The role of acceleration and vorticity in relativistic hydrodynamics

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Based on

- *G. P., O. V. Teryaev and V. I. Zakharov, Phys. Rev. D 99, no. 7, 071901 (2019).*
- *G. P., O. V. Teryaev and V. I. Zakharov, JHEP 1902, 146 (2019).*
- *G. P., O. Teryaev and V. Zakharov, Phys. Rev. D 98, no. 7, 071901 (2018).*

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- Introduction.
- Third-order corrections in **acceleration** and **vorticity** $\omega^2\omega_\mu$, $a^2\omega_\mu$ to the **Chiral Vortical Effect (CVE)**.

  *The concept of **vorticity** as a **(real) chemical potential**.*

- Fourth-order corrections a^4 in **acceleration** in the energy-momentum tensor.

  ***Unruh effect** for fermions.*

  *The concept of **acceleration** as an **imaginary chemical potential**.*

  ***Intability** at the Unruh Temperature.*

- Connection with **polarization**.
- Conclusions.

Methods

- Quantum-statistical density operator of Zubarev. *(D. N. Zubarev, A. V. Prozorkevich, S. A. Smolyanskii, Theoret. and Math. Phys., 40:3 (1979), 821-831).*
- Covariant Wigner Function *(F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32)*.

Zubarev density operator

In the state of global thermodynamic equilibrium with thermal vorticity, the medium is described by the density operator of the form

D. N. Zubarev, A. V. Prozorkevich, S. A. Smolyanskii, Theoret. and Math. Phys., 40:3 (1979), 821-831.

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091.

$$\hat{\rho} = \frac{1}{Z} \exp\left\{ -\beta_\mu(x)\hat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\hat{J}_x^{\mu\nu} + \zeta\hat{Q} \right\}$$

Effects associated with the vorticity and acceleration are described by the term with thermal vorticity

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$$

Quantum statistical mean values can be found within the framework of perturbation theory. In particular, **CVE was received**

$$\langle j_\mu^5 \rangle = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} \right)\omega_\mu$$

Corrections to the chiral vortical effect

Parity allows the appearance of 3 types of terms in the third order of perturbation theory

$$\langle \hat{j}_5^\lambda(x) \rangle^{(3)} = A_1 \omega^2 \omega^\lambda + A_2 a^2 \omega^\lambda + A_3 (\omega a) a^\lambda$$

These coefficients were found based on the density operator for the case of massive fermions and in the chiral limit

G. P., O. V. Teryaev and V. I. Zakharov, JHEP 1902, 146 (2019).

$$A_1 \to -\frac{1}{24\pi^2}\,, \quad A_2 \to -\frac{1}{8\pi^2}\,, \quad A_3 = 0$$

in the chiral limit

Thus, in the **third order** of perturbation theory, we have

$$\langle j^5_\mu \rangle = \Big(\frac{1}{6}\big[T^2 - \frac{\omega^2}{4\pi^2}\big] + \frac{\mu^2}{2\pi^2} - \frac{a^2}{8\pi^2} \Big) \omega_\mu + \mathcal{O}(\varpi^5)$$

Corrections to the chiral vortical effect

$$\langle j^5_\mu \rangle = \Big(\frac{1}{6} \big[T^2 - \frac{\omega^2}{4\pi^2} \big] + \frac{\mu^2}{2\pi^2} - \frac{a^2}{8\pi^2} \Big) \omega_\mu + \mathcal{O}(\varpi^5)$$

Term, cubic in vorticity

A. Vilenkin, Phys. Rev. D 21 (1980) 2260.

Second order terms in the presence of axial chemical potential

M. Buzzegoli and F. Becattini, JHEP 1812, 002 (2018).

Since the third order term $A_3(\omega a) a^\lambda = 0$ the axial charge is conserved

$$\partial^\mu \langle j^5_\mu \rangle = 0$$

Exact nonperturbative formula

In the case $a = 0$ the next nonperturbative formula was derived on the basis of Wigner function

$$\langle \mathbf{j}^5 \rangle_W = \int \frac{d^3p}{(2\pi)^3} \Big\{ n_F(E_p - \mu - \frac{\Omega}{2}) - n_F(E_p - \mu + \frac{\Omega}{2}) + \\ + n_F(E_p + \mu - \frac{\Omega}{2}) - n_F(E_p + \mu + \frac{\Omega}{2}) \Big\} \mathbf{e}_\Omega \,,$$

A **nonperturbative formula** at global equilibrium derived from the **Wigner function** ***coincides*** with the result obtained from the **density operator** up to the third order of the perturbation theory in the general case of massive fermions and if $a = 0$

$$\langle \boldsymbol{j}^5 \rangle_W^{(3)} = \langle \boldsymbol{j}^5 \rangle_\rho^{(3)} \qquad m \neq 0$$

Angular velocity as (real) chemical potential

$$\langle \mathbf{j}^5 \rangle_W = \int \frac{d^3p}{(2\pi)^3} \Big\{ n_F(E_p - \mu - \frac{\Omega}{2}) - n_F(E_p - \mu + \frac{\Omega}{2}) + $$
$$+ n_F(E_p + \mu - \frac{\Omega}{2}) - n_F(E_p + \mu + \frac{\Omega}{2}) \Big\} \mathbf{e}_\Omega$$

