$\Upsilon(3S)$ and $\chi_b(3P)$ production and polarization in the NRQCD with k_T-factorization

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arXiv:1909.05141

- In recent years, the production processes of quarkonia J/ψ and $\Upsilon(nS)$ have been actively studied after the discovery of a strong discrepancy between theoretical predictions within the framework of the color singlet model (CS) and the data obtained at the Tevatron.
- A theoretical framework for the description of heavy quarkonia production and decays is provided by the non-relativistic QCD (NRQCD) factorization.

  G. Bodwin, E. Braaten, G. Lepage, Phys. Rev. D51, 1125 (1995);

  P. Cho, A.K. Leibovich, Phys. Rev. D53, 150 (1996); Phys. Rev. D53, 6203 (1996)
- NRQCD is an effective field theory, which is used to describe the production of heavy quarkonia, based on the expansion in υ and α_s:

$$|\psi_Q\rangle = O(1)|Q\bar{Q}[^3S_1^{(1)}]\rangle + O(\upsilon)|Q\bar{Q}[^3P_J^{(8)}]g\rangle + O(\upsilon^2)|Q\bar{Q}[^3S_1^{(1,8)}]gg\rangle + \\ + O(\upsilon^2)|Q\bar{Q}[^1S_0^{(8)}]g\rangle + O(\upsilon^2)|Q\bar{Q}[^3D_J^{(1,8)}]gg\rangle + \ldots$$

- This formalism implies a separation of perturbatively calculated short-distance cross-sections for the production of $Q\bar{Q}$ pair in an intermediate Fock state ${}^{2S+1}L_J^{(a)}$ and long-distance non-perturbative matrix elements (NMEs), which describe the transition of that intermediate $Q\bar{Q}$ state into a physical quarkonium via soft gluon radiation.
- They are assumed to be universal (process- and energy-independent), not dependent on the quarkonium momentum and obeying certain hierarchy in powers of the relative heavy quark velocity $v_Q \sim \log^{-1} m_Q/\Lambda_{QCD}$.
- The color octet (CO) NMEs are not calculable within the theory and have to be only extracted from the data.

- The cross sections of prompt S -and P-wave quarkonia production in pp collisions are known at the NLO NRQCD and the dominant tree-level NNLO* corrections to the color-singlet production mechanism have been calculated.

  P. Artoisenet et al., Phys. Rev. Lett. 101, 152001 (2008)
- With properly adjusted values of NMEs, one can achieve a good agreement between the NLO NRQCD predictions and the experimental data on the quarkonia transverse momenta distributions.
- However, the extracted NMEs strongly depend on the minimal quarkonia transverse momentum p_T used in the fits and are almost incompatible with each other when obtained from fitting different data sets.
- Moreover, none of the fits can simultaneously describe data on the polarization and the quarkonia production.

- A possible solution to the "polarization puzzle" has been proposed recently in the framework of a model that interprets the soft final state gluon radiation as a series of color-electric dipole transitions.

  S.P. Baranov, Phys. Rev. D 93, 054037 (2016)

- The proposed approach leads to unpolarized or only weakly polarized quarkonia either because of the cancellation between the ${}^3P_1^{(8)}$ and ${}^3P_2^{(8)}$ contributions or as a result of two successive color-electric $E1$ dipole transitions in the chain ${}^3S_1^{(8)} \to {}^3P_J^{(8)} \to {}^3S_1^{(1)}$ giving us the possibility to simultaneously describe the polarization parameters and production for mesons.

- The main goal of this research is a detailed study of the bottomonia production processes and their polarization properties at the LHC.
- The data has been measured recently by the CMS, ATLAS and LHCb Collaborations. And polarization of $\Upsilon(nS)$ mesons has been investigated by the CMS and also by the CDF Collaboration at Tevatron.
- Due to heavier masses of bottomonia and smaller relative velocity v_b of b quarks in the bottomonium rest frame ($v_b \simeq 0.08$ against $v_c \simeq 0.23$), these processes could be even a more suitable case to apply because of a more faster convergence of the double NRQCD expansion in strong coupling α_s and v_Q.
- We are taking into account the latest measurements on the $\chi_b(mP)$ production. The latter have been observed recently by the LHCb Collaboration for the first time and found to be rather significant (up to 40%).

- To describe the perturbative production of the $b\bar{b}$ pair, the k_T-factorization approach of QCD is used by taking into account both CS and CO contributions.
- This approach is based on the Balitsky-Fadin-Kuraev-Lipatov (BFKL) or Ciafaloni-Catani-Fiorani-Marchesini (CCFM) evolution equations, which resum large logarithmic terms proportional to $\ln s \sim \ln(1/x)$, important at high energies.
- The k_T-factorization approach has certain technical advantages in the ease of including higher-order radiative corrections (namely, leading part of NLO + NNLO + ... terms) in the form of transverse momentum dependent (TMD, or unintegrated) gluon density function in a proton $f(x, \mathbf{k}_T^2, \mu^2)$.

