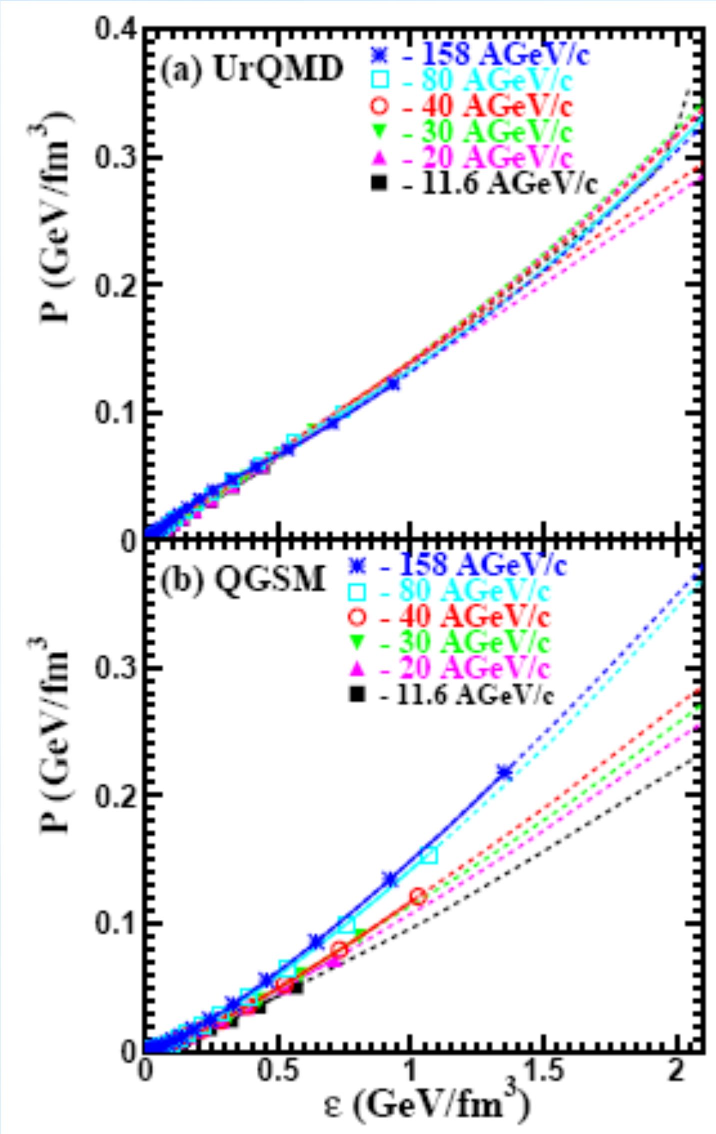
Particle yields



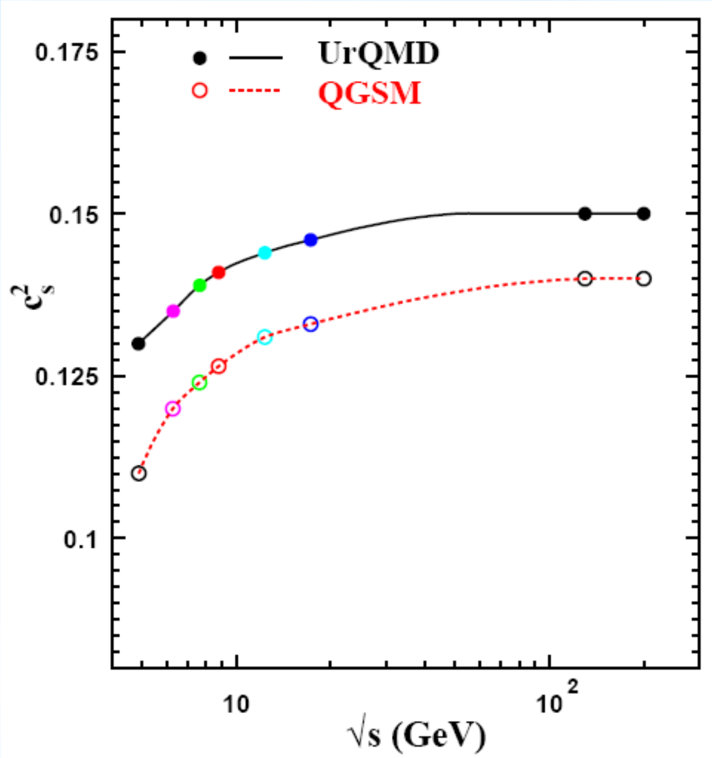
Thermal and chemical equilibrium seems to be reached

Equation of State in the cell

pressure vs. energy



sound velocity



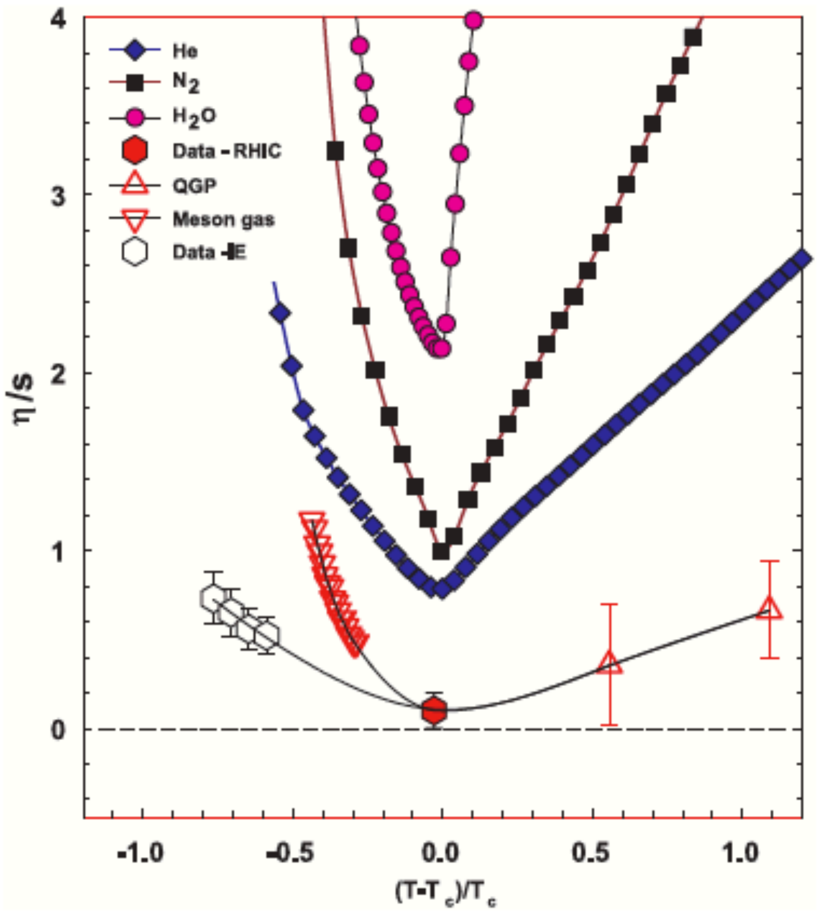
P/ε = const very early (!)

$P/\varepsilon = 0.13(\mathrm{AGS}),\ 0.14(40),\ 0.146(\mathrm{SPS}),\ 0.15(\mathrm{RHIC})$

Conclusions (part 1)

- *MC models favor early pre-equilibration of hot and dense nuclear matter already at $t \approx 2$ fm/c*
- *After that the expansion of matter in the central cell proceeds **isentropically** with constant s/ρ_B (hydro!)*
- *The **EOS** has a simple form: $P/\varepsilon = const$ (hydro!) even at far-from-equilibrium stage*
- *The speed of sound C^2_s varies from 0.12 (AGS) to 0.14 (40 AGeV), and to 0.15 (SPS & RHIC) => saturation*
- *Good agreement between the cell and box results*

Motivation



taken from

R.Rapp, H.Hees. arXiv:0803.0901[hep-ph]

- P.Kovtun, D.T.Son, O.Starinets. PRL 94, 111601 (2005)
- A.Muronga. PRC 69, 044901 (2004)
- L.Csernai, J.Kapusta, L.McLerran. PRL 97, 152303 (2006)
- P.Romatschke, U.Romatschke. PRL 99, 172301 (2007)
- S.Plumari et al. PRC 86, 054902 (2012)
- ALICE collaboration, CERNCOURIER (14.10.2016)
- ...

Theory

Green-Kubo: shear viscosity η may be defined as:

$$\eta(t_0) = \frac{1}{\hbar}\frac{V}{T}\int_{t_0}^{\infty} \mathrm{d}t \langle \pi(t)\pi(t_0)\rangle_t = \frac{\tau}{\hbar}\frac{V}{T}\langle \pi(t_0)\pi(t_0)\rangle,$$

where

$$\begin{aligned}\langle \pi(t)\pi(t_0)\rangle_t &= \frac{1}{3}\sum_{\substack{i,j=1\\ i\neq j}}^{3} \lim_{t_{\max}\to\infty} \frac{1}{t_{\max}-t_0}\int_{t_0}^{t_{\max}} \mathrm{d}t' \pi^{ij}(t+t')\pi^{ij}(t') \\ &= \langle \pi(t_0)\pi(t_0)\rangle \exp\left(-\frac{t-t_0}{\tau}\right)\end{aligned}$$

with

$$\pi^{ij}(t) = \frac{1}{V}\sum_{\text{particles}} \frac{p^i(t)\,p^j(t)}{E(t)}$$

t_0: initial cut-off time to start with

Model setup: cell calculations

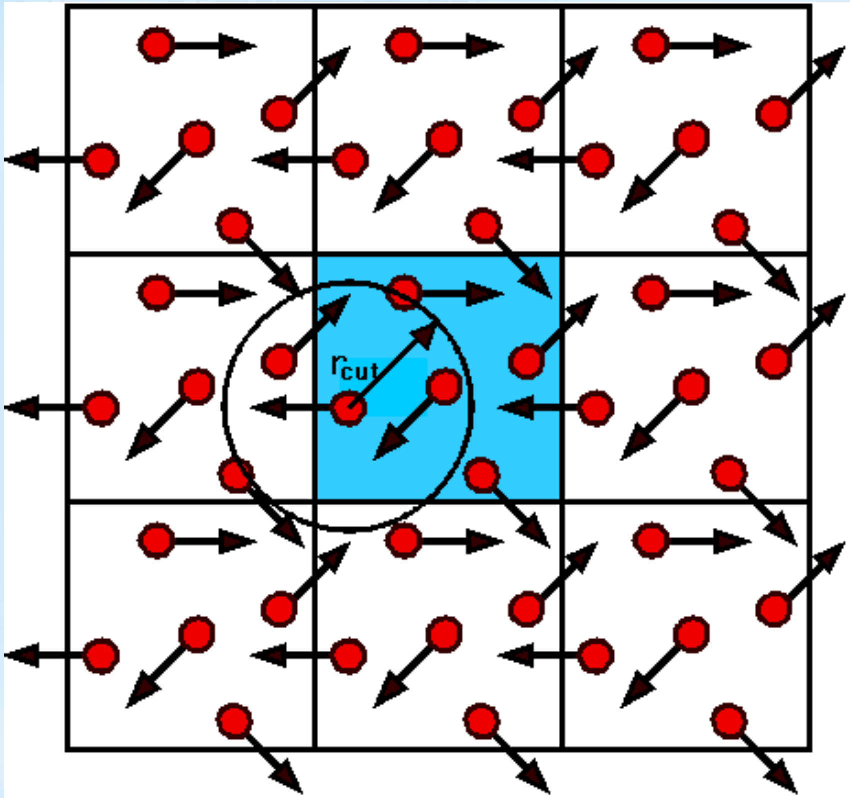
- UrQMD calculations, central Au+Au collisions at energies $E \in [10, 20, 30, 40]$ AGeV of the projectile, 51200 events per each
- central cell $5 \times 5 \times 5$ fm^3 $\Rightarrow$ $\{\varepsilon, \rho_{\rm B}, \rho_{\rm S}\}$ at times $t_{\rm cell} = 1 \div 20$ fm/c
- statistical model (SM): $\{\varepsilon, \rho_{\rm B}, \rho_{\rm S}\}$ $\Rightarrow$ $\{T, s, \mu_{\rm B}, \mu_{\rm S}\}$