The angular velocity enters the formula as a (real) chemical potential

$$\mu \to \mu \pm \frac{\Omega}{2}$$

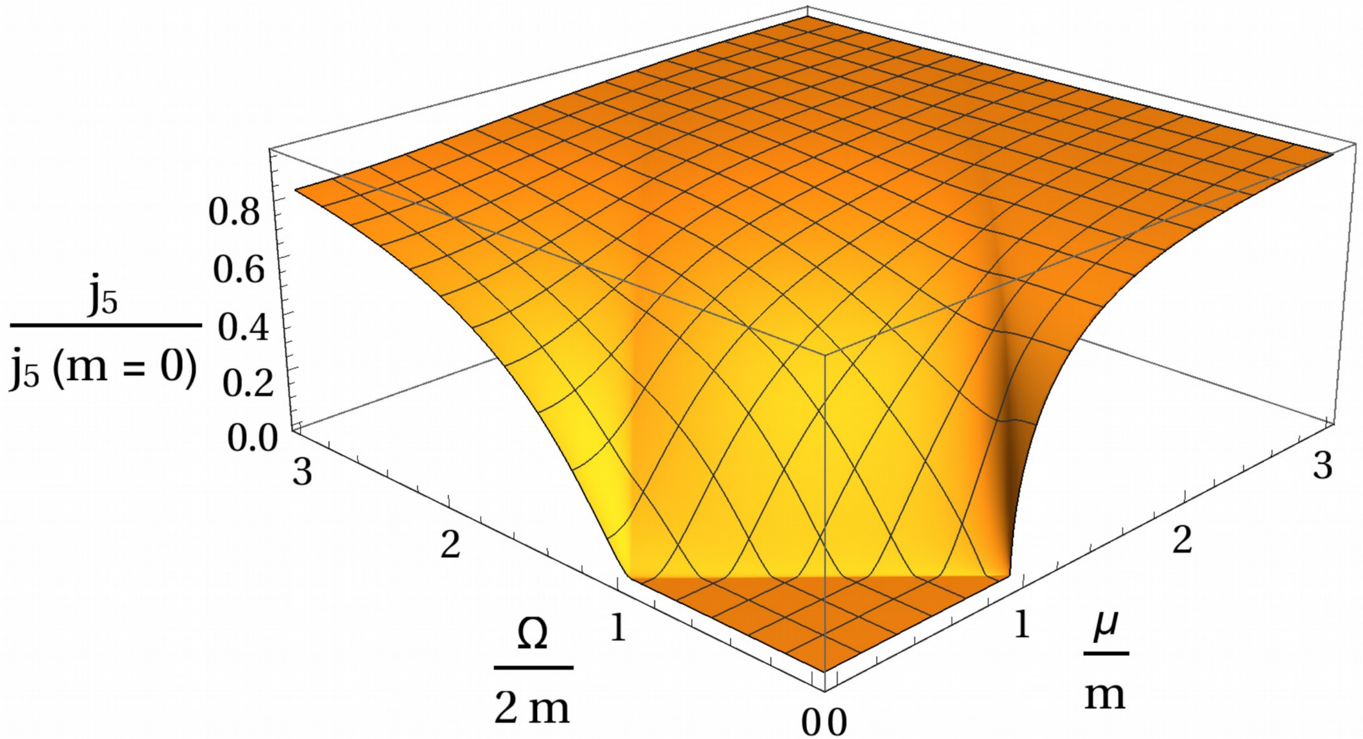
Analogy between angular velocity and chemical potential:

- *W. Florkowski, B. Friman, A. Jaiswal and E. Speranza, Phys. Rev. C 97, no. 4, 041901 (2018)*.
- *S. Ebihara, K. Fukushima and K. Mameda, Phys. Lett. B 764, 94 (2017).*

Since the angular velocity behaves like a new chemical potential, it should be expected that there will be an effect similar to the vanishing of physical quantities at $|\mu| < m$ and $T \to 0$.

Angular velocity as (real) chemical potential

Such an effect really exists



Axial current at $T = 0$ is zero in the region $\Omega < 2(m - |\mu|)$.

Angular velocity as real chemical potential and acceleration as imaginary potential

The angular velocity enters the formula as a real chemical potential

$$\mu \to \mu \pm \frac{\Omega}{2}$$

If we consider also the acceleration, then

$$\mu \to \mu \pm \frac{\Omega}{2} \pm \frac{i|\boldsymbol{a}|}{2}$$

acceleration appears as ***imaginary chemical potential:***

G. P., O. Teryaev and V. Zakharov, Phys. Rev. D 98, no. 7, 071901 (2018).

Unruh effect from the point of view of quantum statistical mechanics

The meaning of the Unruh effect is that the accelerated observer sees the Minkowski vacuum as a medium filled with particles with a Unruh temperature proportional to the acceleration

$$T_U = \frac{a}{2\pi}$$

W. G. Unruh, Phys. Rev. D 14, 870 (1976).

Thus, the values of observables in the laboratory frame should be equal to zero when the proper temperature, measured by comoving observer equals to the Unruh temperature.

- It has been shown for **scalar particles** by calculating quantum correlators

F. Becattini, Phys. Rev. D 97, no. 8, 085013 (2018).

For **fermions**, a similar effect was observed based on the Wigner function in the Boltzmann limit at a temperature $2T_U$

W. Florkowski, E. Speranza and F. Becattini, Acta Phys. Polon. B 49, 1409 (2018).

- This is due to the *approximate* nature of the Wigner function used.

Unruh effect for fermions: energy-momentum tensor

We will study the effects associated with acceleration based on the density operator.

We will consider case $\mu = \mu_5 = 0$, $\omega^\mu = 0$ and $m = 0$.