- Our consideration is based on the off-shell gluon-gluon fusion subprocesses that represent the true LO in QCD:

$$g^*(k_1) + g^*(k_2) \to \Upsilon[^3S_1^{(1)}](p) + g(k),$$

$$g^*(k_1) + g^*(k_2) \to \Upsilon[^1S_0^{(8)}, {}^3S_1^{(8)}, {}^3P_J^{(8)}](p).$$

$$g^*(k_1) + g^*(k_2) \to \chi_{bJ}(p)[^3P_J^{(1)}, {}^3S_1^{(8)}] \to \Upsilon(p_1) + \gamma(p_2),$$

- To obtain the production amplitudes for $b\bar{b}$ states with required quantum numbers from the ones for an unspecified $b\bar{b}$ state we use the appropriate projection operators. These operators for the spin-singlet and spin-triplet states read:

$$\Pi_0 = (\hat{p}_{\bar{b}} - m_b)\gamma_5(\hat{p}_b + m_b)/m^{3/2},$$

$$\Pi_1 = (\hat{p}_{\bar{b}} - m_b)\hat{\epsilon}(S_z)(\hat{p}_b + m_b)/m^{3/2},$$

where $m = 2m_b$, $p_b = p/2 + q$ and $p_{\bar{b}} = p/2 - q$.

- To calculate off-shell production amplitudes we integrate the product of the hard scattering amplitude $A(q)$ expanded in a series around $q = 0$ and meson bound state wave function $\Psi^{(a)}(q)$ with respect to q:

$$A(q)\Psi^{(a)}(q) = A|_{q=0}\Psi^{(a)}(q) + q^\alpha(\partial A/\partial q^\alpha)|_{q=0}\Psi^{(a)}(q) + \dots$$

- A term-by-term integration of this series employs the identities:

$$\int \frac{d^3q}{(2\pi)^3}\Psi^{(a)}(q) = \frac{1}{\sqrt{4\pi}}\mathcal{R}^{(a)}(0),$$

$$\int \frac{d^3q}{(2\pi)^3}q^\alpha\Psi^{(a)}(q) = -i\epsilon^\alpha(L_z)\frac{\sqrt{3}}{\sqrt{4\pi}}\mathcal{R}'^{(a)}(0),$$

where $\mathcal{R}^{(a)}(x)$ is the radial wave function in the coordinate representation.

- The corresponding NMEs are directly related to the wave functions $\mathcal{R}^{(a)}(x)$ and their derivatives:

$$\langle \mathcal{O}^{\mathcal{Q}}[^{2S+1}L_J^{(a)}] \rangle = 2N_c(2J+1)|\mathcal{R}^{(a)}(0)|^2/4\pi,$$

$$\langle \mathcal{O}^{\mathcal{Q}}[^{2S+1}L_J^{(a)}] \rangle = 6N_c(2J+1)|\mathcal{R}'^{(a)}(0)|^2/4\pi$$

where $N_c = 3$.

- Additionally, the NMEs obey the multiplicity relations coming from heavy quark spin symmetry (HQSS) at LO:

$$\langle \mathcal{O}^{\mathcal{Q}}[^3P_J^{(a)}] \rangle = (2J+1)\langle \mathcal{O}^{\mathcal{Q}}[^3P_0^{(a)}] \rangle.$$

- The summation over polarizations of the incoming off-shell gluons is performed according the BFKL prescription:

$$\sum \epsilon^{\mu}\epsilon^{*\nu} = \boldsymbol{k}_T^{\mu}\boldsymbol{k}_T^{\nu}/\boldsymbol{k}_T^2$$

- The spin density matrix of the S-wave quarkonia is expressed in terms of the momenta l_1 and l_2 of the decay leptons:

$$\sum \epsilon^{\mu}\epsilon^{*\nu} = 3(l_1^{\mu}l_2^{\nu} + l_1^{\nu}l_2^{\mu} - \frac{m^2}{2}g^{\mu\nu})/m^2$$

  This expression is equivalent to the standard expression

$$\sum \epsilon^{\mu}\epsilon^{*\nu} = -g^{\mu\nu} + p^{\mu}p^{\nu}/m^2$$

- In all other respects the evaluation follows the standard QCD Feynman rules.
- The obtained results have been explicitly tested for gauge invariance. We have observed their gauge invariance even with off-shell initial gluons.

- To describe the transition of an unbound octet $b\bar{b}$ quark pair to an observed singlet state we employ the mechanism by S. Baranov.
- It was already used for the prompt charmonia production.
  S.P. Baranov, A.V. Lipatov, Eur. Phys. J. C79, 621 (2019);
  S.P. Baranov, A.V. Lipatov, arXiv:1906.07182 [hep-ph]
- In this approach, a soft gluon with a small energy $E \sim \Lambda_{QCD}$ is emitted after the hard interaction is over, bringing away the unwanted color and changing other quantum numbers of the produced CO system.
- Thus, having small energy of the emitted gluons gives us the confidence that we do not enter the confinement or perturbative domains.
- In our calculations such soft gluon emission is described by a classical multipole expansion, in which the electric dipole ($E1$) transition dominates.