Model setup: box calculations

- UrQMD box calculations at $\{\varepsilon, \rho_{\rm B}, \rho_{\rm S}\}$ for every energy and cell time $t_{\rm cell}$ from cell calculations, 80 points in total, 12800 events per each
  $\rho_{\rm B}$ is included as $N_p : N_n = 1 : 1$
  $\rho_{\rm S}$ is included via kaons K^-
  box size: $10 \times 10 \times 10$ fm^3
  box boundaries: transparent
- $\pi^{ij}(t)$ data extraction: $t = 1 \div 1000$ fm/c in box time, all types of hadrons are taken into account

Box with periodic boundary conditions

M.Belkacem et al., PRC 58, 1727 (1998)

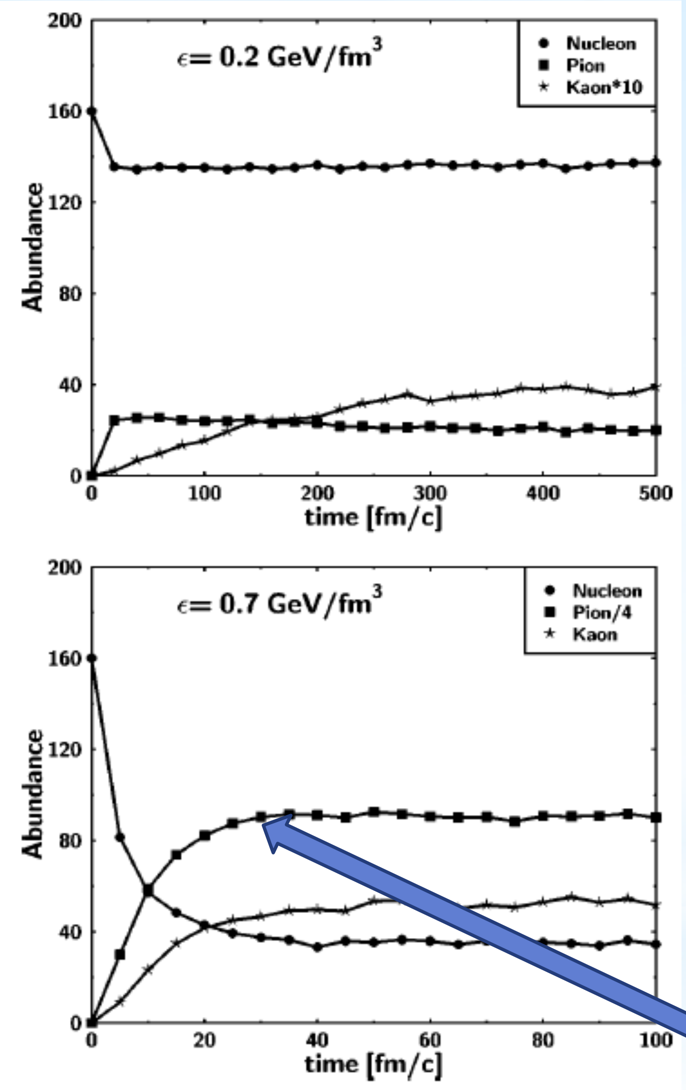


Initialization: (i) nucleons are uniformly distributed in a configuration space;
(ii) Their momenta are uniformly distributed in a sphere with random radius and then rescaled to the desired energy density.

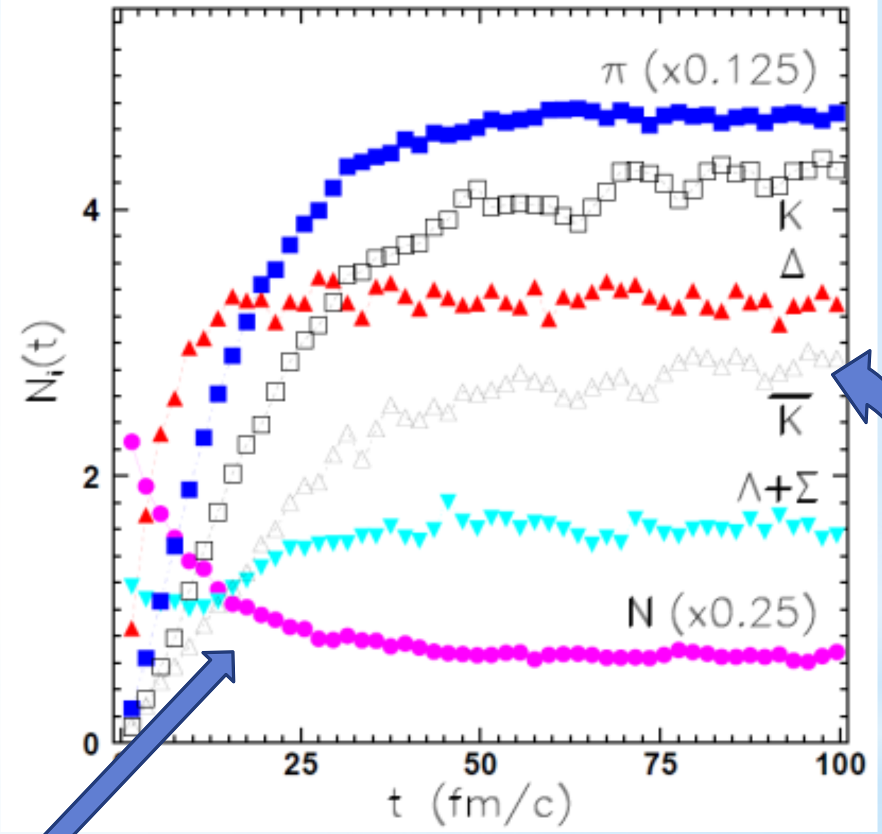
Model employed: UrQMD
55 different baryon species (N, Δ, hyperons and their resonances with $m \leq 2.25$ GeV/c^2)
32 different meson species (including resonances with $m \leq 2$ GeV/c^2) and their respective antistates.
For higher mass excitations a string mechanism is invoked.

Test for equilibrium: particle yields and energy spectra

Box: particle abundances



M.Belkacem et al., PRC 58, 1727 (1998)



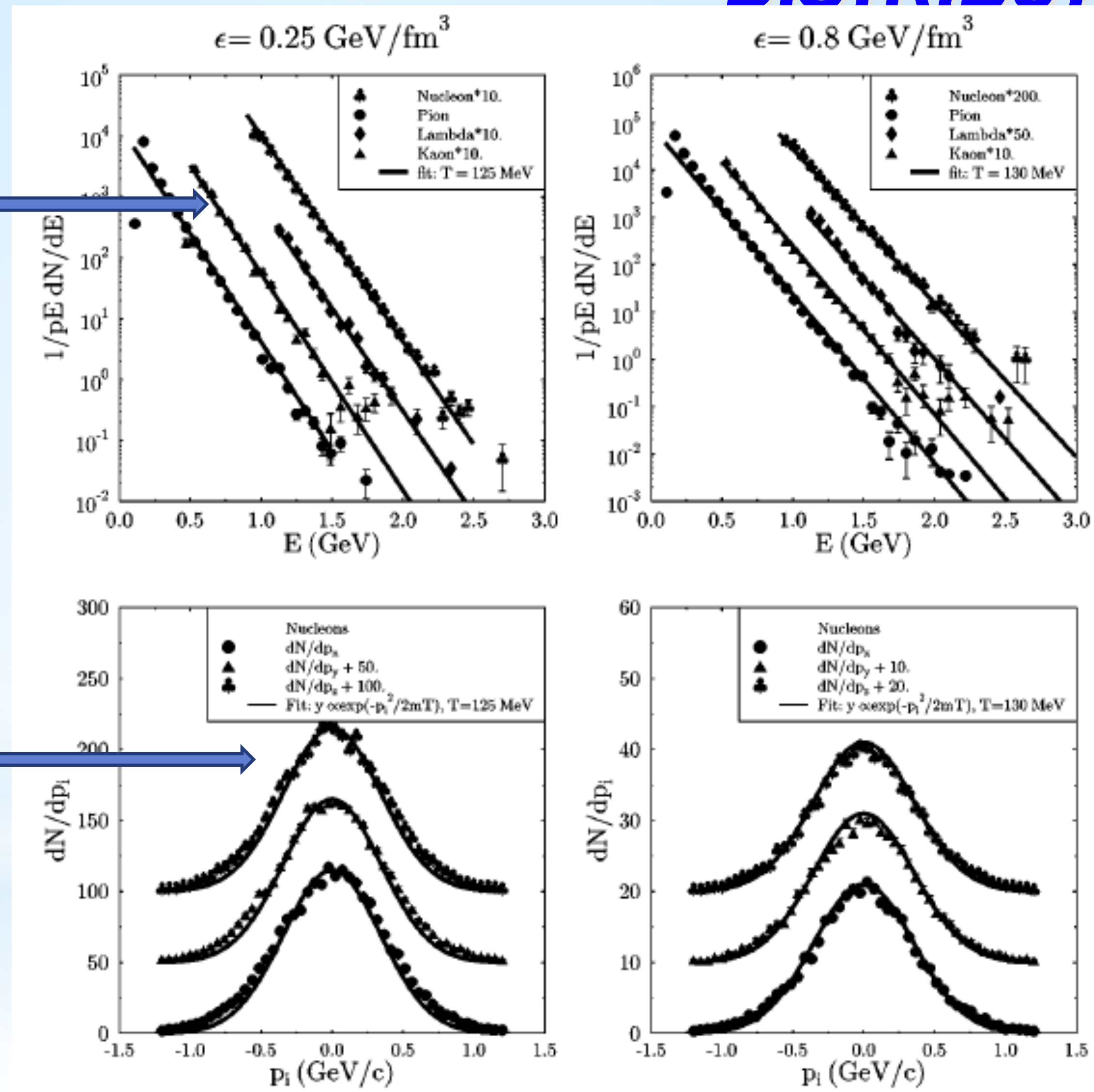
L.Bravina et al., PRC 62, 064906 (2000)

Saturation of yields after a certain time. Strange hadrons are saturated longer than others (at not very high energy densities)