Effects associated with acceleration are described by a term with a boost operator

$$\hat{\rho} = \frac{1}{Z} \exp\Big\{ -\beta_\mu \hat{P}^\mu - \alpha_\mu \hat{K}^\mu_x \Big\}$$

In the fourth order of the perturbation theory, the energy-momentum tensor has the form

$$\begin{aligned} \langle \hat{T}^{\mu\nu} \rangle \; &= \; (\rho_0 + A_1 a^2 T^2 + A_2 a^4) u^\mu u^\nu - (p_0 + B_1 a^2 T^2 + B_2 a^4) \Delta^{\mu\nu} \\ &+ (A_3 T^2 + A_4 a^2) a^\mu a^\nu + \mathcal{O}(a^6) \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \end{aligned}$$

The coefficients up to the second order for fermions $\rho_0,\ p_0,\ A_1,\ B_1,\ A_3$ were calculated previously.

Unruh effect for fermions: energy-momentum tensor

4-order terms were calculated in the **chiral limit:**

$$A_2 = -\frac{17}{960\pi^2}\,, \quad B_2 = \frac{A_2}{3} = -\frac{17}{2880\pi^2}\,, \quad A_4 = 0$$

in particular, the energy density takes the form

$$\rho_{Den} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} + \mathcal{O}(a^6)$$
$$= \frac{1}{240}\left(T^2 - \left(\frac{a}{2\pi}\right)^2\right)(17a^2 + 28\pi^2 T^2) + \mathcal{O}(a^6)$$

This result fully agrees (J. Dowker, Class.Quant.Grav. 11 (1994)), where energy density was evaluated in ***non-trivial metric****.*

The energy density is 0 at proper temperature $T = \frac{a}{2\pi}$, if we take into account the terms of 4 order. When substituting $T = \frac{a}{2\pi}$, we get $\frac{a^4}{960\pi^2}(7 + 10 - 17) = 0$.

Unruh effect for fermions: energy-momentum tensor

The same is true, in general, for the energy-momentum tensor

$$\langle \hat{T}^{\mu\nu} \rangle = 0 \qquad (T = T_U)$$

The remaining observables are 0 due to parity considerations and due to the conditions

$$\mu = \mu_5 = 0 \quad , \quad \omega^\mu = 0 \; .$$

Thus, the **Minkowski vacuum** corresponds to the **proper temperature** measured by the comoving observer equal to the **Unruh temperature**.

- **Essence of the Unruh effect.**

Integral representation, acceleration as imaginary chemical potential

It can be shown that the next mathematical equality is exactly fulfilled

$$\rho = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} = 2\int \frac{d^3p}{(2\pi)^3}\Big(\frac{|\mathbf{p}| + ia}{1+e^{\frac{|\mathbf{p}|}{T}+\frac{ia}{2T}}} + \frac{|\mathbf{p}| - ia}{1+e^{\frac{|\mathbf{p}|}{T}-\frac{ia}{2T}}}\Big)$$
$$+4\int \frac{d^3p}{(2\pi)^3}\frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{a}}-1} \qquad (T > T_U)$$

Thus, at temperatures above T_U, the calculation result based on the density operator can be represented as **integrals** with **Fermi distributions** at temperature T and **Bose at temperature** T_U.

- In the first integral, the acceleration enters as an ***imaginary chemical potential*** $\pm\frac{ia}{2}$.

Motivation:

- Exact match with the fundamental result from the density operator at $T > T_U$
- True limit at $a \to 0$.
- Acceleration enters as an **imaginary chemical potential** into **axial current.**

Comparison with Wigner function

- From the **Wigner function**, we obtain the following expression for the Fermi gas energy density

$$\rho_{Wig} = 2\int \frac{d^3p}{(2\pi)^3}\varepsilon\Big(\frac{1}{1+e^{\frac{\varepsilon}{T}+\frac{ia}{2T}}} + \frac{1}{1+e^{\frac{\varepsilon}{T}-\frac{ia}{2T}}}\Big) \qquad \rho_{Wig}(T_U) \neq 0$$

where **acceleration** also appears as an **imaginary chemical potential**.

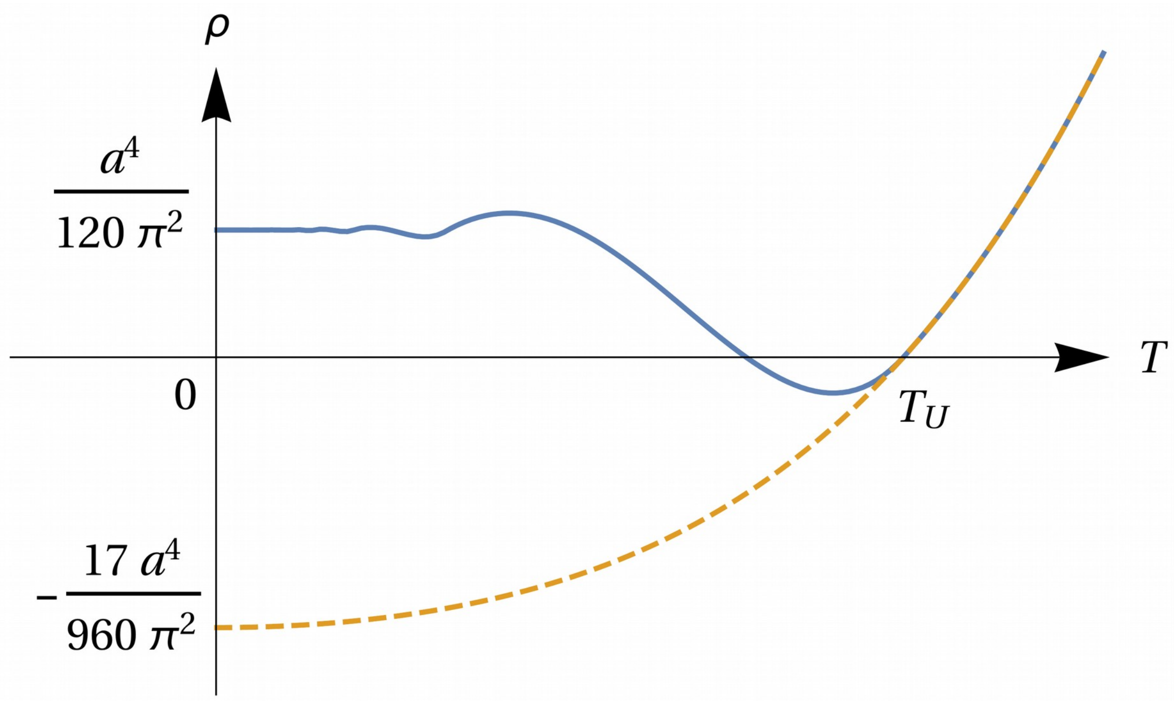
The difference from the result obtained from the density operator