- Only a single $E1$ transition is needed to transform a P-wave state into an S-wave state and the structure of the respective ${}^3P_J^{(8)} \to {}^3S_1^{(1)} + g$ amplitudes is given by:

$$A({}^3P_0^{(8)} \to \Upsilon + g) \sim k_\mu^{(g)} p^{(\mathrm{CO})\mu} \epsilon_\nu^{(\Upsilon)} \epsilon^{(g)\nu},$$

$$A({}^3P_1^{(8)} \to \Upsilon + g) \sim e^{\mu\nu\alpha\beta} k_\mu^{(g)} \epsilon_\nu^{(\mathrm{CO})} \epsilon_\alpha^{(\Upsilon)} \epsilon_\beta^{(g)},$$

$$A({}^3P_2^{(8)} \to \Upsilon + g) \sim p_\mu^{(\mathrm{CO})} \epsilon_{\alpha\beta}^{(\mathrm{CO})} \epsilon_\alpha^{(\Upsilon)} \left[k_\mu^{(g)} \epsilon_\beta^{(g)} - k_\beta^{(g)} \epsilon_\mu^{(g)} \right],$$

  where $e^{\mu\nu\alpha\beta}$ is the fully antisymmetric Levi-Civita tensor.
- The transformation of color-octet S-wave state into the color-singlet S-wave state is treated as two successive $E1$ transitions ${}^3S_1^{(8)} \to {}^3P_J^{(8)} + g$ and ${}^3P_J^{(8)} \to {}^3S_1^{(1)} + g$.
- All the expressions above are the same for gluons and photons, therefore can be used to calculate the polarization variables in radiative decays in feed-down process $\chi_b(3P) \to \Upsilon(3S) + \gamma$.

- The cross sections of $\Upsilon(3S)$ and $\chi_b(3P)$ production in the k_T-factorization approach are calculated as a convolution of the off-shell partonic cross sections and TMD gluon densities in a proton:

$$\sigma = \int \frac{1}{8\pi(x_1 x_2 s)F} f_g(x_1, \boldsymbol{k}_{1T}^2, \mu^2) f_g(x_2, \boldsymbol{k}_{2T}^2, \mu^2) \times$$
$$\times \overline{|A(g^* + g^* \to Q\bar{Q} + g)|^2} d\boldsymbol{p}_T^2 d\boldsymbol{k}_{1T}^2 d\boldsymbol{k}_{2T}^2 dy dy_g \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi},$$

$$\sigma = \int \frac{2\pi}{x_1 x_2 s F} f_g(x_1, \boldsymbol{k}_{1T}^2, \mu^2) f_g(x_2, \boldsymbol{k}_{2T}^2, \mu^2) \times$$
$$\times \overline{|A(g^* + g^* \to Q\bar{Q})|^2} d\boldsymbol{k}_{1T}^2 d\boldsymbol{k}_{2T}^2 dy \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi}$$

- In this paper we use several TMD gluon distribution functions to describe the cross sections of the inclusive production $\Upsilon(3S)$: A0, JH'2013 set 1 and KMR.
- The renormalization μ_R and factorization μ_F scales for CCFM-evolved gluon densities were set to

  $\mu_R^2 = m_\Upsilon^2 + p_T^2,$

  $\mu_F^2 = \hat{s} + \mathbf{Q}_T^2$
- In the KMR calculations, we used standard choice:

  $\mu_R^2 = \mu_F^2 = m_\Upsilon^2 + p_T^2$
- We set the masses

  $m_{\Upsilon(3S)} = 10.3552$ GeV,

  $m_{\chi_{b\mathbf{1}}(3P)} = 10.512$ GeV,

  $m_{\chi_{b\mathbf{2}}(3P)} = 10.522$ GeV

- The branching ratios
  $B(\Upsilon(3S) \to \mu^+\mu^-) = 0.0218$,
  $B(\chi_{b1}(3P) \to \Upsilon(3S) + \gamma) = 0.1044$,
  $B(\chi_{b2}(3P) \to \Upsilon(3S) + \gamma) = 0.0611$
- We use the one-loop formula for the coupling α_s with $n_f = 4(5)$ quark flavours at $\Lambda_{\rm QCD} = 250(167)$ MeV for A0 (KMR) gluon density and two-loop expression for α_s with $n_f = 4$ and $\Lambda_{\rm QCD} = 200$ MeV for JH'2013 set 1.
- As a commonly adopted choice, we set CS NMEs
  $\langle \mathcal{O}(\Upsilon[^3S_1^{(1)}]) \rangle = 3.54$ GeV^3,
  $\langle \mathcal{O}(\chi[^3P_0^{(1)}]) \rangle = 2.83$ GeV^5.
  These values were obtained in the potential model calculations.
  E.J. Eichten, C. Quigg, arXiv:1904.11542 [hep-ph]

