Test of the Hypothesis of Realism Using a Modified Version of Wigner Inequality

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Introduction

Tests of the Local Realism using the Wigner inequality do not include any dependence on time.

Tests of the Macroscopic Realism using time-dependent Leggett–Garg inequalities require the technique of non-invasive (soft) measurements.

In the current talk we propose a new time-dependent inequality for tests of Hypothesis of Realism. These tests do not require non-invasive measurements.

NSC and NSIT

The "No-signaling condition" (NSC) is written in the following form:

$$\sum_{a} w(a,\, b_\beta,\, \ldots \,|\, A,\, B,\, \ldots) = w(b_\beta,\, \ldots \,|\, B,\, \ldots),$$

where A is an observable selected for measurement, a is the measured value of the observable A, and $\sum_{a}$ sums all possible values of the observable A. The same notation is used for the observable B.

The "No-signaling in time" condition (NSIT) demands that the probability $w(q_j,\, q_i,\, \ldots \,|\, t_j,\, t_i,\, \ldots)$ of measurement of an observable Q at times t_i, $t_j > t_i$ and so on, does not depend on the state of the observable Q at time $t_k \neq \{t_i,\, t_j,\, \ldots\}$. Denoting $Q(t_i)$ as q_i, no-signaling in time condition may be written as follows.

$$\sum_{q_k} w(q_j,\, q_k,\, q_i,\, \ldots \,|\, t_j,\, t_k,\, t_i,\, \ldots) = w(q_j,\, q_i,\, \ldots \,|t_j,\, t_i,\, \ldots).$$

Hypothesis of Realism

1) At any time t_i a system is in a "real physical state" which exists impartially and independently of any observer. "Real physical states" are distinguished from each other by the values of observables that characterize the system under study. We do not suppose these values to be jointly measurable by any macroscopic device.

2) Observable physical states of a system are distinguished by the values of variables which can be jointly measurable in the system at time t_i.

3) For the considered system the NSIT condition and/or NSC are hold.

4) The experimentalist has free will to plan, perform, and analyze the results of the experiments on the system.

Test of the hypothesis of realism – I

Consider a physical system which consists of two subsystems, "1" and "2". In each of the subsystems there is a variable $Q^{(\eta)}(t)$, where $\eta = \{1, 2\}$ is the subsystem index. At any time t_i both variables $Q^{(\eta)}(t)$ must have only a two defined values $q_i^{(\eta)} = \pm 1$ (so called dichotomic variables).

Let us consider three moments of time, $t_3 > t_2 > t_1$. At time t_1 there is an anticorrelation between dichotomic variables $Q^{(1)}(t)$ and $Q^{(2)}(t)$ like $Q^{(1)}(t_1) = -Q^{(2)}(t_1)$, or

$$q_{1\,\pm}^{(1)} = -\,q_{1\,\mp}^{(2)}.$$

If at time t_1 a measurement of $Q^{(\eta)}(t_1)$ occured, then at times t_2 and t_3 there is no correlation between $Q^{(1)}(t)$ and $Q^{(2)}(t)$.

If at time t_1 there is no measurement of $Q^{(\eta)}(t_1)$, then the anticorrelation will hold at t_2. Note, that by definition at t_3 the anticorrelation between the observables $Q^{(1)}(t)$ and $Q^{(2)}(t)$ cannot be observed under any conditions.

Test of the hypothesis of realism – II

We introduce a space of elementary outcomes $\omega^{(\widetilde{LG})} \in \Omega^{(\widetilde{LG})}$, which consists of the aggregates

$$\{q^{(2)}_{3\,\alpha},\, q^{(2)}_{2\,\beta},\, q^{(2)}_{1\,\gamma},\, q^{(1)}_{3\,\alpha'},\, q^{(1)}_{2\,\beta'},\, q^{(1)}_{1\,\gamma'=-\gamma}\},$$

where the indices $\{\alpha,\, \beta,\, \gamma,\, \alpha',\, \beta',\, \gamma'\} = \{+,\, -\}$, and the anticorrelation condition is taken into account. Denote an elementary event as:

$$\mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3\,\alpha},\, q^{(2)}_{2\,\beta},\, q^{(2)}_{1\,\gamma},\, q^{(1)}_{3\,\alpha'},\, q^{(1)}_{2\,\beta'},\, q^{(1)}_{1\,\gamma'=-\gamma}} \subseteq \Omega^{(\widetilde{LG})}$$

.