$$\rho = \frac{7\pi^2T^4}{60} + \frac{T^2a^2}{24} - \frac{17a^4}{960\pi^2} = 2\int \frac{d^3p}{(2\pi)^3}\Big(\frac{|\mathbf{p}|+ia}{1+e^{\frac{|\mathbf{p}|}{T}+\frac{ia}{2T}}} + \frac{|\mathbf{p}|-ia}{1+e^{\frac{|\mathbf{p}|}{T}-\frac{ia}{2T}}}\Big)$$

$$+4\int \frac{d^3p}{(2\pi)^3}\frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{a}}-1} \qquad (T > T_U)$$

in red: modifications compared to the Wigner function

Comparison of perturbative formula and integral representation

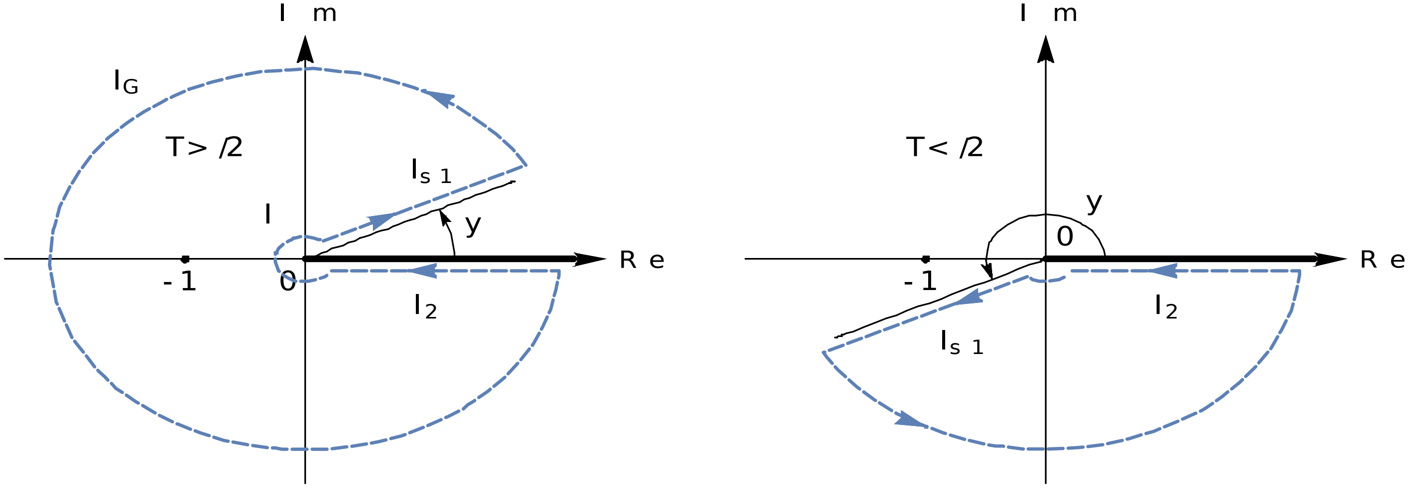


The solid blue line is the energy density as a function of temperature, corresponding to the integral representation. *The dashed orange line* - the result of the fundamental calculation based on the density operator.

- There is a **difference** in the area $T < T_U$, which can be forbidden.

Acceleration as imaginary chemical potential → source of instability

Integration is made along the contour in the complex plane. The appearance of acceleration as an **imaginary** chemical potential leads to the **rotation of the contour** of integration with respect to the real axis by an angle of $y = \frac{a}{2T}$



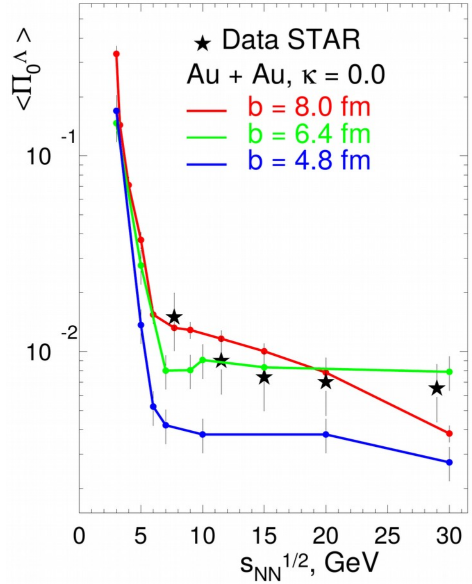
$T > T_U$ $T < T_U$

The integrand contains a **pole** at $\eta = -1$ and a cut along the real axis. At $T = \frac{a}{2\pi}$, the contour **crosses the pole**, which leads to **instability**.