BOX: ENERGY SPECTRA AND MOMENTUM DISTRIBUTIONS

Fit to Boltzmann distributions ~exp(-E/T)

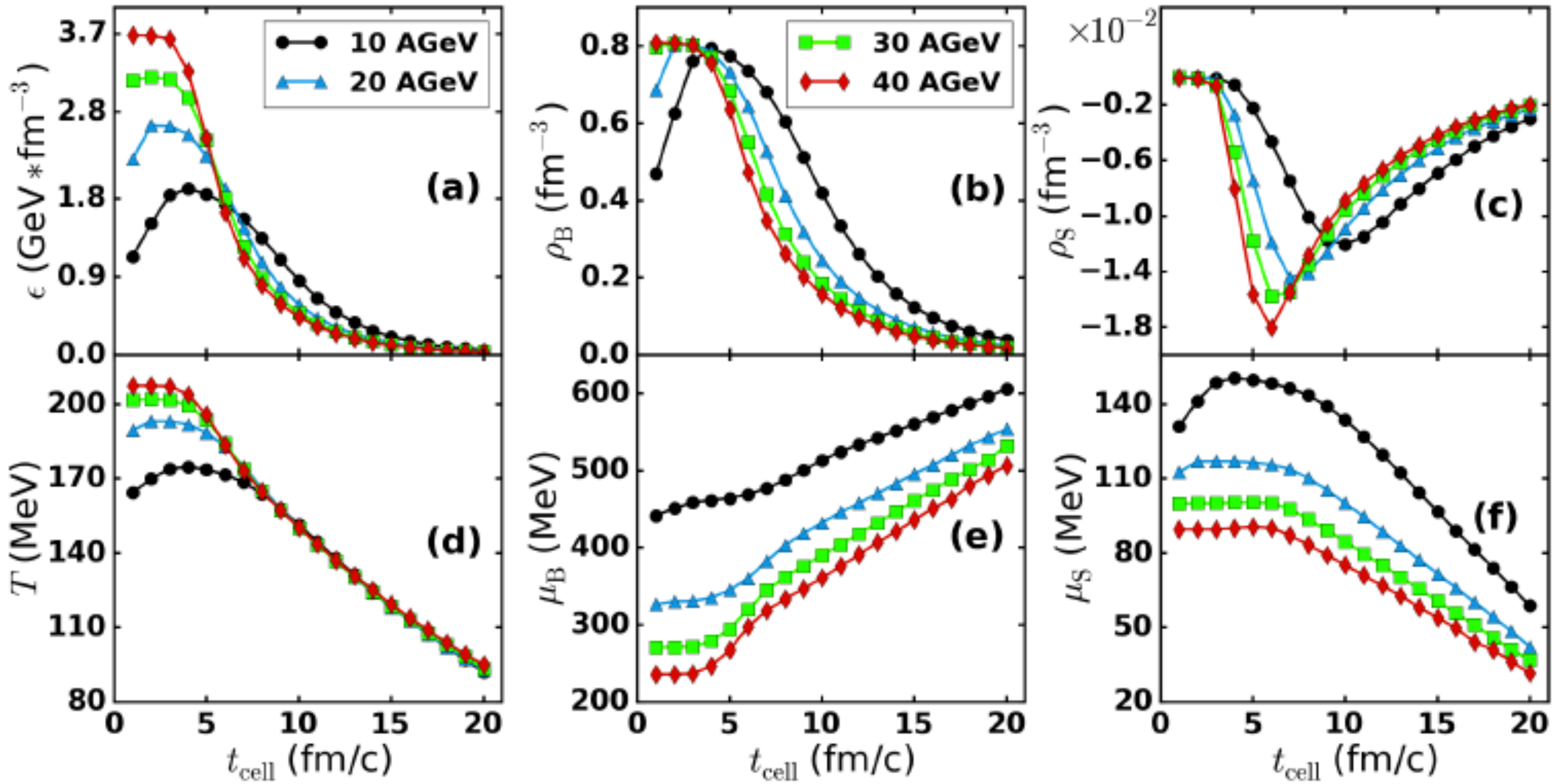
Fit to Gaussian distributions ~exp(-p²/2mT)



M.Belkacem et al., PRC 58, 1727 (1998)

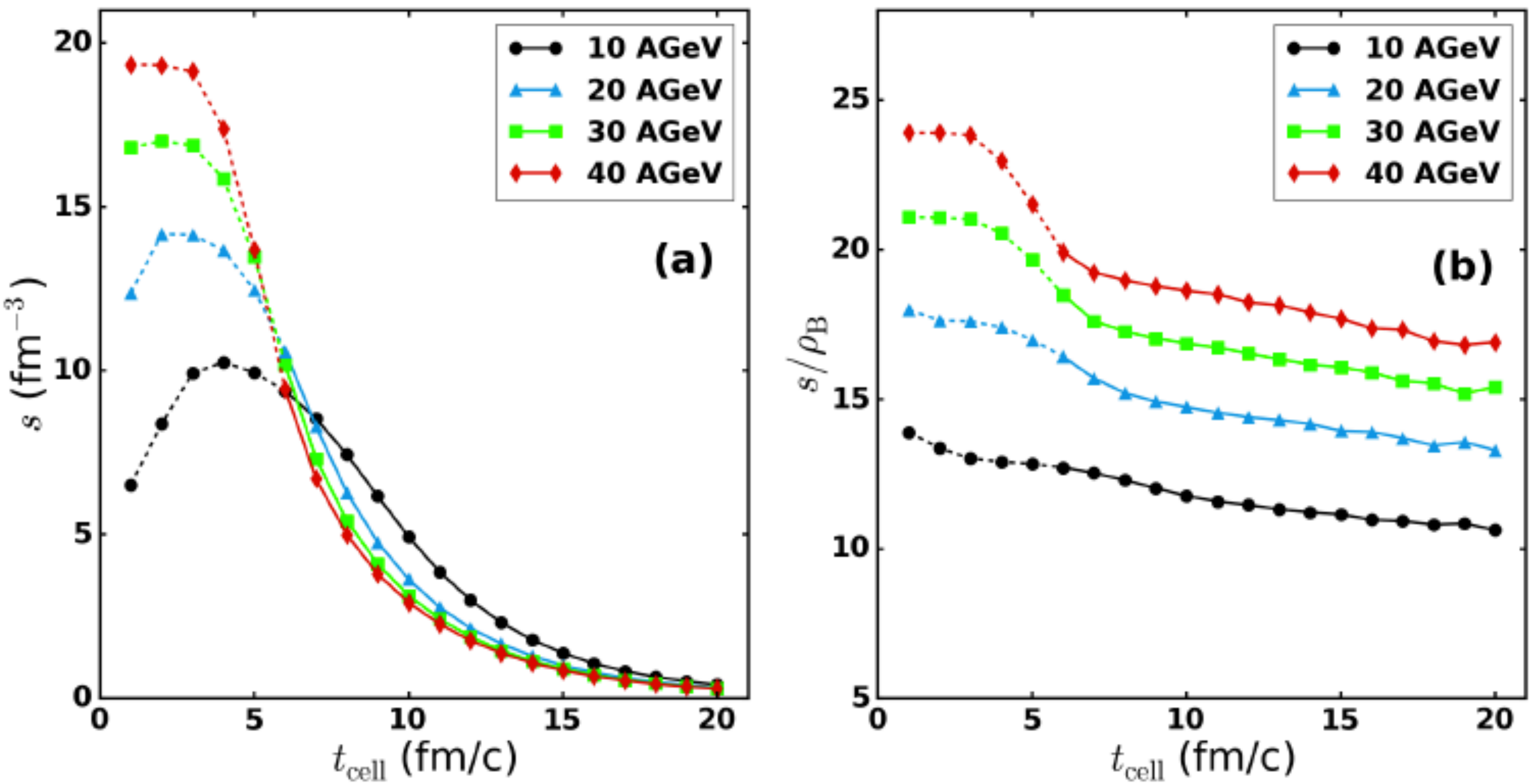
Nearly the same temperature and complete isotropy of dN/dp_T

Cell + SM



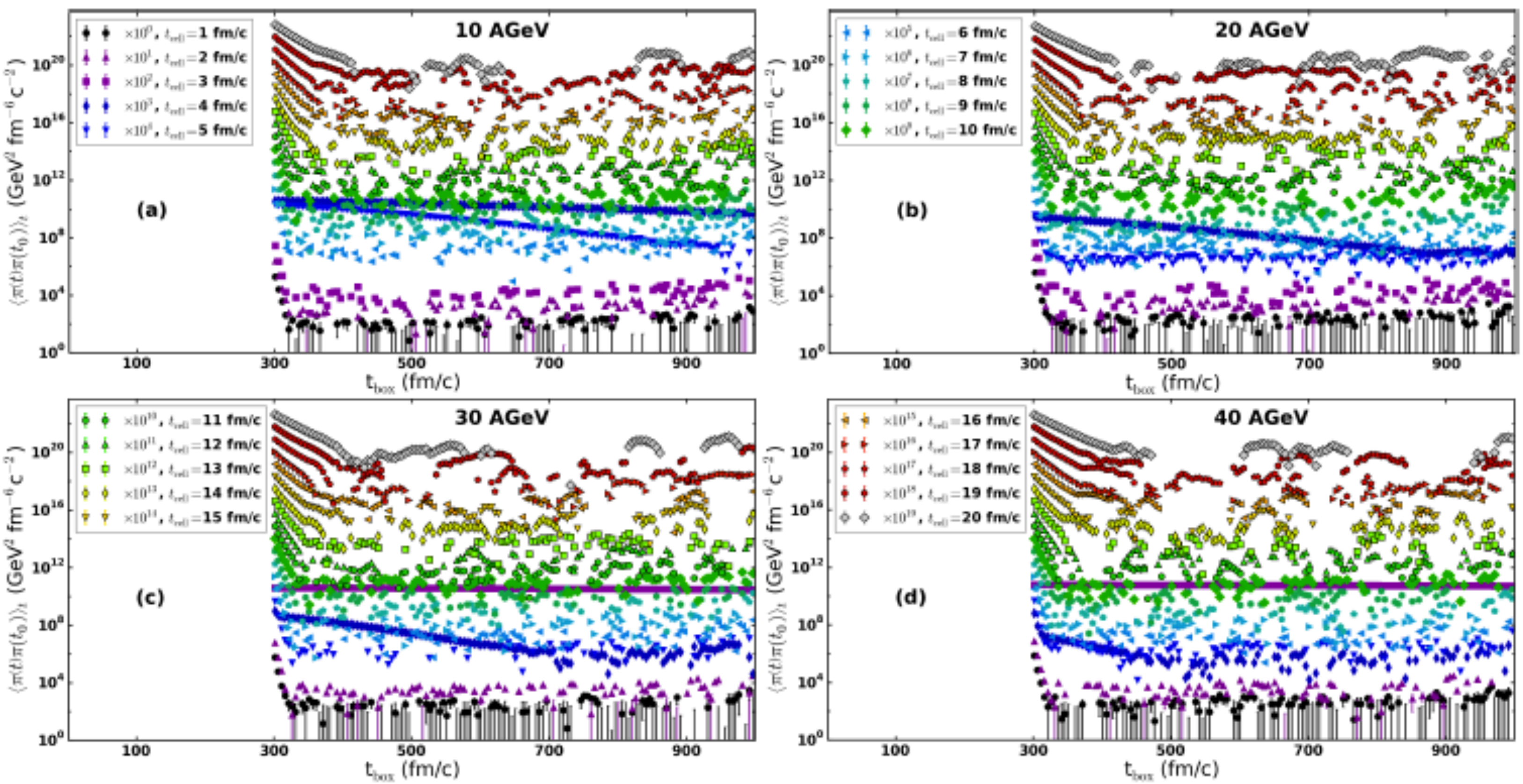
Dependence of $\varepsilon, \rho_B, \rho_S$ (from cell) and of T, μ_B, μ_S (from SM) on t_{cell}