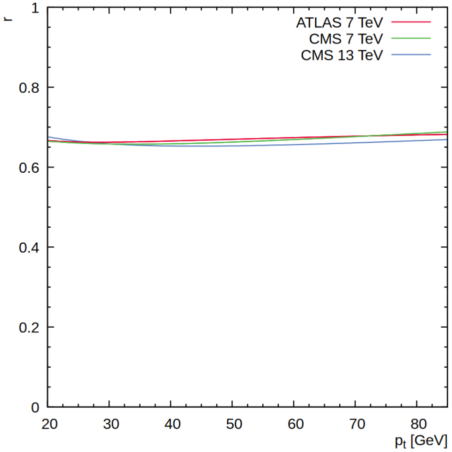
- We have performed a global fit to the $\Upsilon(3S)$ production data at the LHC and determined the corresponding NMEs for both $\Upsilon(3S)$ and $\chi_b(3P)$ mesons.
- We have included in the fitting procedure the $\Upsilon(3S)$ transverse momentum distributions measured by the CMS and ATLAS Collaborations at $\sqrt{s} = 7$ and 13 TeV and central rapidities, where our k_T-factorization calculations are most relevant due to essentially low-x region probed.
- We have excluded from our fit low p_T region and consider only the data at $p_T > p_T^{\rm cut} = 10$ GeV, where the NRQCD formalism is believed to be most reliable.

- To determine NMEs for $\chi_b(3P)$ mesons, we also included into the fit the recent LHCb data on the radiative $\chi_b(3P) \to \Upsilon(3S) + \gamma$ decays taken at $\sqrt{s} = 7$ and 8 TeV.
- We found that the p_T shape of the direct $\Upsilon[^3S_1^{(8)}]$ and feed-down $\chi_b[^3S_1^{(8)}]$ contributions is almost the same in all kinematical regions probed by the LHC and Tevatron experiments, i.e. the ratio

$$r = \frac{\sum_{J=0}^{2} (2J+1)\, B(\chi_{bJ}(3P) \to \Upsilon(3S) + \gamma) d\sigma[\chi_{bJ}(3P), {}^3S_1^{(8)}]/dp_T}{d\sigma[\Upsilon(3S), {}^3S_1^{(8)}]/dp_T}$$

can be well approximated by a constant for a wide $\Upsilon(3S)$ transverse momentum p_T and rapidity y ranges at different energies.

- We estimate the mean-square average $r = 0.654 \pm 0.005$, which is practically independent on the TMD gluon density in a proton.



- Since up to now there are no experimental data on the $\chi_b(3P)$ transverse momentum distributions, we cannot separately determine the values of $\langle \mathcal{O}^{\Upsilon(3S)}[^3S_1^{(8)}]\rangle$ and $\langle \mathcal{O}^{\chi_{b0}(3P)}[^3S_1^{(8)}]\rangle$ from the available $\Upsilon(3S)$ data.
- Instead, we introduce the linear combination

$$M_r = \langle \mathcal{O}^{\Upsilon(3S)}[^3S_1^{(8)}]\rangle + r\langle \mathcal{O}^{\chi_{b0}(3P)}[^3S_1^{(8)}]\rangle,$$

  which can be extracted from the measured $\Upsilon(3S)$ transverse momentum distributions.
- Then we use recent LHCb data on the fraction of $\Upsilon(3S)$ mesons originating from the $\chi_b(3P)$ radiative decays measured at $\sqrt{s} = 7$ and 8 TeV.
- The LHCb Collaboration reported the ratio

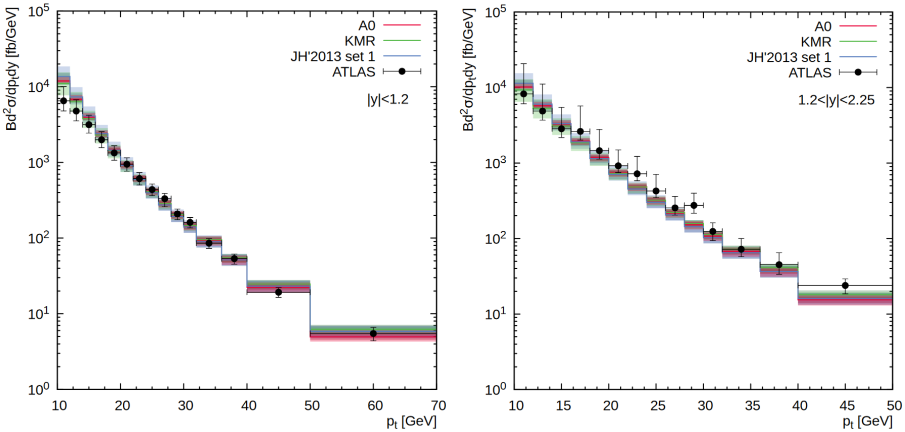
$$R^{\chi_b(3P)}_{\Upsilon(3S)} = \sum_{J=1}^{2} \frac{\sigma(pp \to \chi_{bJ}(3P) + X)}{\sigma(pp \to \Upsilon(3S) + X)} \times B(\chi_{bJ} \to \Upsilon(3S) + \gamma),$$