2 approaches to the calculation of the polarization of hyperons

1-st approach

The fall of polarization of hyperons with increasing collision energy.



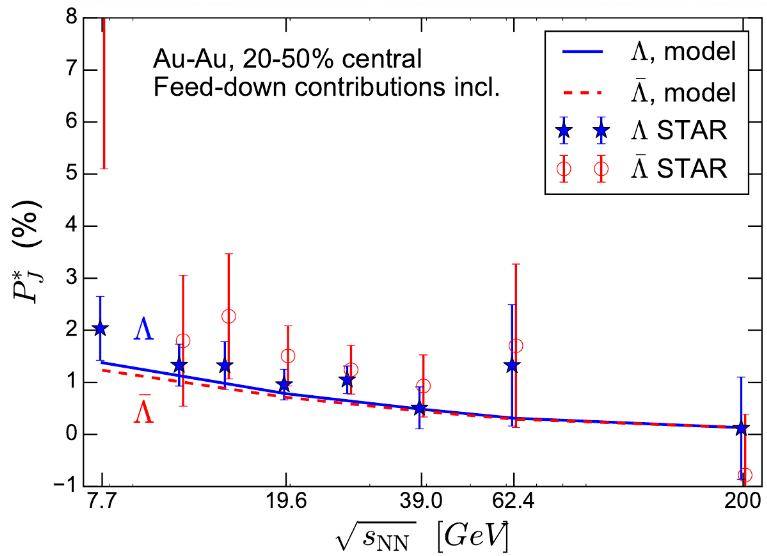
- *Sorin, Alexander et al. Phys.Rev.C95(2017)no.1,011902.*
- *M. Baznat,K. Gudima,A. Sorin and O. Teryaev, EPJ Web Conf. 138 (2017) 01008.*

2-nd approach

Karpenko, Iu. et al. Nucl.Phys. A967 (2017) 764-767 arXiv:1704.02142 [nucl-th]

Based on the Wigner function

$$S^\mu(x,p) = -\frac{1}{8m}(1-f(x,p))\epsilon^{\mu\nu\rho\sigma}p_\sigma\varpi_{\nu\rho}$$



The polarization is determined by the axial charge of strange quarks ⇒ from CVE ⇒ Axial charge depends on chemical potential of strange quarks ⇒ The **fall of polarization** is caused by the **decrease of the strange chemical potential**

2 approaches to the calculation of the polarization of hyperons

It was proved that the Wigner function used in the second approach leads to CVE. CVE underlies the first approach. Thus, CVE is essential for both approaches.

⇩

There is a connection between the two approaches to the calculation of the polarization of hyperons.

This could be one of the sources of the same polarization behaviour in them.

Conclusions

- The **third-order** corrections in acceleration and vorticity to CVE are calculated using the fundamental Zubarev density operator.
- This calculation confirms a **non-perturbative formula** for axial current derived from the Wigner function, in which the **vorticity** enters as a **(real) chemical potential**.
- The **fourth-order** corrections are calculated in the energy-momentum tensor of massless fermions.
- It is shown that, taking into account these corrections, the energy-momentum tensor vanishes at a proper temperature measured by the comoving observer equal to the Unruh temperature. **This is the essence of the Unruh effect** and is a generalization of the previous result for the case of fermions.
- An integral representation for the energy density is proposed, in which the **acceleration** enters as an **imaginary chemical potential**.
- **Instability** arises due to the singularity of the Fermi distribution in the complex momentum plane.
- *The connection of various approaches to calculating **hadron polarization** when considering the axial current is shown.*

Thank you for your attention!

Quantum Field Effects of Acceleration and Rotation

Quantum field effects of rotation and magnetic fields: CVE and Chiral Magnetic Effect (CME)

[1] A. Vilenkin, Phys. Rev. D 21 (1980) 2260; Phys. Rev. D 22 (1980) 3080.

[2] D. T. Son and P. Surowka, Phys. Rev. Lett. 103 (2009) 191601.

[3] A. V. Sadofyev, V. I. Shevchenko and V. I. Zakharov, Phys. Rev. D 83 (2011) 105025.

[4] K. Landsteiner, E. Megias, F. Pena-Benitez. Phys. Rev. Lett. 107, 021601 (2011).

[5] M. Stone and J. Kim, Phys. Rev. D98, no. 2, 025012 (2018).

[6] D. E. Kharzeev, K. Landsteiner, A. Schmitt and H. U. Yee, Lect. Notes Phys. 871 (2013).

Some of the early works

Connection with quantum-field anomalies

Quantum field effects of acceleration: Unruh Effect

[7] W. G. Unruh, Phys. Rev. D 14, 870 (1976).

[8] P. Castorina, D. Kharzeev, and H. Satz, Eur. Phys. J. C 52, 187 (2007).