SM, Boltzmann entropy s



Dynamics of Boltzmann entropy density s and of $s/\rho_{\rm B}$ in cell

Results: $\langle \pi(t)\,\pi(t_0) \rangle_t$ at $E \in [10, 20, 30, 40]$ AGeV

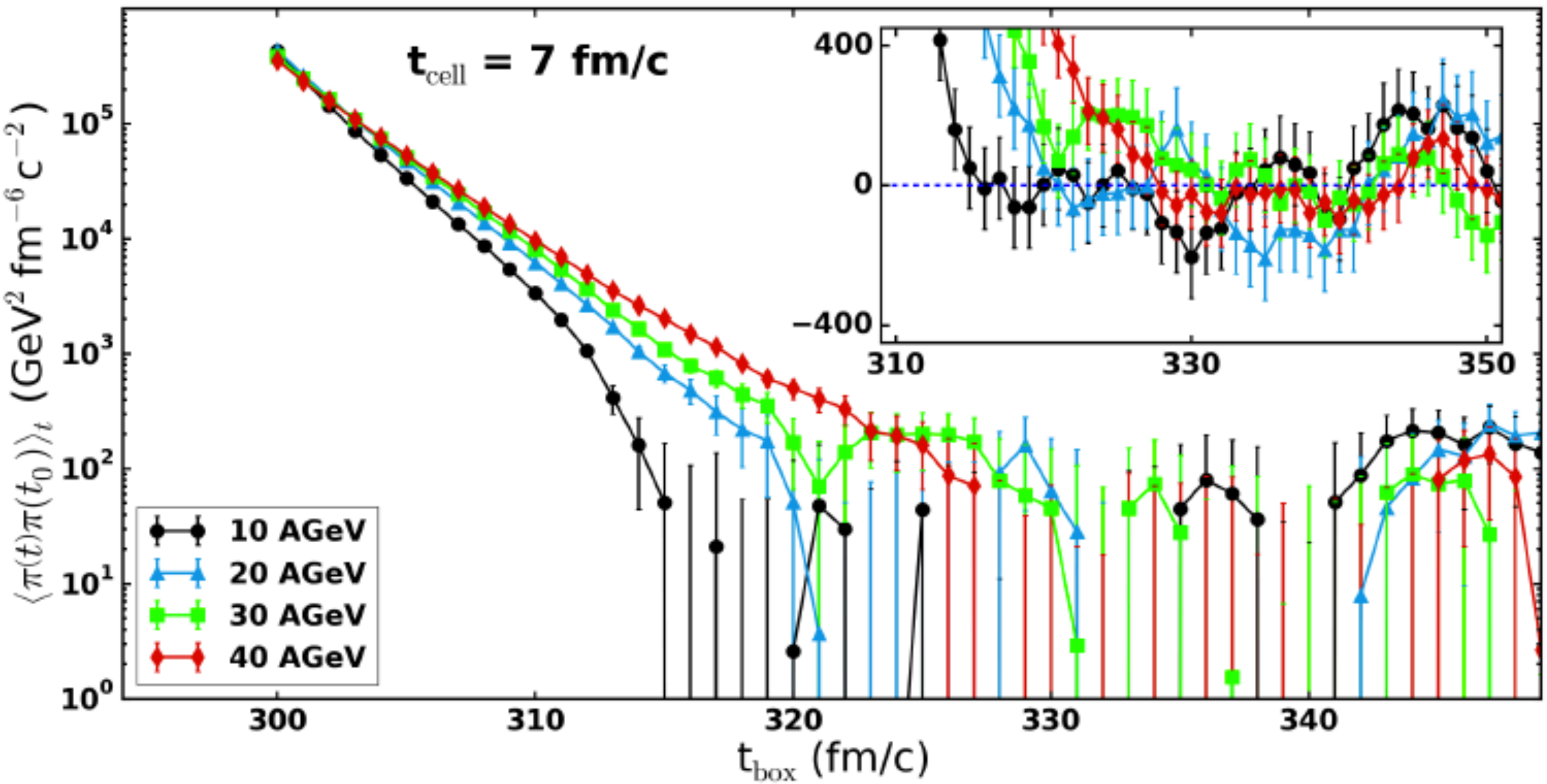


Time dependence of correlators $\langle \pi(t)\,\pi(t_0) \rangle_t$

$t_0 = 300$ fm/c

$t_{\mathrm{cell}} \in \{1 \div 20\}$ fm/c

Results: $\langle \pi(t)\pi(t_0) \rangle_t$ at fixed t_{cell}



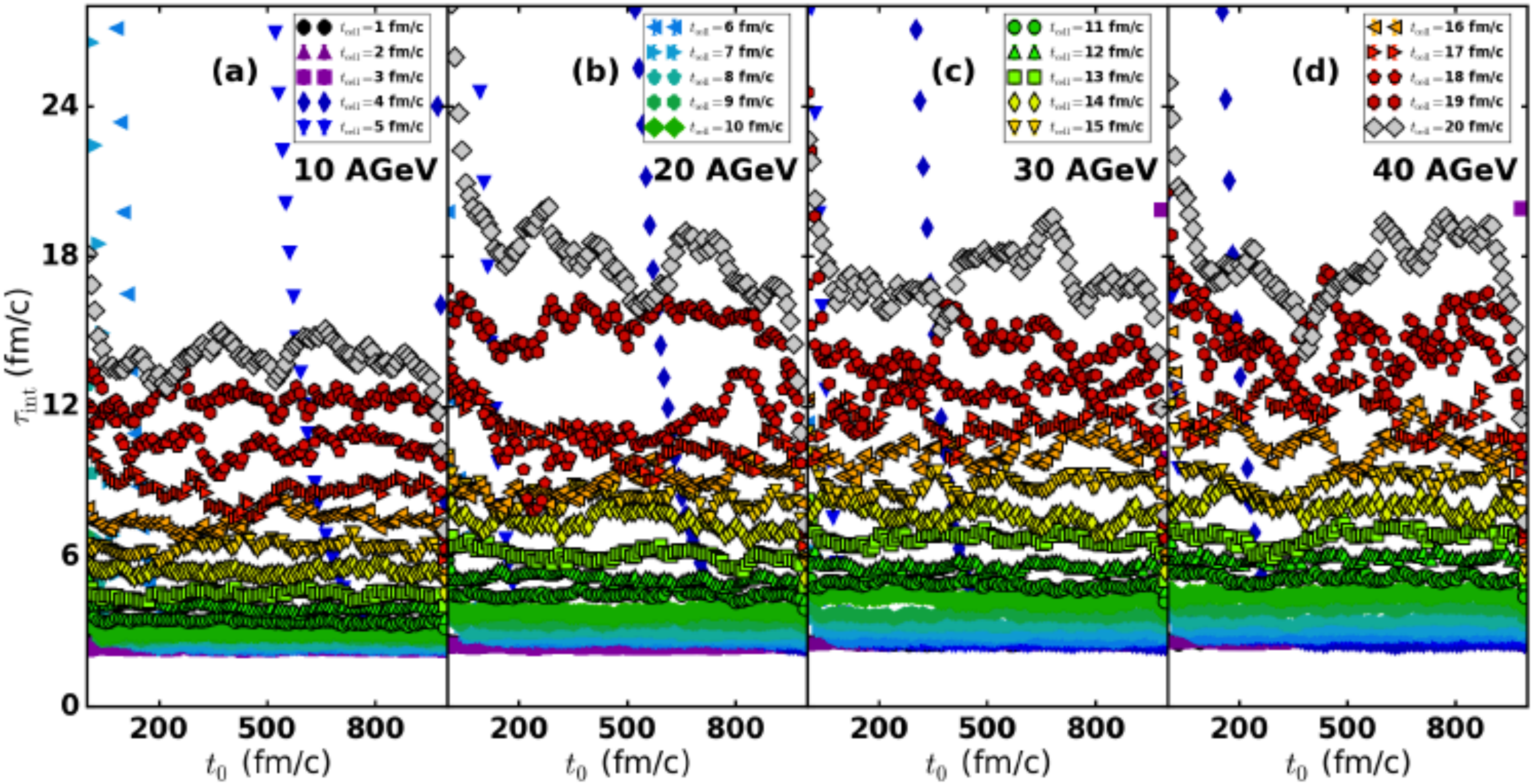
Time dependence of correlators $\langle \pi(t)\pi(t_0) \rangle_t$