- The corresponding uncertainties are estimated in the conventional way using Student's t-distribution at the confidence level $P = 80\%$

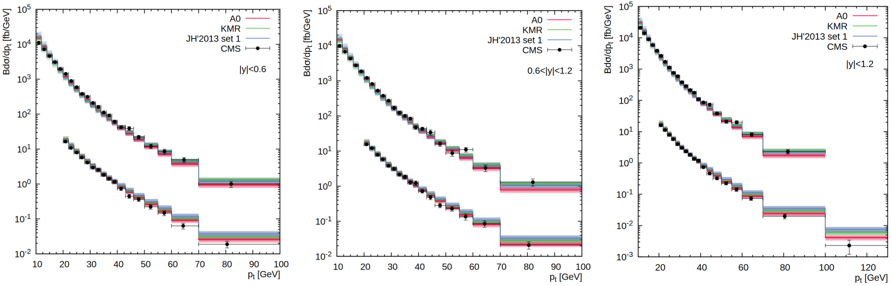
	A0	JH'2013 set 1	KMR	NLO NRQCD
$\langle \mathcal{O}^{\Upsilon(3S)}[^3S_1^{(1)}]\rangle/\mathrm{GeV}^3$	3.54	3.54	3.54	3.54
$\langle \mathcal{O}^{\Upsilon(3S)}[^1S_0^{(8)}]\rangle/\mathrm{GeV}^3$	0.0	0.0	0.0	-0.0107 ± 0.0107
$\langle \mathcal{O}^{\Upsilon(3S)}[^3S_1^{(8)}]\rangle/\mathrm{GeV}^3$	0.018 ± 0.001	0.007 ± 0.002	0.006 ± 0.001	0.0271 ± 0.0013
$\langle \mathcal{O}^{\Upsilon(3S)}[^3P_0^{(8)}]\rangle/\mathrm{GeV}^5$	0.0	0.09 ± 0.03	0.073 ± 0.006	0.0039 ± 0.0023
$\langle \mathcal{O}^{\chi_{b0}(3P)}[^3P_0^{(1)}]\rangle/\mathrm{GeV}^5$	2.83	2.83	2.83	2.83
$\langle \mathcal{O}^{\chi_{b0}(3P)}[^3S_1^{(8)}]\rangle/\mathrm{GeV}^3$	0.016 ± 0.003	0.009 ± 0.001	0.005 ± 0.001	—

Y. Feng, B. Gong, L.-P. Wan, J.-X. Wang, H.-F. Zhang, Chin. Phys. C 39, 123102 (2015)

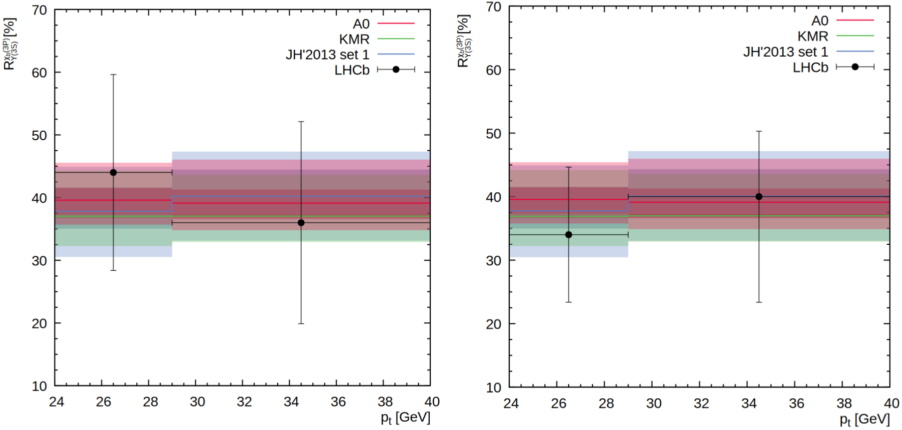
- Transverse momentum distribution of inclusive $\Upsilon(3S)$ production calculated at $\sqrt{s} = 7$ TeV. The experimental data are from ATLAS.



- Transverse momentum distribution of inclusive $\Upsilon(3S)$ production calculated at $\sqrt{s} = 7$ TeV (upper histograms) and $\sqrt{s} = 13$ TeV (lower histograms, divided by 100). The experimental data are from CMS.