The full aggregate of such events forms a σ–algebra $\mathcal{F}^{(\widetilde{LG})}$. On $\left(\Omega^{(\widetilde{LG})},\, \mathcal{F}^{(\widetilde{LG})}\right)$ let us introduce a non-negative σ–additive measure $w\left(\omega^{(\widetilde{LG})},\, q^{(2)}_{3\,\alpha},\, q^{(2)}_{2\,\beta},\, q^{(2)}_{1\,\gamma},\, q^{(1)}_{3\,\alpha'},\, q^{(1)}_{2\,\beta'},\, q^{(1)}_{1\,\gamma'=-\gamma} \,|\, t_3,\, t_2,\, t_1\right)$. The triplet $\left(\Omega^{(\widetilde{LG})},\, \mathcal{F}^{(\widetilde{LG})},\, w(\ldots)\right)$ is a probabilistic model, which will be used to test the hypothesis of realism.

Test of the hypothesis of realism – III

We introduce the first event:

$$\mathcal{K}_{32}^{(\widetilde{LG})} = \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\, q^{(2)}_{2-},\, q^{(2)}_{1+},\, q^{(1)}_{3+},\, q^{(1)}_{2+},\, q^{(1)}_{1-}} \cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\, q^{(2)}_{2-},\, q^{(2)}_{1+},\, q^{(1)}_{3-},\, q^{(1)}_{2+},\, q^{(1)}_{1-}} \cup$$

$$\cup\ \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\, q^{(2)}_{2-},\, q^{(2)}_{1-},\, q^{(1)}_{3+},\, q^{(1)}_{2+},\, q^{(1)}_{1+}} \cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\, q^{(2)}_{2-},\, q^{(2)}_{1-},\, q^{(1)}_{3-},\, q^{(1)}_{2+},\, q^{(1)}_{1+}}.$$

In this event we takes into account that the variables $Q^{(1)}(t)$ and $Q^{(2)}(t)$ are anticorrelated at time t_1, as well as at time t_2, because there has been no measurement at t_1. Then, taking into account NSIT, we may write:

$$w\left(q^{(2)}_{3+},\, q^{(1)}_{2+}\,|\, t_3,\, t_2\right) = \sum_{\omega^{(\widetilde{LG})}_{32} \in \mathcal{K}^{(\widetilde{LG})}_{32}} \sum_{q^{(1)}_3} \sum_{q^{(1)}_1} \sum_{q^{(2)}_1} \delta_{-q^{(1)}_1\, q^{(2)}_1}\, w\left(\omega^{(\widetilde{LG})}_{32},\, q^{(2)}_{3+},\, q^{(2)}_{2-},\, q^{(2)}_1,\, q^{(1)}_3,\, q^{(1)}_{2+},\, q^{(1)}_1\,|\, t_3,\, t_2,\, t_1\right),$$

where δ_{ij} is the Kronecker simbol.

Test of the hypothesis of realism – IV

Let us introduce second event:

$$\begin{aligned}
\mathcal{K}_{31}^{(\widetilde{LG})} &= \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\,q^{(2)}_{2+},\,q^{(2)}_{1-},\,q^{(1)}_{3+},\,q^{(1)}_{2+},\,q^{(1)}_{1+}} \cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\,q^{(2)}_{2+},\,q^{(2)}_{1-},\,q^{(1)}_{3+},\,q^{(1)}_{2-},\,q^{(1)}_{1+}} \cup \\
&\cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\,q^{(2)}_{2+},\,q^{(2)}_{1-},\,q^{(1)}_{3-},\,q^{(1)}_{2+},\,q^{(1)}_{1+}} \cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\,q^{(2)}_{2+},\,q^{(2)}_{1-},\,q^{(1)}_{3-},\,q^{(1)}_{2-},\,q^{(1)}_{1+}} \cup \\
&\cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\,q^{(2)}_{2-},\,q^{(2)}_{1-},\,q^{(1)}_{3+},\,q^{(1)}_{2+},\,q^{(1)}_{1+}} \cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\,q^{(2)}_{2-},\,q^{(2)}_{1-},\,q^{(1)}_{3+},\,q^{(1)}_{2-},\,q^{(1)}_{1+}} \cup \\
&\cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\,q^{(2)}_{2-},\,q^{(2)}_{1-},\,q^{(1)}_{3-},\,q^{(1)}_{2+},\,q^{(1)}_{1+}} \cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\,q^{(2)}_{2-},\,q^{(2)}_{1-},\,q^{(1)}_{3-},\,q^{(1)}_{2-},\,q^{(1)}_{1+}}.
\end{aligned}$$

For this event we define probability:

$$\begin{aligned}
w\left(q^{(2)}_{3+},\, q^{(1)}_{1+} \,|\, t_3,\, t_1\right) &= \sum_{\omega^{(\widetilde{LG})}_{31} \in \mathcal{K}^{(\widetilde{LG})}_{31}} \sum_{q^{(1)}_3} \sum_{q^{(1)}_2} \sum_{q^{(2)}_2} \\
&\quad w\left(\omega^{(\widetilde{LG})}_{31},\, q^{(2)}_{3\,+},\, q^{(2)}_{2},\, q^{(2)}_{1\,-},\, q^{(1)}_{3},\, q^{(1)}_{2},\, q^{(1)}_{1\,+} \,|\, t_3,\, t_2,\, t_1\right).
\end{aligned}$$