Subplot: the same but at linear scale

$t_0 = 300$ fm/c

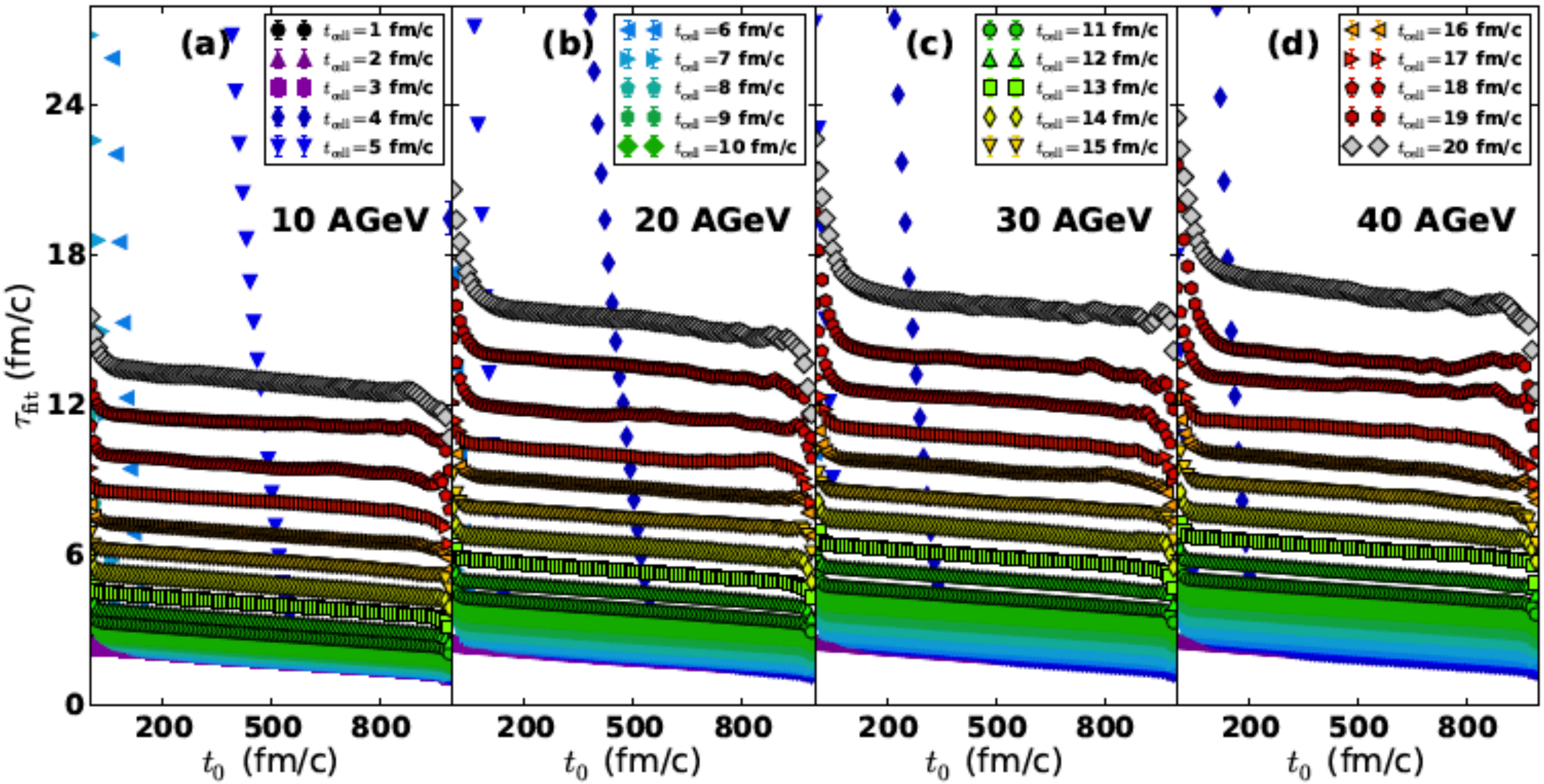
$t_{\text{cell}} = 7$ fm/c

Results: $\tau\,(t_0)$



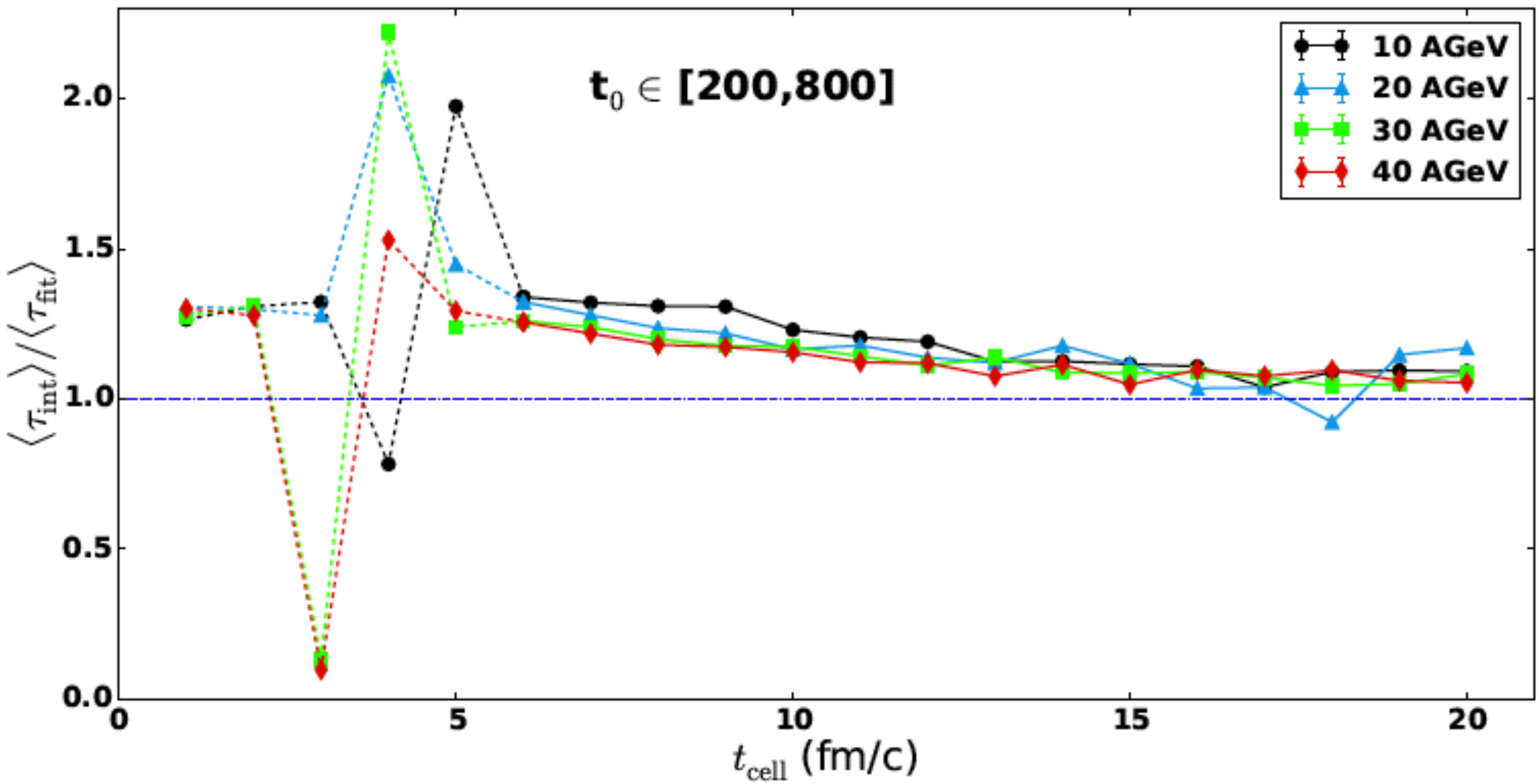
Dependence of τ on t_0

Results: τ from the fit



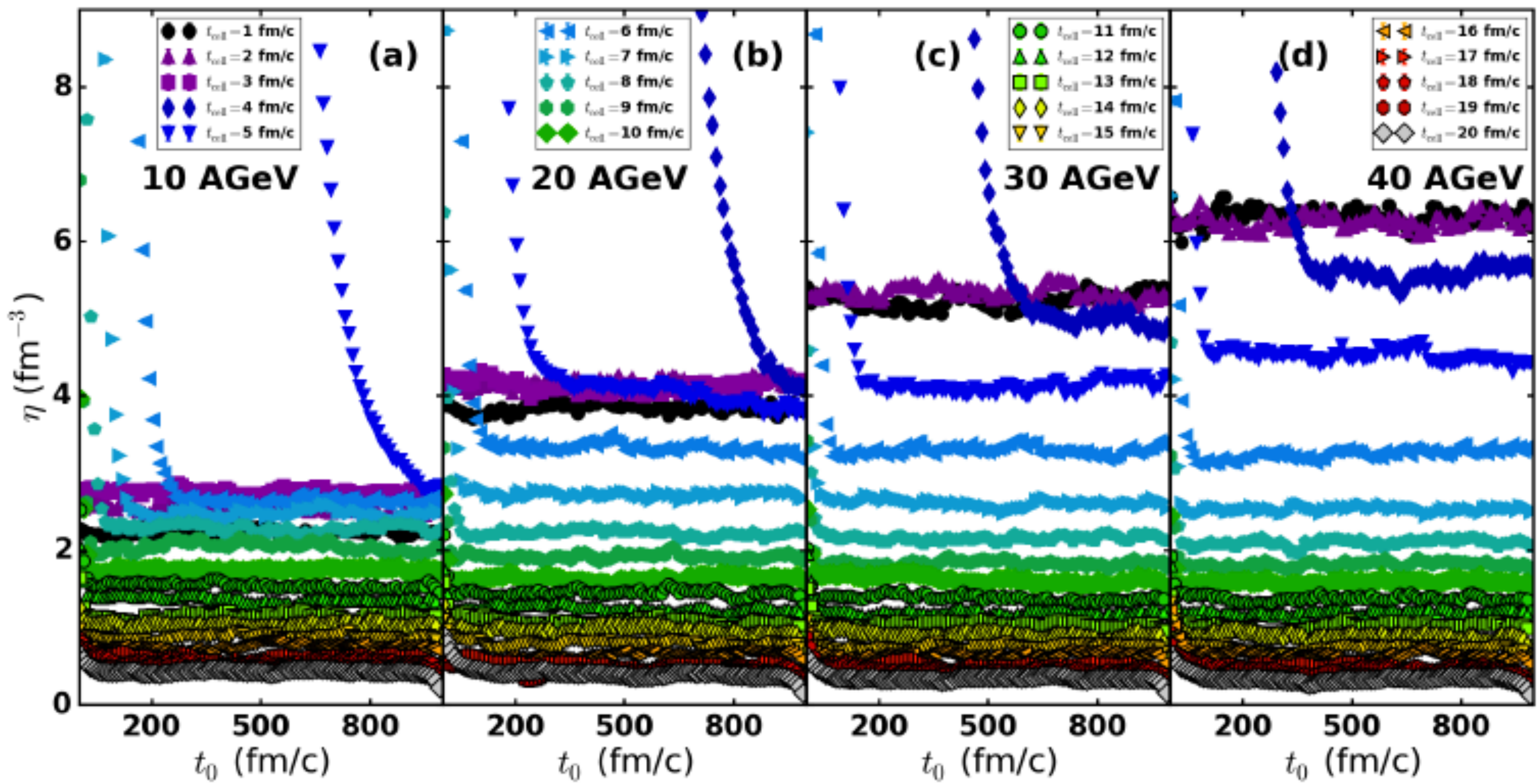
Dependence of τ_{fit} on t_0

Results: Comparison of τ_{int} and τ_{fit}



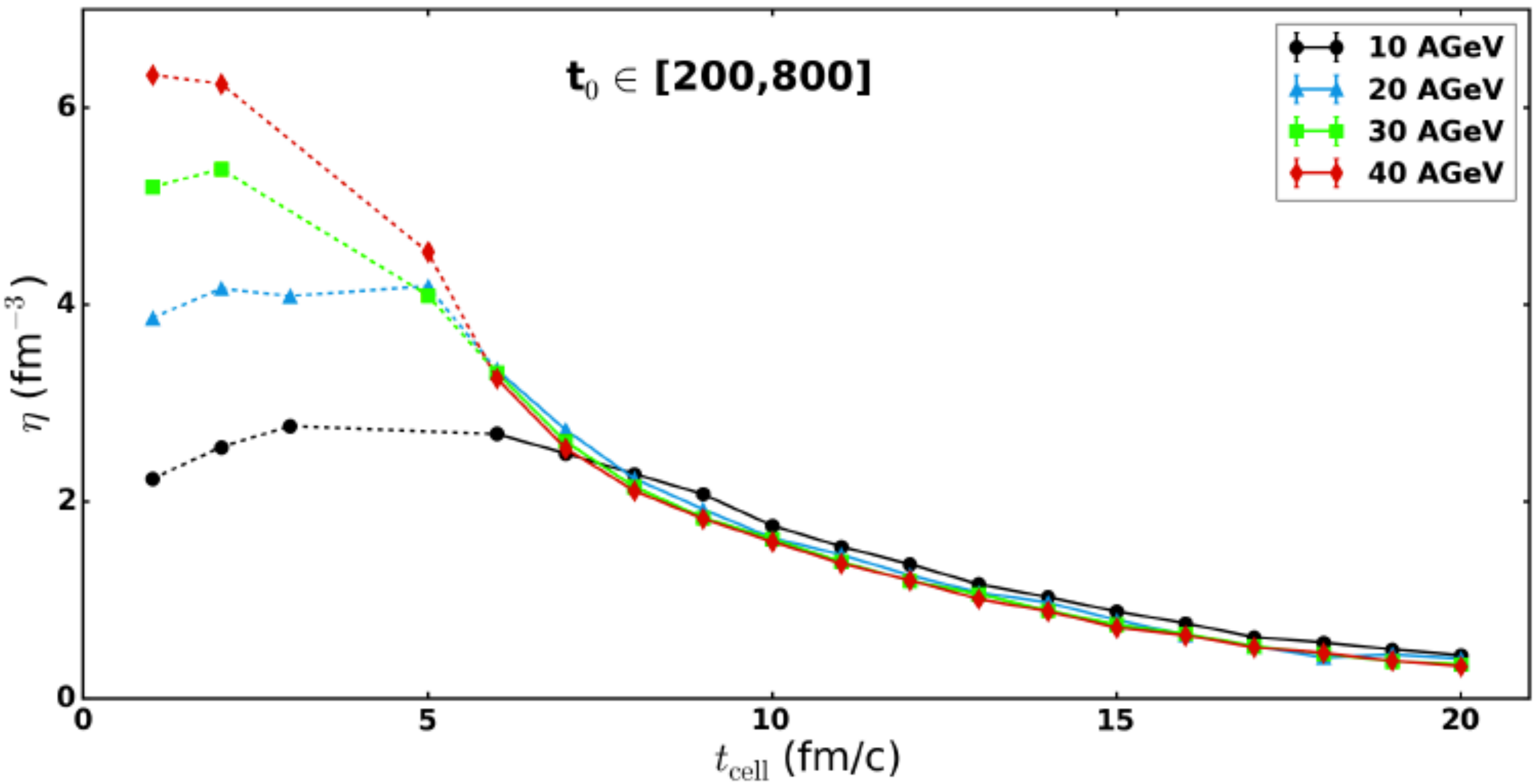