- The ratio $R^{\chi_b(3P)}_{\Upsilon(3S)}$ calculated as function of $\Upsilon(3S)$ transverse momentum at $\sqrt{s} = 7$ TeV (left panel) and $\sqrt{s} = 8$ TeV (right panel). The experimental data are from LHCb.



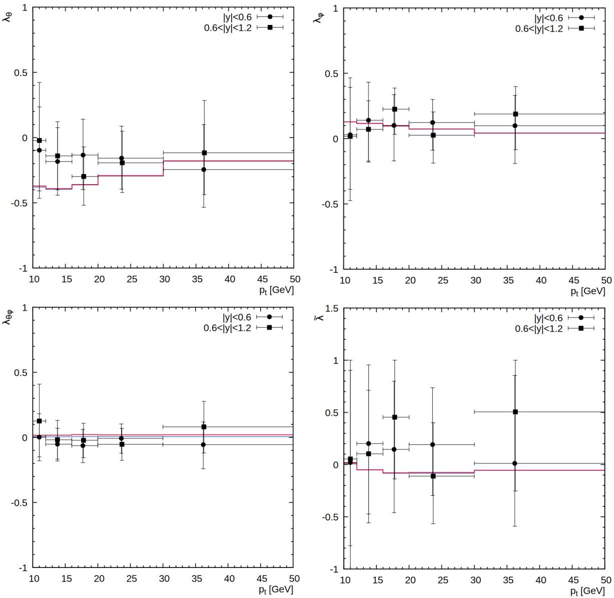
- The polarization of any vector meson can be described with three parameters λ_θ, λ_ϕ and $\lambda_{\theta\phi}$.
- The double differential angular distribution of the decay leptons can be written as:

$$\frac{d\sigma}{d\cos\theta^* d\phi^*} \sim \frac{1}{3+\lambda_\theta}(1+\lambda_\theta\cos^2\theta^* + \lambda_\phi\sin^2\theta^*\cos 2\phi^* + \lambda_{\theta\phi}\sin 2\theta^*\cos\phi^*)$$

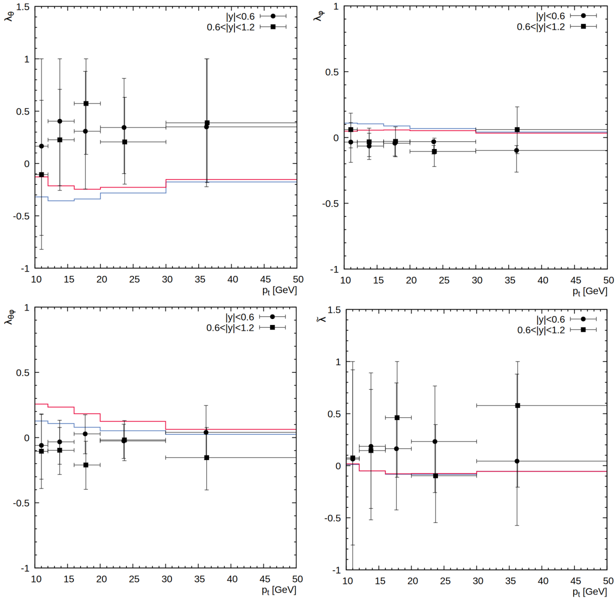
- The case of $(\lambda_\theta,\ \lambda_\phi,\ \lambda_{\theta\phi}) = (0,0,0)$ corresponds to unpolarized state, while $(\lambda_\theta,\ \lambda_\phi,\ \lambda_{\theta\phi}) = (1,0,0)$ and $(\lambda_\theta,\ \lambda_\phi,\ \lambda_{\theta\phi}) = (-1,0,0)$ refer to fully transverse and fully longitudinal polarizations.

- The CMS Collaboration has measured all of these parameters as functions of $\Upsilon(3S)$ transverse momentum in three complementary frames: the Collins-Soper, helicity and perpendicular helicity ones at $\sqrt{s} = 7$ TeV.
- The CDF Collaboration also measured these parameters in the helicity frame at $\sqrt{s} = 1.96$ TeV.
- Additionally, the frame-independent parameter $\tilde{\lambda} = (\lambda_\theta + 3\lambda_\phi)/(1 - \lambda_\phi)$ has been studied.