The Unruh effect as a source of thermalization in particle collisions

Thermal vorticity as a source of hadron polarization in heavy-ion collisions

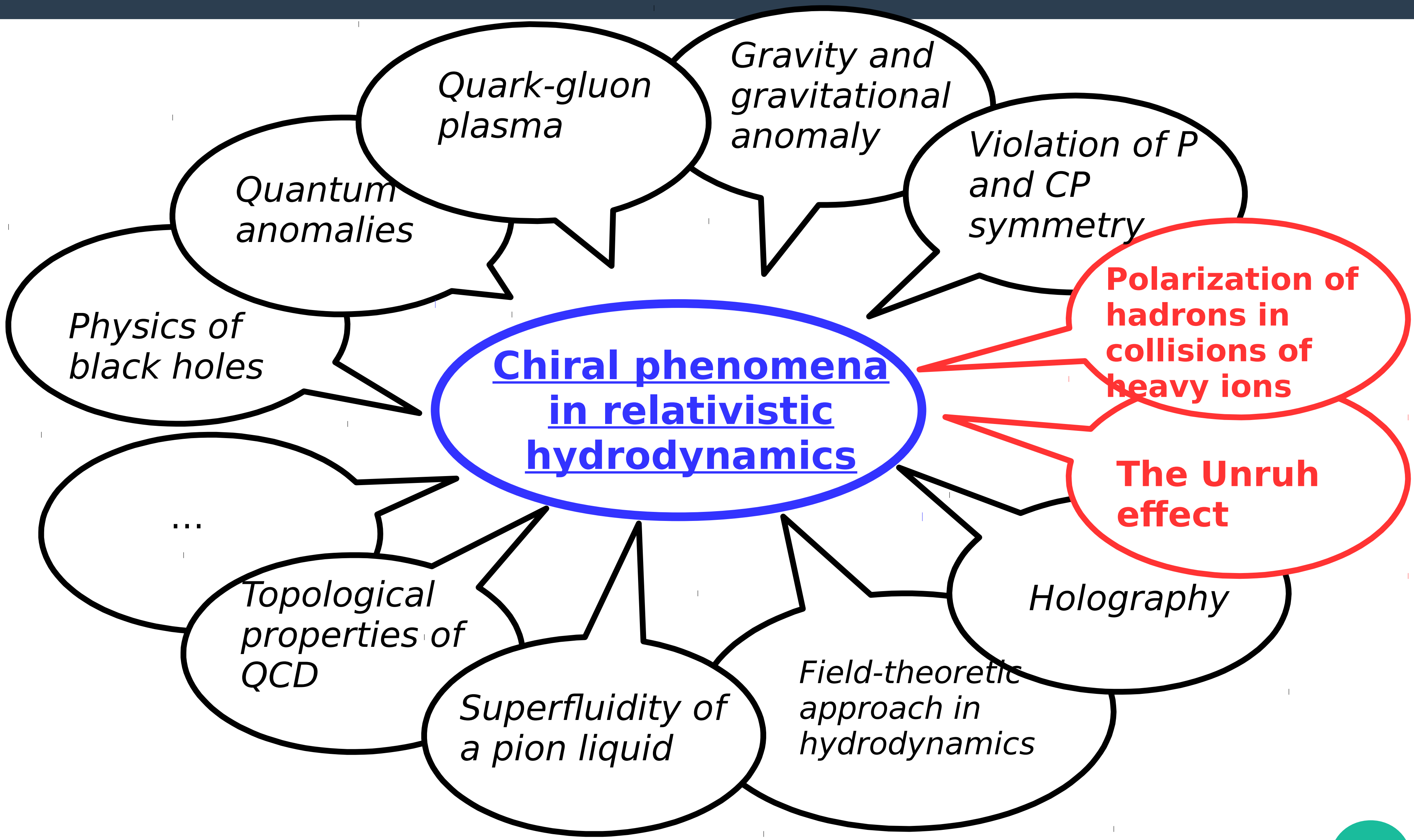
[9] O. Rogachevsky, A. Sorin, and O. Teryaev, Phys. Rev. C 82, 054910 (2010).

[10] J. H. Gao, Z. T. Liang, S. Pu, Q. Wang and X. N. Wang, Phys. Rev. Lett. 109 (2012) 232301.

[11] F. Becattini and I. Karpenko, Phys. Rev. Lett. 120, no. 1, 012302 (2018).

Different approaches to the calculation of polarization

Areas of physics manifested in chiral phenomena



Corrections to the chiral vortical effect

Parity allows the appearance of **3 types** of terms in the third order of perturbation theory

$$\langle \hat{j}_5^\lambda(x)\rangle^{(3)} = A_1\omega^2\omega^\lambda + A_2 a^2\omega^\lambda + A_3(\omega a)a^\lambda$$

These coefficients can be found based on the density operator for the case of massive fermions and in the chiral limit. It is necessary to calculate **quantum correlators with three angular momentum or boost operators** and one current operator.

For example for A_2 :

$$A_2 = -\frac{1}{6|\beta|^3}\Bigg(\int_0^{|\beta|} d\tau_1 d\tau_2 d\tau_3 \langle T_\tau \big(\hat{K}^1_{-i\tau_1 u}\hat{J}^3_{-i\tau_2 u} + \hat{J}^3_{-i\tau_1 u}\hat{K}^1_{-i\tau_2 u}\big)\hat{K}^1_{-i\tau_3 u}\hat{j}^3_5(0)\rangle_{\beta(x),c} + \\ + \int_0^{|\beta|} d\tau_1 d\tau_2 d\tau_3 \langle T_\tau \hat{K}^1_{-i\tau_1 u}\hat{K}^1_{-i\tau_2 u}\hat{J}^3_{-i\tau_3 u}\hat{j}^3_5(0)\rangle_{\beta(x),c}\Bigg),$$

It includes integration over **9** spatial and **12** momentum variables, **3** integrations over the inverse temperature, and **4** summations over the Matsubara frequencies.