Ratio of τ_{int}/τ_{fit}

Results: viscosity $\eta\,(t_0)$



Dependence of η on t_0

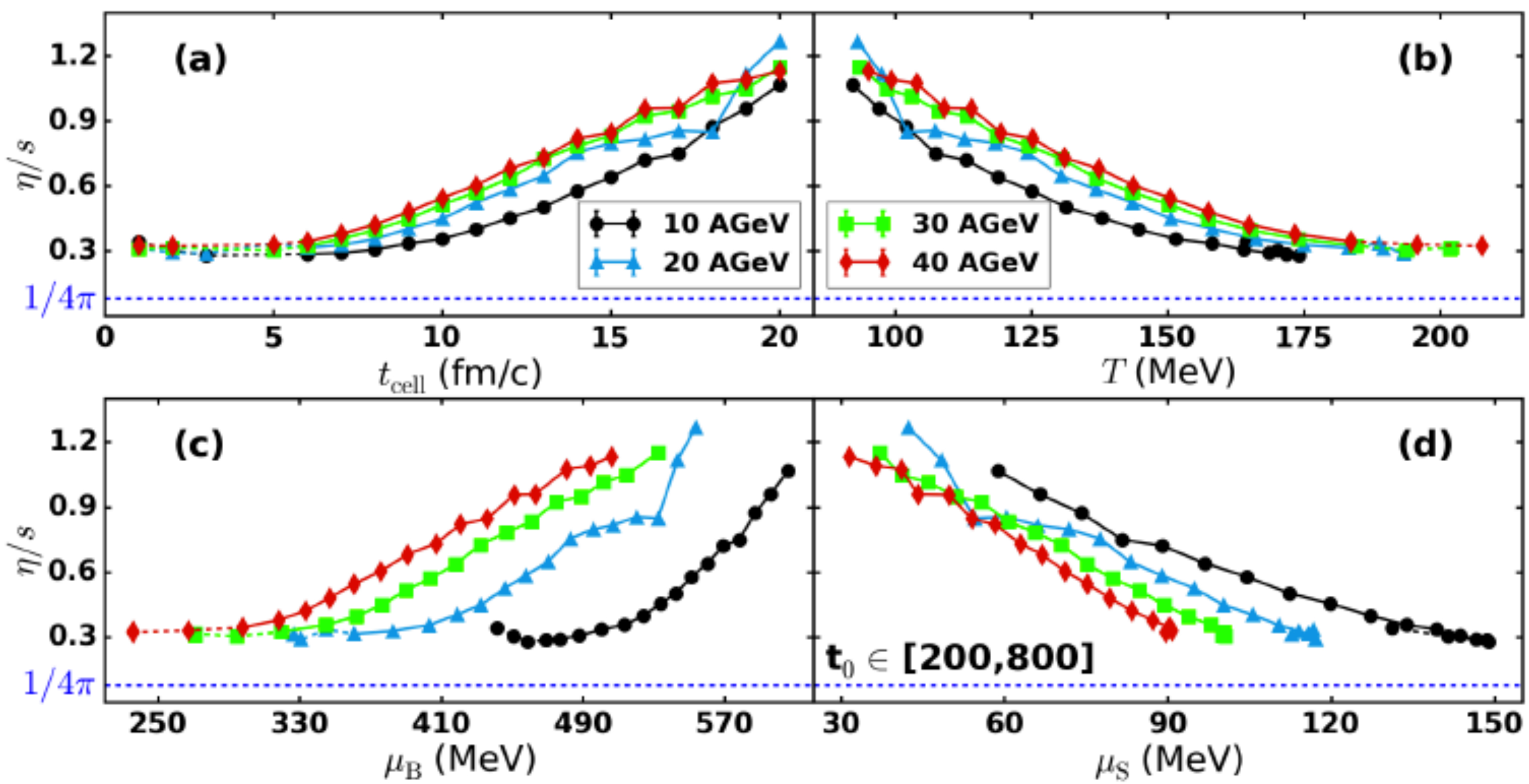
Results: viscosity $\eta\,(t_{\text{cell}})$



Dynamics of η in cell

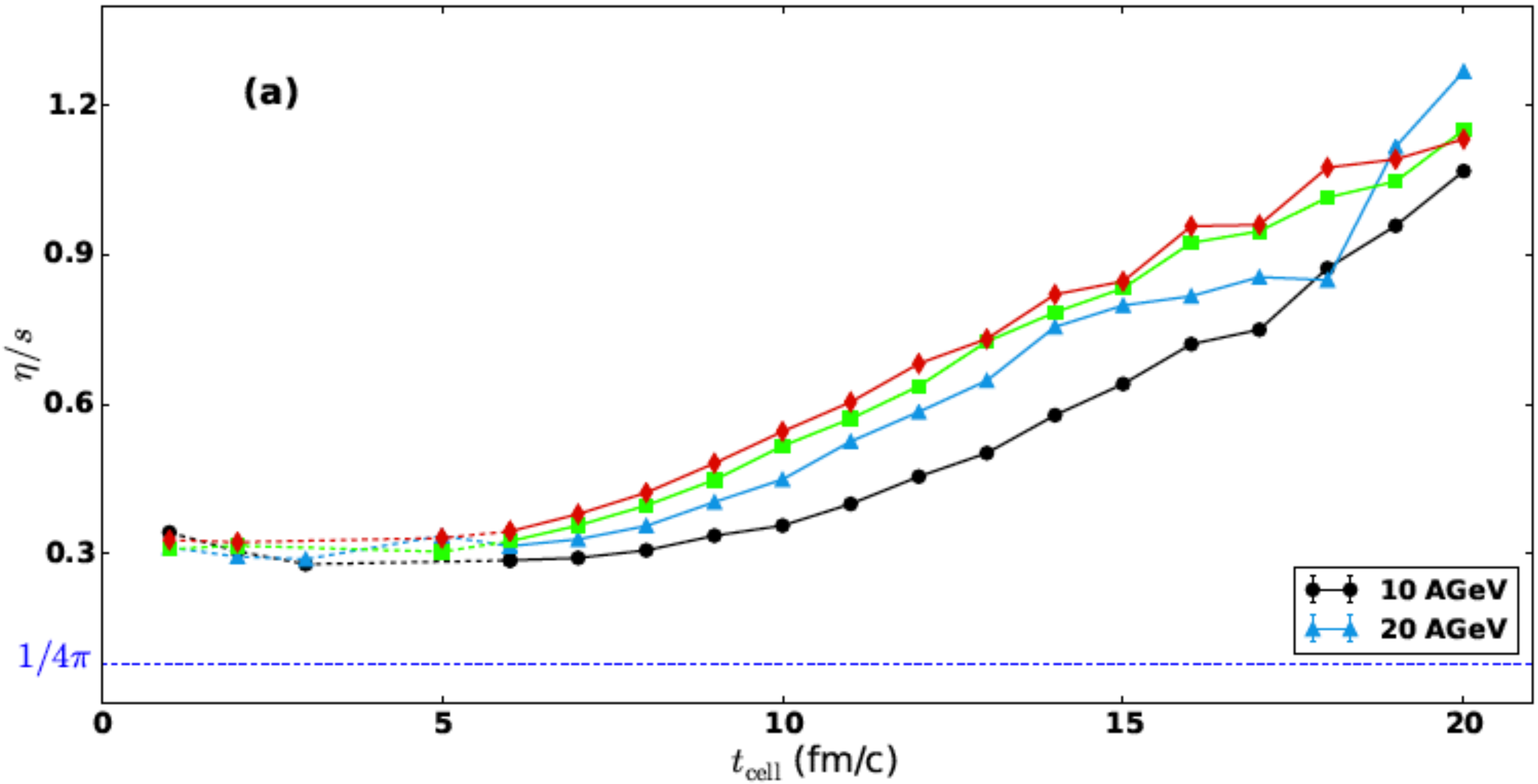
All curves sit on the top of each other for $t_{\text{cell}} \geq 7$ fm/c

Results: η / s_{SM}



Dynamics of η / s_{SM} in cell
as function of time, T, $\mu_{\rm B}$, $\mu_{\rm S}$