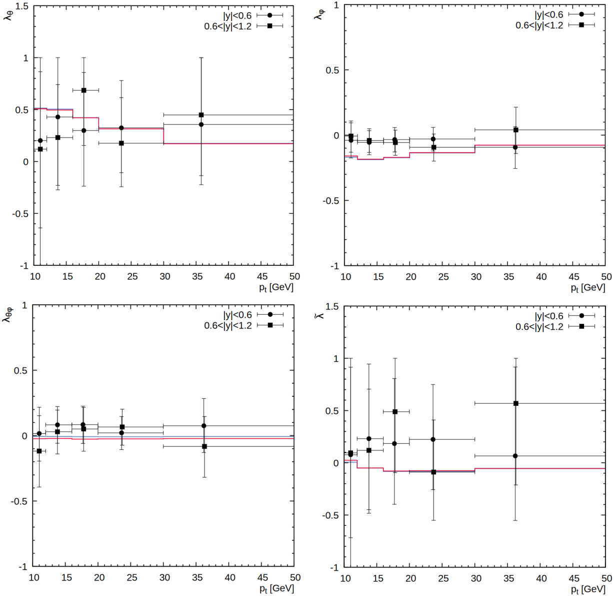
- The polarization parameters calculated in the CS frame at $\sqrt{s} = 7$ TeV.



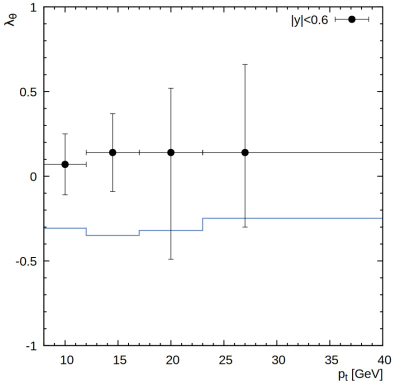
- The polarization parameters calculated in the helicity frame at $\sqrt{s} = 7$ TeV.

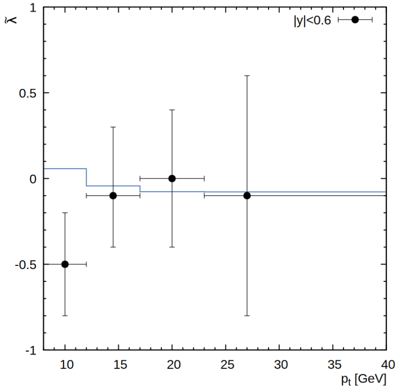


- The polarization parameters calculated in the perpendicular helicity frame at $\sqrt{s} = 7$ TeV.



- The polarization parameters calculated in the helicity frame at $\sqrt{s} = 1.96$ TeV.



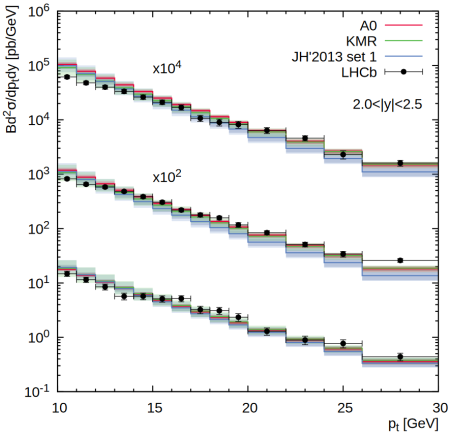


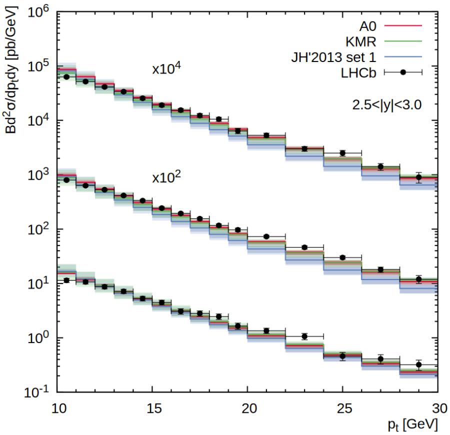
- We have considered the $\Upsilon(3S)$ production at the Tevatron and LHC in the framework of k_T-factorization approach. Our consideration was based on the off-shell production amplitudes for hard partonic subprocesses, NRQCD formalism for the formation of bound states and TMD gluon densities in a proton.
- Treating the nonperturbative color octet transitions in terms of multipole radiation theory and taking into account feed-down contributions from the radiative $\chi_b(3P)$ decays, we extracted $\Upsilon(3S)$ and $\chi_b(3P)$ NMEs in a fit to $\Upsilon(3S)$ transverse momentum distributions measured by the CMS and ATLAS Collaborations at $\sqrt{s} = 7$ and 13 TeV.
- We have inspected the extracted NMEs with the available Tevatron and LHC data taken in the different kinematical regions and demostrated that these NMEs do not contradict the data.