Test of the hypothesis of realism – V

Let us introduce third event:

$$\begin{aligned}
\mathcal{K}_{12}^{(\widetilde{LG})} &= \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\, q^{(2)}_{2+},\, q^{(2)}_{1+},\, q^{(1)}_{3+},\, q^{(1)}_{2+},\, q^{(1)}_{1-}} \cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\, q^{(2)}_{2+},\, q^{(2)}_{1+},\, q^{(1)}_{3-},\, q^{(1)}_{2+},\, q^{(1)}_{1-}} \cup \\
&\cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\, q^{(2)}_{2-},\, q^{(2)}_{1+},\, q^{(1)}_{3+},\, q^{(1)}_{2+},\, q^{(1)}_{1-}} \cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3+},\, q^{(2)}_{2-},\, q^{(2)}_{1+},\, q^{(1)}_{3-},\, q^{(1)}_{2+},\, q^{(1)}_{1-}} \cup \\
&\cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3-},\, q^{(2)}_{2+},\, q^{(2)}_{1+},\, q^{(1)}_{3+},\, q^{(1)}_{2+},\, q^{(1)}_{1-}} \cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3-},\, q^{(2)}_{2+},\, q^{(2)}_{1+},\, q^{(1)}_{3-},\, q^{(1)}_{2+},\, q^{(1)}_{1-}} \cup \\
&\cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3-},\, q^{(2)}_{2-},\, q^{(2)}_{1+},\, q^{(1)}_{3+},\, q^{(1)}_{2+},\, q^{(1)}_{1-}} \cup \mathcal{K}^{(\widetilde{LG})}_{q^{(2)}_{3-},\, q^{(2)}_{2-},\, q^{(2)}_{1+},\, q^{(1)}_{3-},\, q^{(1)}_{2+},\, q^{(1)}_{1-}} .
\end{aligned}$$

For last event we define probability:

$$\begin{aligned}
w\left(q^{(2)}_{1+},\, q^{(1)}_{2+} \,|\, t_2,\, t_1\right) &= \sum_{\omega^{(\widetilde{LG})}_{12} \in \mathcal{K}^{(\widetilde{LG})}_{12}} \sum_{q^{(2)}_2} \sum_{q^{(2)}_3} \sum_{q^{(1)}_3} \\
&\quad w\left(\omega^{(\widetilde{LG})}_{12},\, q^{(2)}_3,\, q^{(2)}_2,\, q^{(2)}_{1+},\, q^{(1)}_3,\, q^{(1)}_{2+},\, q^{(1)}_{1-} \,|\, t_3,\, t_2,\, t_1\right).
\end{aligned}$$

Test of the hypothesis of realism – VI

The sum of the second and third events defines the event

$$\mathcal{K}_{321}^{(\widetilde{LG})} = \mathcal{K}_{31}^{(\widetilde{LG})} \cup \mathcal{K}_{12}^{(\widetilde{LG})}.$$

The event $\mathcal{K}_{321}^{(\widetilde{LG})}$ also contains the event $\mathcal{K}_{32}^{(\widetilde{LG})}$.

Taking into account the non-negativity of the probability we find that for event $\mathcal{K}_{321}^{(\widetilde{LG})}$ the following is satisfied:

$$w\left(q_{3+}^{(2)},\, q_{2+}^{(1)} \,|\, t_3,\, t_2\right) \leq w\left(q_{3+}^{(2)},\, q_{1+}^{(1)} \,|\, t_3,\, t_1\right) + w\left(q_{1+}^{(2)},\, q_{2+}^{(1)} \,|\, t_2,\, t_1\right).$$

This main inequality is obtained using the hypothesis of realism and NSIT condition.

An example of inequality violation – I

Consider a pair of neutral pseudoscalar mesons, which at time $t_1 = 0$ are in Bell-state

$$\left|\,\Psi^+\,\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\,M^{(2)}\,\right\rangle \otimes \left|\,\bar{M}^{(1)}\,\right\rangle + \left|\,\bar{M}^{(2)}\,\right\rangle \otimes \left|\,M^{(1)}\,\right\rangle\right).$$

This state is anticorrelated by flavor of the pair, but is correlated by CP-parity (defined as $\left|\,M_1^{(i)}\,\right\rangle$ and $\left|\,M_2^{(i)}\,\right\rangle$) and mass/lifetime (defined as $\left|\,M_H^{(i)}\,\right\rangle$ and $\left|\,M_L^{(i)}\,\right\rangle$).

Let us choose as an observable $Q^{(\eta)}(t)$ the flavor of pseudoscalar meson. $Q = +1$, corresponds to meson with flavor "M", while $Q = -1$ – to meson flavor "$\bar{M}$".