Results: η/s_{SM}



Dynamics of η/s_{SM} in cell

η/s increases with time for $t_{\text{cell}} \geq 6$ fm/c for all four energies

Minimum - for 10 AGeV, corresponding to 4.5 GeV in c.m. frame

Entropy density of nonequilibrium state

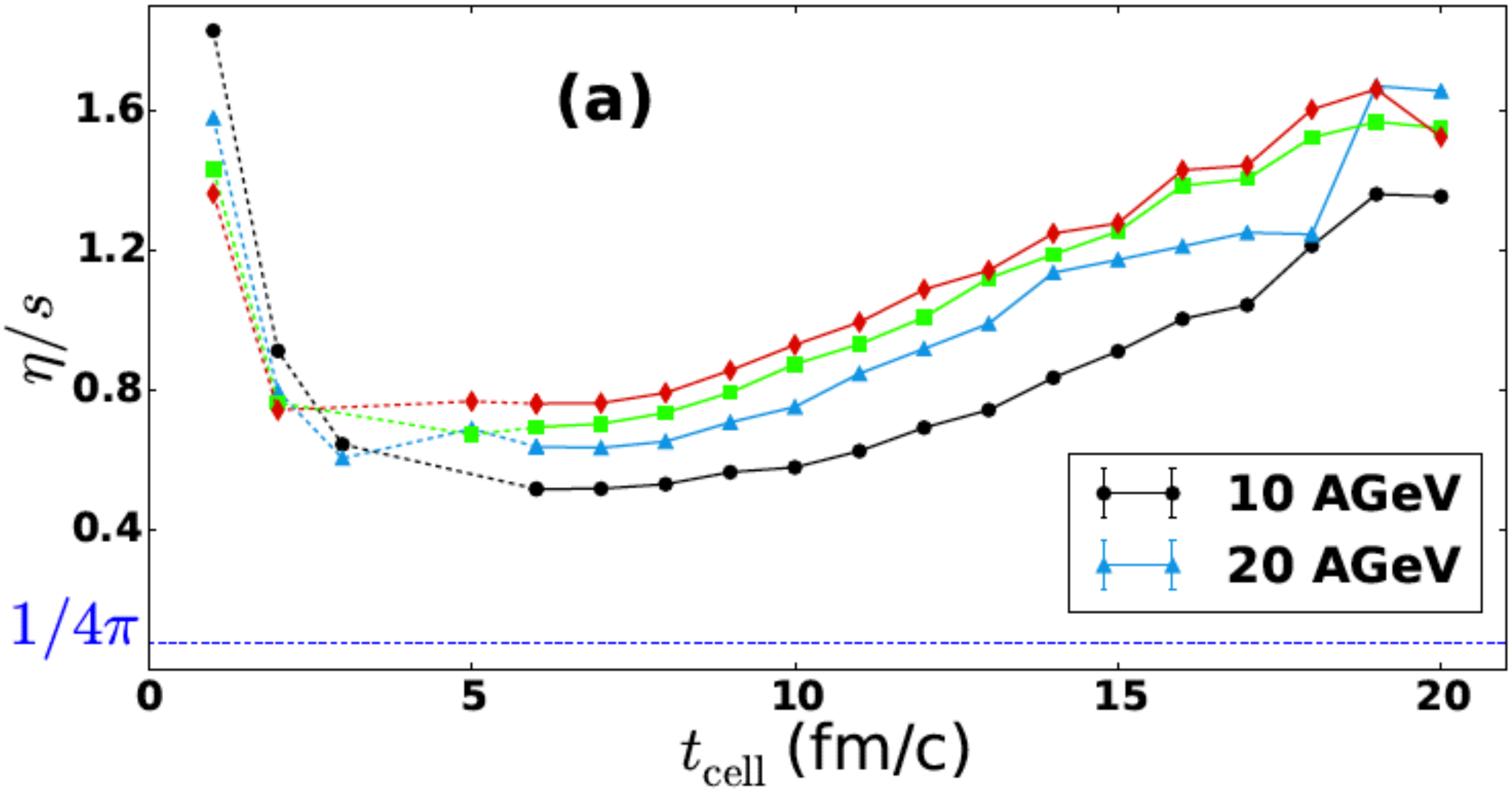
Entropy density

$$s = -\sum_i \frac{g_i}{(2\pi\hbar)^3} \int_0^\infty f_i(p, m_i)\ [\ln f_i(p, m_i) - 1]\ d^3p$$

Microscopic distribution function

$$f_i^{\mathrm{mic}}(p) = \frac{(2\pi\hbar)^3}{V g_i} \frac{dN_i}{d^3p}$$

Results: $\eta/s_{noneq.}$

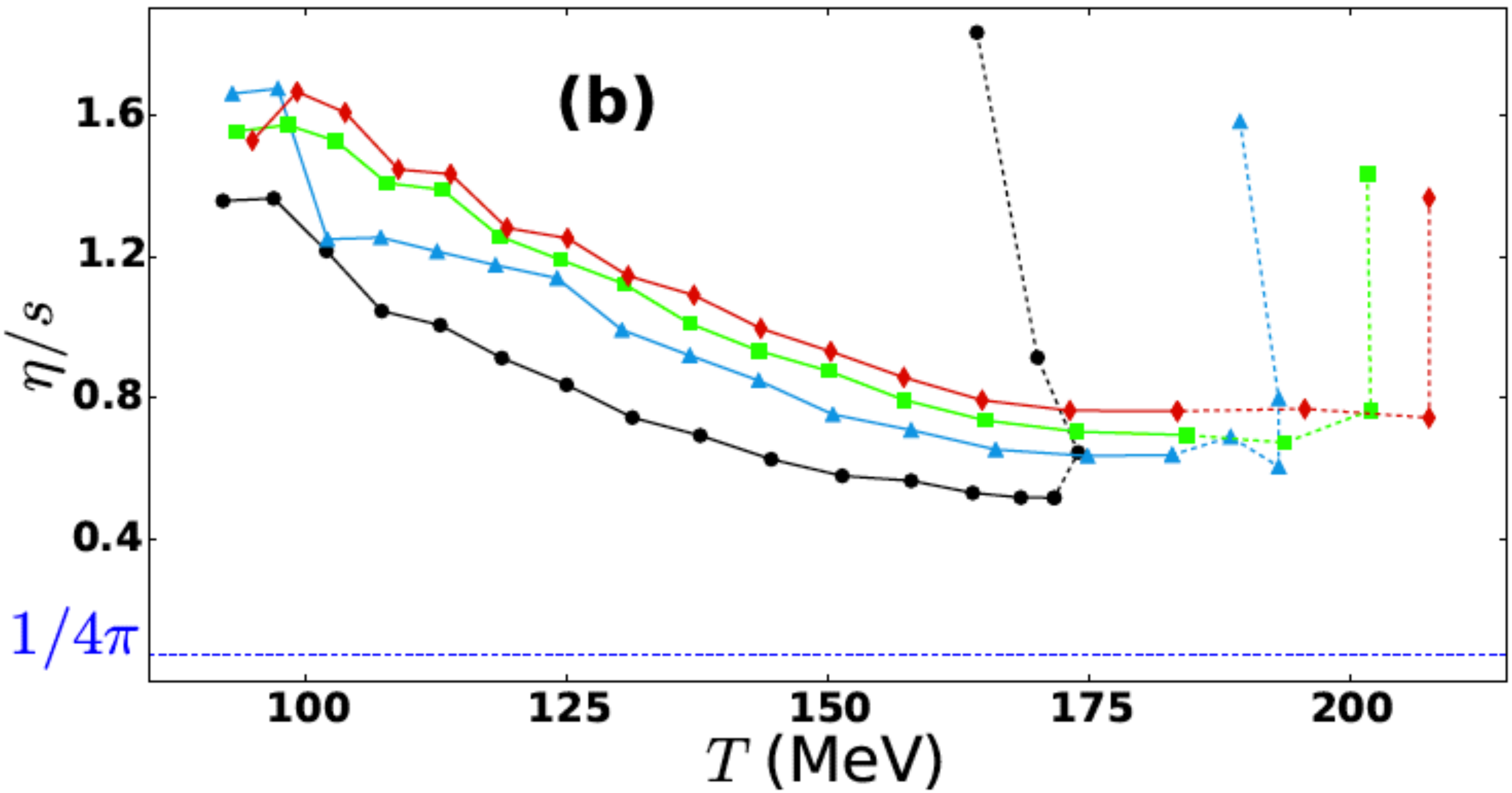


Dynamics of $\eta/s_{noneq.}$ in cell

η/s drops with time for $t_{\rm cell} \leq 6$ fm/c. Then it increases for all four energies

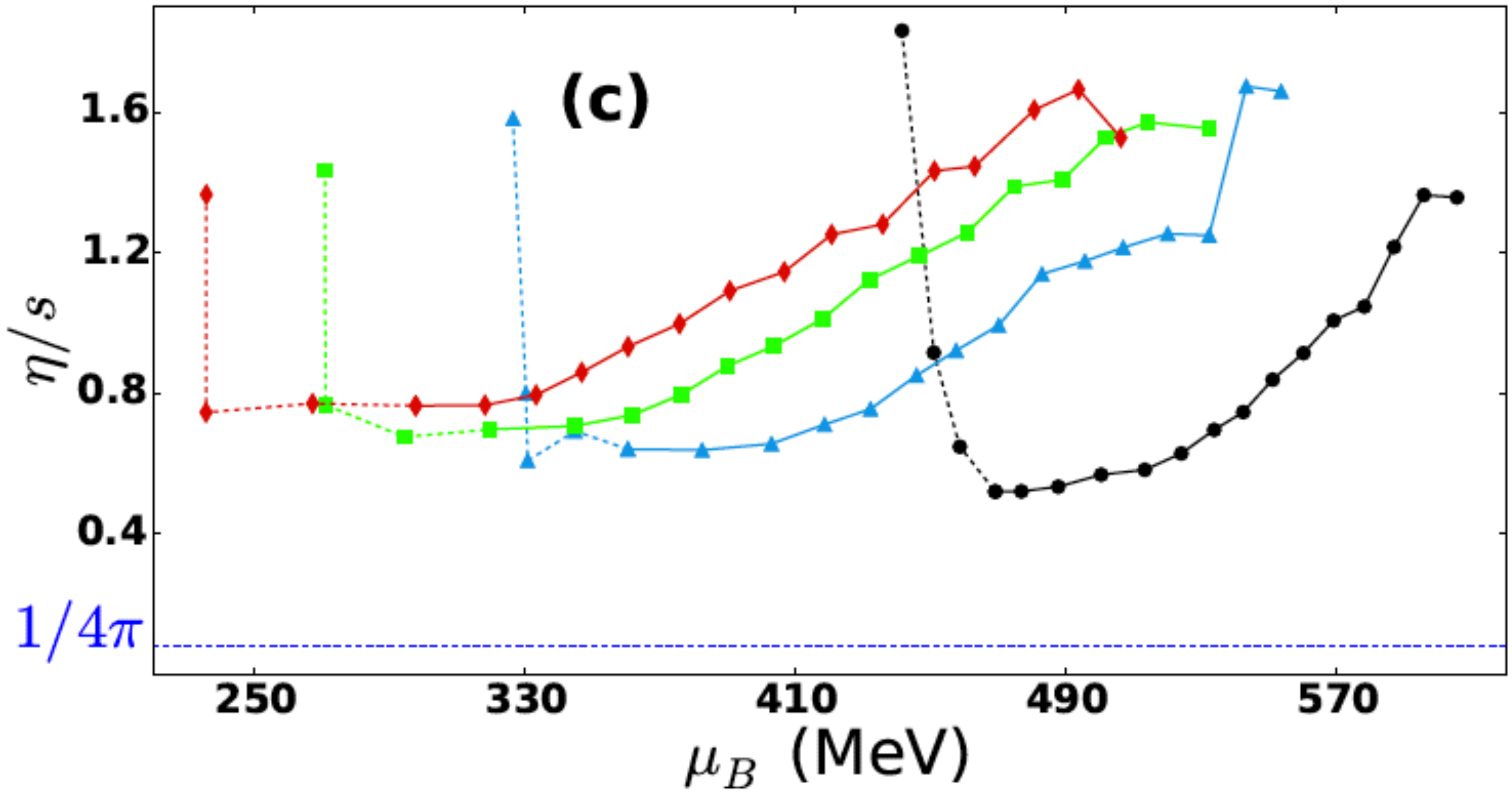
Pronounced minima for all reactions

Results: $\eta/s_{noneq.}$



Dynamics of $\eta/s_{noneq.}$ in cell

η/s increases with temperature drop at $t_{\rm cell} \geq 6$ fm/c for all four energies