Corrections to the chiral vortical effect

These coefficients were found analytically for the case of massive fermions and in the chiral limit *(G. P., O. V. Teryaev and V. I. Zakharov, JHEP 1902, 146 (2019)*. In the case of massive ferA_2ons, for example, for one obtains:

$$A_2 = \frac{1}{16\pi^2|\beta|^3}\int_0^{\infty} dp\Big(n_F'''(E_p-\mu)+n_F'''(E_p+\mu)\Big)\Big(p^2+\frac{m^2}{3}\Big)$$

where the result is presented as derivatives of the Fermi distribution. In the chiral limit the last momentum integral can be found analytically through integration in the complex momentum plane. We give the final result right away:

$$A_1 \to -\frac{1}{24\pi^2}\,, \quad A_2 \to -\frac{1}{8\pi^2}\,, \quad A_3 = 0$$

Thus, in the third order of perturbation theory, we have

$$\langle j_\mu^5\rangle = \Big(\frac{1}{6}\big[T^2-\frac{\omega^2}{4\pi^2}\big]+\frac{\mu^2}{2\pi^2}-\frac{a^2}{8\pi^2}\Big)\omega_\mu+\mathcal{O}(\varpi^5)$$

Exact nonperturbative formula

The nonperturbative formula for axial current can be derived on the basis of the **covariant Wigner function** for particles with spin ½ :

$$X(x,p) = \left(\exp[\beta_\mu p^\mu - \zeta] \exp\left[-\frac{1}{2} \varpi_{\mu\nu} \Sigma^{\mu\nu} \right] + I \right)^{-1}$$

where the effects of vorticity and acceleration are contained in the term $\varpi_{\mu\nu}\Sigma^{\mu\nu}$, where $\Sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$. The procedure for obtaining *accurate nonperturbative values* for mean values of physical quantities was described in *(G.P. and O. Teryaev, Phys. Rev. D 97, no. 7, 076013 (2018))*. For axial current one needs to **sum up an infinite series** of the terms with the traces of the products of Dirac matrices

$$\mathrm{tr}(X\Sigma^{\nu\beta}) = \sum_{n=0}^{\infty} (-1)^n \exp\left[t(n+l)(\beta \cdot p - \xi)\right] \sum_{m=0}^{\infty} \frac{1}{m!} \Big(t(n+l)(-\frac{1}{2})\Big)^m \mathrm{tr}\Big((\varpi : \Sigma)^m \Sigma^{\nu\beta}\Big)$$

which **can be done analytically** using the properties of Dirac matrices.

Exact nonperturbative formula

$$\langle \mathbf{j}^5 \rangle_W = \int \frac{d^3p}{(2\pi)^3} \Big\{ n_F(E_p - \mu - \frac{\Omega}{2}) - n_F(E_p - \mu + \frac{\Omega}{2}) + $$
$$+ n_F(E_p + \mu - \frac{\Omega}{2}) - n_F(E_p + \mu + \frac{\Omega}{2}) \Big\} \mathbf{e}_\Omega \, ,$$

This formula leads to **correct limiting cases**:

- It describes the **effects of mass** in nonperturbative way and reproduces the result *(A. Flachi and K. Fukushima, Phys. Rev. D 98, no. 9, 096011 (201*m^2in the first order in

$$\langle \hat{j}_5^\mu \rangle = (\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{m^2}{4\pi^2}) \omega^\mu$$

- Gives correct answer, including integrand, for **massless case** *(A. Vilenkin, Phys. Rev. D 21 (1980) 2260, M. Stone and J. Kim, Phys. Rev. D 98 (2018) 025012).*

Unruh effect for fermions: energy-momentum tensor

4-order terms were calculated in *(G. P., O. V. Teryaev and V. I. Zakharov, Phys. Rev. D 99, no. 7, 071901 (2019)).* To define them, it is necessary to calculate quantum correlators with **4 boost operators** and **1 operator of the energy-momentum tensor**. In particular, for A_2 we need to calculate:

$$A_2 = \frac{1}{4!}\int_0^{|\beta|} d\tau_x d\tau_y d\tau_z d\tau_f \langle T_\tau \hat{K}^3_{-i\tau_x u}\hat{K}^3_{-i\tau_y u}\hat{K}^3_{-i\tau_z u}\hat{K}^3_{-i\tau_f u}\hat{T}^{00}(0)\rangle_{\beta(x),c}$$

Having expressed the boost operator through the energy-momentum tensor, we reduce the problem to the calculation of quantities of the form:

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\alpha_7\alpha_8|\alpha_9\alpha_{10}|ijkl} = \int_0^{|\beta|} d\tau_x d\tau_y d\tau_z d\tau_f d^3x d^3y d^3z d^3f$$
$$\times x^i y^j z^k f^l \langle T_\tau \hat{T}^{\alpha_1\alpha_2}(\tau_x,\mathbf{x})\hat{T}^{\alpha_3\alpha_4}(\tau_y,\mathbf{y})\hat{T}^{\alpha_5\alpha_6}(\tau_z,\mathbf{z})\hat{T}^{\alpha_7\alpha_8}(\tau_f,\mathbf{f})\hat{T}^{\alpha_9\alpha_{10}}(0)\rangle_{\beta(x),c}$$

where the energy-momentum tensor must be represented in a *split form:*

$$\hat{T}^{\alpha\beta}(X) = \lim_{X_1,X_2\to X}\mathcal{D}^{\alpha\beta}_{ab}(\partial_{X_1},\partial_{X_2})\bar{\Psi}_a(X_1)\Psi_b(X_2),$$

$$\mathcal{D}^{\alpha\beta}_{ab}(\partial_{X_1},\partial_{X_2}) = \frac{i^{\delta_{0\alpha}+\delta_{0\beta}}}{4}[\tilde{\gamma}^\alpha_{ab}(\partial_{X_2}-\partial_{X_1})^\beta + \tilde{\gamma}^\beta_{ab}(\partial_{X_2}-\partial_{X_1})^\alpha]$$