An example of inequality violation – II

For the subsequent calculations let us use the following definitions:

$$\Delta m = m_H - m_L, \qquad \Delta\Gamma = \Gamma_H - \Gamma_L, \qquad \Gamma \;=\; \frac{1}{2}\,(\Gamma_H + \Gamma_L).$$

The probability $w\left(q_{3+}^{(2)},\, q_{2+}^{(1)}\,|\,t_3,\, t_2\right)$ **may be written as:**

$$w\left(q_{3+}^{(2)},\, q_{2+}^{(1)}\,|\,t_3,\, t_2\right) \;=\; \frac{1}{4}\,e^{-2\Gamma \boldsymbol{t_3}}\,\mathrm{ch}\,\left(\frac{\Delta\Gamma\,\Delta t_{32}}{2}\right) \left[\mathrm{ch}\,\left(\frac{\Delta\Gamma\,(t_2+t_3)}{2}\right) - \cos\left(\Delta m\,(t_2+t_3)\right)\right].$$

In analogy we obtain:

$$w\left(q_{3+}^{(2)},\, q_{1+}^{(1)}\,|\,t_3,\, t_1\right) \;=\; \frac{1}{4}\,e^{-2\Gamma \boldsymbol{t_3}}\,\mathrm{ch}\,\left(\frac{\Delta\Gamma\,\Delta t_3}{2}\right)\left[\mathrm{ch}\,\left(\frac{\Delta\Gamma\,t_3}{2}\right) - \right.$$

$$\left. -\;\; \cos\left(\Delta m\,t_3\right)\right]$$

An example of inequality violation – III

and

$$w\left(q_{1+}^{(2)},\, q_{2+}^{(1)} \,|\, t_2,\, t_1\right) \;=\; \frac{1}{4}\, e^{-2\Gamma t_3}\, \mathrm{ch}\left(\frac{\Delta\Gamma\, \Delta t_{32}}{2}\right) \mathrm{ch}\left(\frac{\Delta\Gamma\; t_3}{2}\right) \left[\mathrm{ch}\left(\frac{\Delta\Gamma\; t_2}{2}\right) - \cos\left(\Delta m\, t_2\right)\right].$$

Denote

$$\kappa \;=\; \frac{\Delta\,\Gamma}{2\,\Delta\,m}, \qquad \alpha \;=\; \Delta m\, t_3, \qquad \beta \;=\; \Delta m\, t_2.$$

Then we find the following inequality:

$$\begin{aligned} &\Big[\mathrm{ch}(\kappa\,(\alpha+\beta)) \;-\; \cos(\alpha+\beta)\Big]\,\mathrm{ch}(\kappa\,(\alpha-\beta)) \;\leq \\ \leq\;\; &\Big[\mathrm{ch}(\kappa\,\alpha) \;-\; \cos(\alpha)\Big]\,\mathrm{ch}(\kappa\,\alpha) + \\ +\;\; &\Big[\mathrm{ch}(\kappa\,\beta) \;-\; \cos(\beta)\Big]\,\mathrm{ch}(\kappa\,(\alpha-\beta))\,\mathrm{ch}(\kappa\,\alpha). \end{aligned}$$

An example of inequality violation – IV

In order to simplify the above inequality let us consider $B_s\,\bar{B}_s$–meson pairs. For B_s–meson $\Delta\Gamma \approx -6.0 \times 10^{-11}$ MeV and $\Delta m \approx 1.2 \times 10^{-8}$ MeV. Hence $\kappa \approx -2.5 \times 10^{-3}$.

I.e. the violation of above inequality may be considered in $\kappa = 0$ regime. In this case our inequality turns into a simple relation:

$$\cos(\alpha) + \cos(\beta) - \cos(\alpha + \beta) \leq 1$$

for $\alpha > \beta > 0$. Choose $\alpha = \dfrac{3\pi}{8}$ and $\beta = \dfrac{3\pi}{10}$. Then $\cos\alpha \approx 0.383$, $\cos\beta \approx 0.588$, and $\cos(\alpha + \beta) \approx -0.522$, which leads to violation of current inequality.

Conclusion

1. **We obtain the inequality**

$$w\left(q^{(2)}_{3+},\, q^{(1)}_{2+} \,|\, t_3,\, t_2\right) \le w\left(q^{(2)}_{3+},\, q^{(1)}_{1+} \,|\, t_3,\, t_1\right) + w\left(q^{(2)}_{1+},\, q^{(1)}_{2+} \,|\, t_2,\, t_1\right).$$

   for test of the Hypothesis of Realism.

2. **We stress the fact that derivation of this inequality requires the NSIT condition.**
3. **We have shown that this inequality is violated in quantum mechanics.**

Thank you!