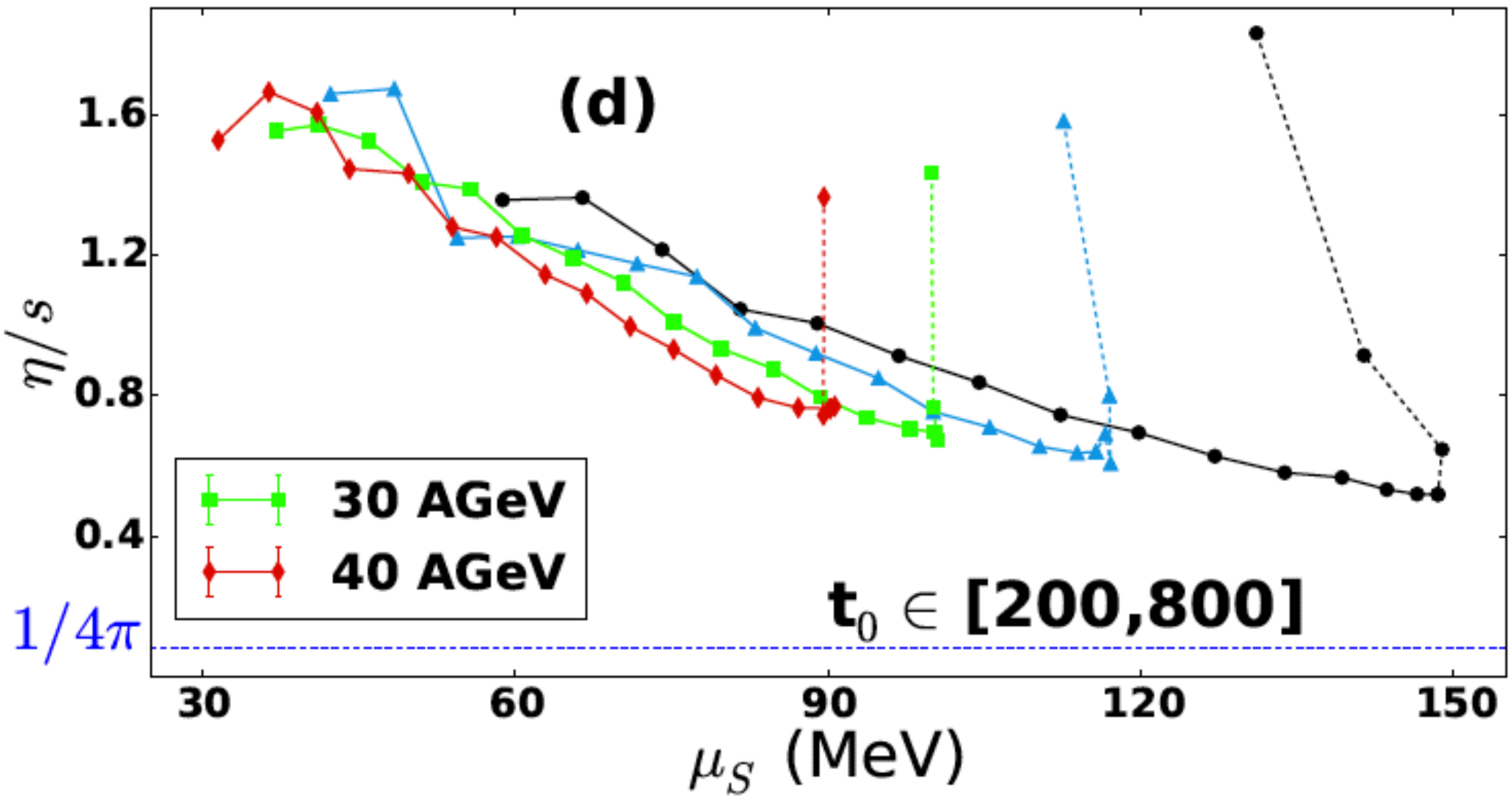


Dynamics of $\eta/s_{noneq.}$ in cell

η/s increases with increase of μ_B for $t_{\rm cell} \geq 6$ fm/c for all four energies

Clear minimum for 10 AGeV

Results: $\eta/s_{noneq.}$

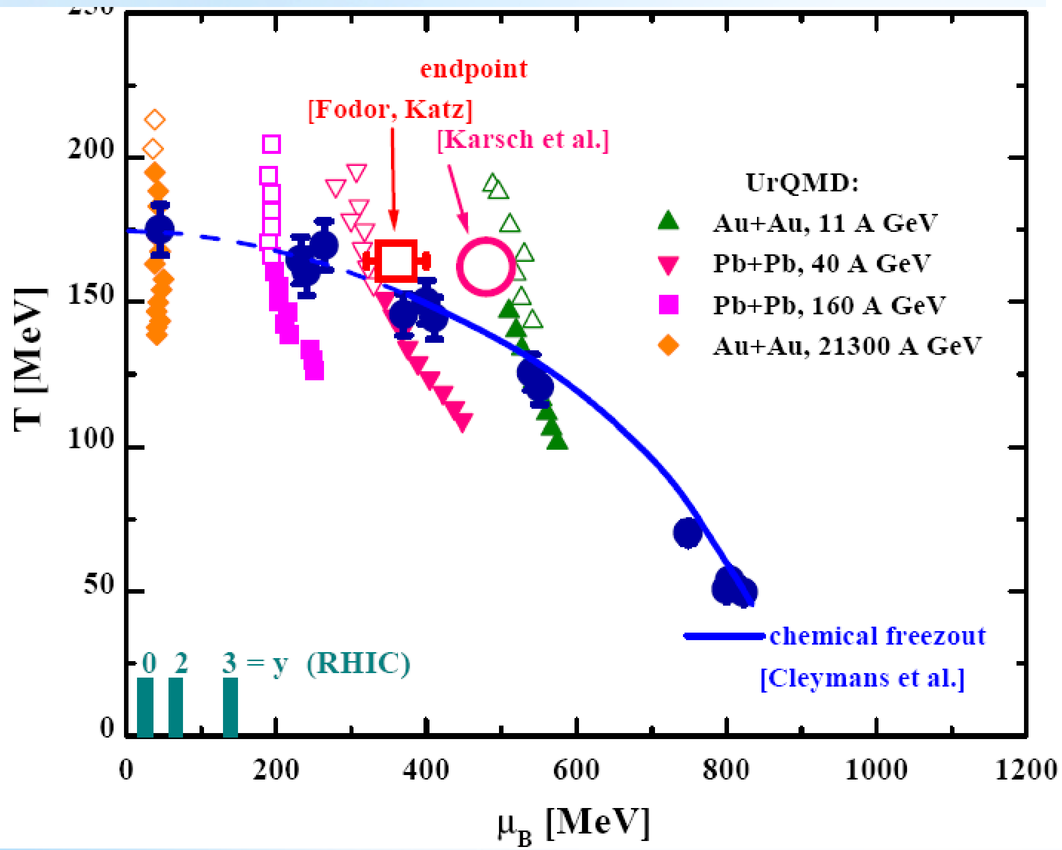


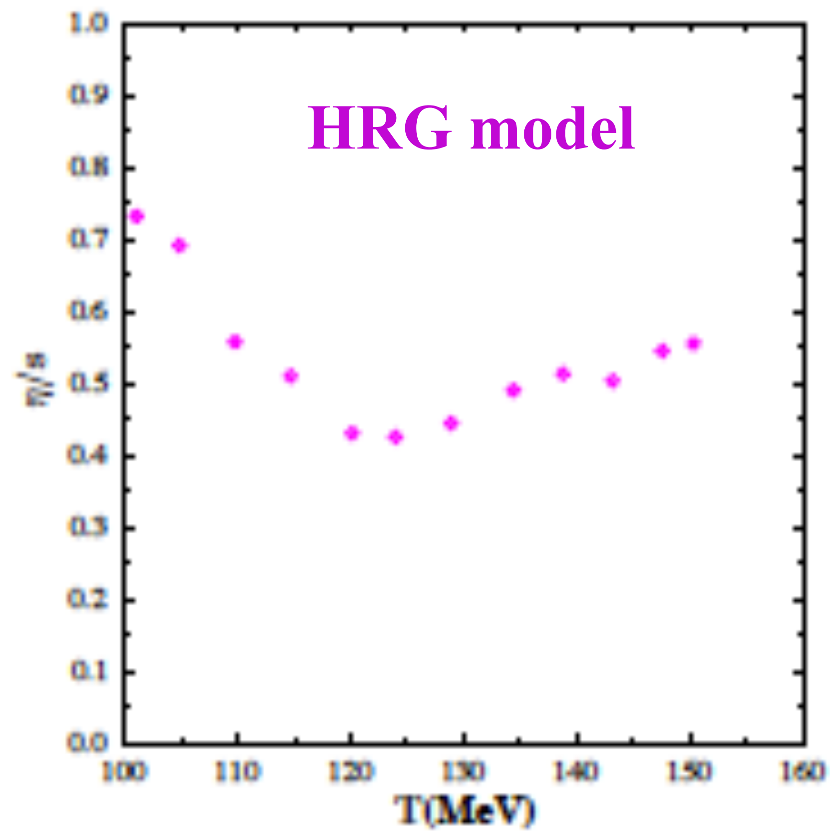
Dynamics of $\eta/s_{noneq.}$ in cell

η/s increases with drop of μ_S for $t_{\rm cell} \geq 6$ fm/c for all four energies

Reliability of obtained results

Central cell: comparison to chemical freeze-out conditions





G.Kadam, S.Pawar, arXiv:1802.01942 [hep-ph]

Conclusions

- data from central cell of UrQMD calculations are used as input for SM to calculate temperature T and entropy density s, and for UrQMD box calculations to estimate shear viscosity η
- box data are taken within the range $200 \le t_0 \le 800$ fm/c because:

  - values at $t_0 < 200$ fm/c are distorted by the initial fluctuation in the box

  - values at $t_0 > 800$ fm/c may be disturbed by the analog of Brownian motion
- it is shown that for all four tested energies η and s in the cell drop with time
- ratios η/s reach minima about 0.3(0.5) at $t \approx 5$ fm/c for all energies. Then, the ratios rise to $1.0 \div 1.2$ $(1.3 \div 1.6)$ at $t = 20$ fm/c
- this increase is accompanied by the simultaneous rise of μ_B and drop of both T and μ_S in the cell