Unruh effect for fermions: energy-momentum tensor

The connected correlator from the product of Fermi fields is transformed to the sum of the products of thermal propagators according to analog of **Wick theorem** for thermal field theory

$$\langle T_\tau \overline{\Psi}_1 \Psi_2 \overline{\Psi}_3 \Psi_4 \overline{\Psi}_5 \Psi_6 \overline{\Psi}_7 \Psi_8 \overline{\Psi}_9 \Psi_{10} \rangle_{\beta(x),c} \to \langle T_\tau \overline{\Psi}_1 \Psi_4 \rangle_{\beta(x)} \langle T_\tau \Psi_2 \overline{\Psi}_5 \rangle_{\beta(x)} \times \ldots (24\,\text{terms})$$

Thermal propagator of Fermi field has a form *(M. Laine and A. Vuorinen, Lect. Notes Phys. 925 (2016)):*

$$\langle T_\tau \Psi_{a_1}(X_1) \overline{\Psi}_{a_2}(X_2) \rangle_{\beta(x)} = \sum\!\!\!\!\!\!\int_P e^{iP^+(X_1-X_2)} (-iP^+_\mu \tilde{\gamma}_\mu + m)_{a_1 a_2} \Delta(P^+)$$

where $P^\pm = (\pi(2n+1)/|\beta| \pm \mu, \mathbf{p})$ and $\Delta(P) = \frac{1}{P^2}$. Summation over the **Matsubara frequencies** can be done using the formula

$$\frac{1}{|\beta|} \sum_{\omega_n} \frac{(\omega_n \pm i\mu)^k e^{i(\omega_n \pm i\mu)\tau}}{(\omega_n \pm i\mu)^2 + E^2} =$$

$$\frac{1}{2E} \sum_{s=\pm 1} (-isE)^k e^{\tau s E} [\theta(-s\tau) - n_F(E \pm s\mu)]$$

Comparison with Wigner function

- The additional motivation for introducing such an integral representation is the result for energy density, obtained using the **Wigner function** (beyond the Boltzmann approximation). **Expanding** the Wigner function into a series, we obtain:

$$\rho_{Wig} = \frac{1}{2}\int \frac{d^3 p}{(2\pi)^3}\,\varepsilon\left(\sum_{n=0}^{\infty}(-1)^n \exp[t(n+l)(\beta\cdot p-\xi)]\right.$$
$$\times \sum_{m=0}^{\infty}\frac{1}{m!}\left(-\frac{1}{2}t(n+l)\right)^m \mathrm{tr}[(\varpi:\Sigma)^m]$$
$$\left. + (\xi\to-\xi,\varpi\to-\varpi)\right)$$

- It is necessary to **find a trace in each term**, decomposing the products of Dirac matrices into basic 4x4 matrices, and then **sum** the resulting **series back**.

Acceleration as imaginary chemical potential → source of instability

- **Acceleration** appears as an **imaginary chemical potential:**

$$\rho = 2\int \frac{d^3p}{(2\pi)^3}\Big(\frac{|\mathbf{p}|+ia}{1+e^{\frac{|\mathbf{p}|}{T}+\frac{ia}{2T}}}+\frac{|\mathbf{p}|-ia}{1+e^{\frac{|\mathbf{p}|}{T}-\frac{ia}{2T}}}\Big)+4\int \frac{d^3p}{(2\pi)^3}\frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{a}}-1}$$

As will now be shown, this leads to **instability at the Unruh temperature** *(G.P., O.V. Teryaev and V.I. Zakharov, arXiv:1906.03529)*. Energy density contains ***symmetric Fermi-integral*** (and similar ***antisymmetric***) of the form

$$I_{s1} = \int_0^\infty \frac{f(x)dx}{e^{x+iy}+1}+\int_0^\infty \frac{f(x)dx}{e^{x-iy}+1}$$

which can be transformed to the integral (“Blankenbecler’s method”)

$$\boxed{I_{s1} = \int_{I_{s1}} \frac{\eta^D d\eta}{(\eta+1)^2}F(-iy)}$$

in the complex plane of variable $\eta = e^x$. Here $D = \frac{\partial}{\partial(-iy)}$ and $F(x) = \int_0^x f(x)dx$, giving polynomial.

Acceleration as imaginary chemical potential → source of instability

Instabilty is manifested in **discontinuities** in $\frac{\partial^2 \rho}{\partial T^2}$:

$$\frac{\partial^2}{\partial T^2}\rho_{T>T_U}(T \to T_U) \neq \frac{\partial^2}{\partial T^2}\rho_{T<T_U}(T \to T_U)$$

Repeated pole crossing leads to instabilities at $T = T_U/(2k+1)\,,\; k = 0, 1..$. Such a situation could be expected in advance, since acceleration appears as an **imaginary chemical potential**. In the theories with an imaginary chemical potential, there are also periodic instabilities called the **Roberge-Weiss phase transitions** *(A. Roberge and N. Weiss, Nucl. Phys. B275, 734 (1986))*.

- Discontinuty at $T = \frac{a}{2\pi}$ should be interpreted as another **manifestation of the Unruh effect.**