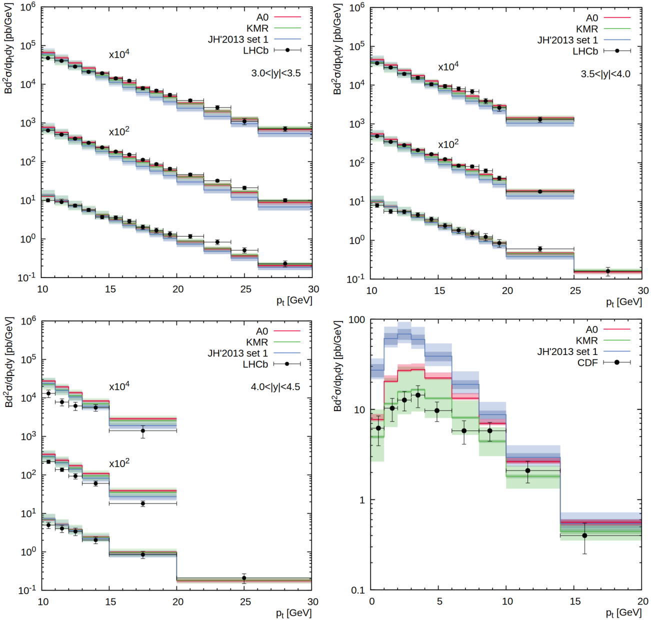
- Then we estimated polarization parameters λ_θ, λ_ϕ, $\lambda_{\theta\phi}$ and frame-independent parameter $\tilde{\lambda}$ which determine the $\Upsilon(3S)$ spin density matrix.
- We show that treating the soft gluon emission as a series of explicit color-electric dipole transitions within the NRQCD leads to unpolarized $\Upsilon(3S)$ production at moderate and large transverse momenta, that is in agreement with the Tevatron and LHC data.

Back up

- Transverse momentum distribution of inclusive $\Upsilon(3S)$ production calculated at $\sqrt{s} = 1.8$, 7, 8 and 13 TeV. The experimental data are from CDF and LHCb.

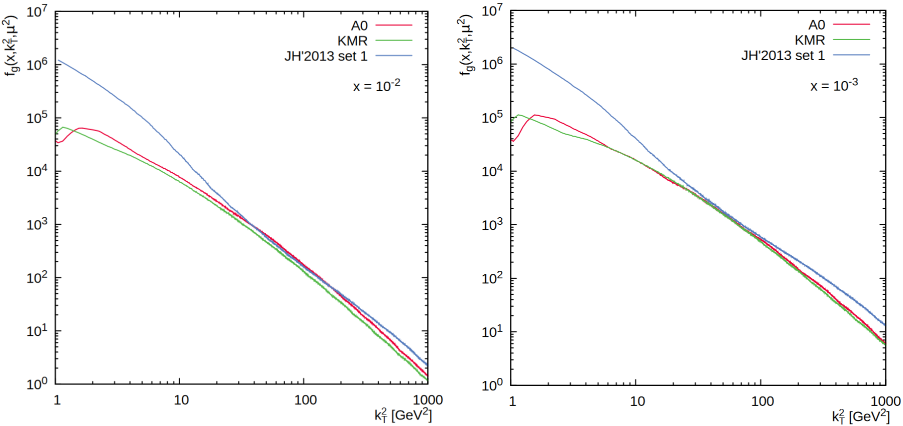






A0
KMR
JH'2013 set 1
LHCb
x10^4
x10^2
3.0<|y|<3.5
3.5<|y|<4.0
4.0<|y|<4.5
CDF
$Bd^2\sigma/dp_tdy$ [pb/GeV]
p_t [GeV]

- The TMD gluon densities in the proton calculated as a function of the gluon transverse momentum $\mathbf{k}_T^2$ at different longitudinal momentum fraction $x = 0.01$ (left) and 0.001 (right).



- First of all, it is important to establish a proper definition of the off-shell flux factor F for $2 \to 1$ subprocesses of the gluon-gluon fusion. The definition of the flux, which is velocity of the off-shell interacting partons, is not clear and can be disputable.
- According to the general definition, the off-shell gluon flux factor is defined as $F = 2\lambda^{1/2}(\hat{s}, k_1^2, k_2^2)$.
- For $2 \to 2$ subprocesses one can use the approximation $\lambda^{1/2}(\hat{s}, k_1^2, k_2^2) \simeq \hat{s} \simeq x_1 x_2 s$. However, it is not suitable for the $2 \to 1$ kinematics because the difference between $\hat{s} \simeq m_\Upsilon^2$ and $x_1 x_2 s = m_\Upsilon^2 + p_T^2$ can make pronounced effect on the p_T spectrum.
- It was argued that such definition leads to a good agreement of calculations based on Equivalent Photon Approximation and exact $\mathcal{O}(\alpha^4)$ results. Contrary, the calculations performed with using conventional (collinear) $2 \to 1$ flux treatment $\lambda^{1/2}(\hat{s}, k_1^2, k_2^2) \simeq x_1 x_2 s$ did not reproduce the latter.

  S.P. Baranov, A. Szcurek, Phys. Rev. D77, 054016 (2008)

- Let us consider the ratio R defined as

$$R = \frac{m^2_{\Upsilon(3S)} \sum_{J=0}^{2} (2J+1)\, d\sigma[\Upsilon(3S), {}^3P_J^{(8)}]/dp_T}{d\sigma[\Upsilon(3S), {}^1S_0^{(8)}]/dp_T}$$