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Effects of light-by-light scattering in the Lamb shift and hyperfine structure of muonic hydrogen.

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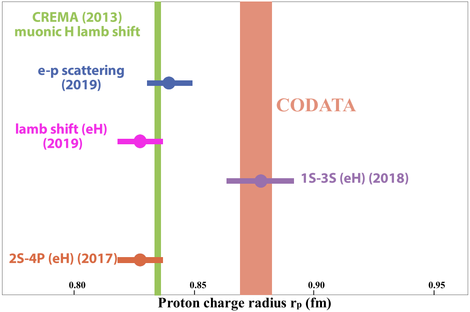
Proton radius puzzle

is a disagreement between the value of the proton charge radius r_p obtained from experiments involving muonic hydrogen and those based on electron-proton systems.

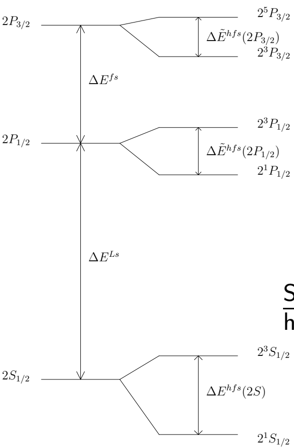


2019 results show that maybe the proton radius puzzle is starting to resolve

- CODATA (ep) summary: $r_p = 0.8775(51)$ fm
- A. Antognini et al., Science **339**, 417 (2013)
  (μp) lamb shift : $r_p = 0.84087(39)$ fm
- A. Beyer, et al., Science **358**, 79–85 (2017)
  (ep) transition frequency 2S-4P : $r_p = 0.8335(95)$ fm
- H. Fleurbaey et al. Phys. Rev. Lett. **120**, 183001 (2018)
  (ep) transition frequency 1S-3S: $r_p = 0.877(13)$ fm
- N. Bezginov et al., Science **365** (6457), 1007-1012(2019)
  (ep) lamb shift: $r_p = 0.833(10)$ fm
- J. M. Alarcon et al., Phys. Rew. C **99**, 044303(2019)
  $(e-p)$ scattering: $r_p = 0.844(7)$ fm

The aim of the work

Laser spectroscopy of muonic atoms can be used for the determination of nuclear parameters with high accuracy. In the coming years different collaborations plan new experiments:



1. A. Adamczak et al., The FAMU experiment at RIKEN-RAL to study the muon transfer rate from hydrogen to other gases, JINST 13 no.12, P12033, (2018)

The measurement of ground state HFS in muonic hydrogen with an accuracy 1-2 ppm.

2. R. Pohl et al., The next generation of laser spectroscopy experiments using light muonic atoms, IOP Conf. Series: Journal of Physics: Conf. Series 1138, 012010 (2018)

The measurement of fine and hyperfine structure in other light muonic ions.

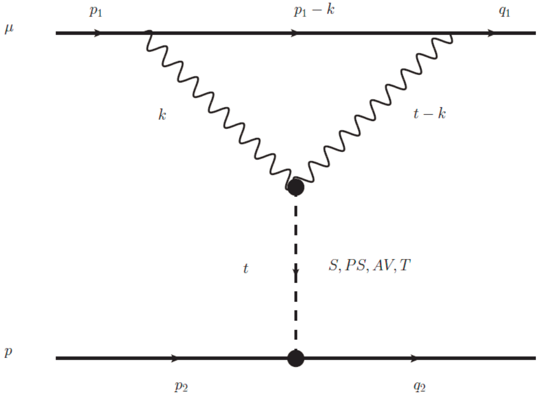
3. The measurement of 1S-2S transition frequency in muonic hydrogen.

So, it is necessary to improve the calculation of muonic hydrogen energy spectrum:

- More accurate theoretical construction of the particle interaction operator.
- Accounting for new contributions, that were not studied.

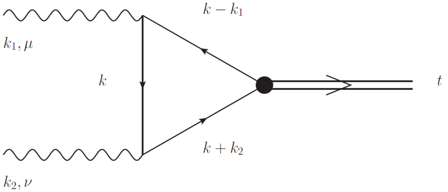
Effective one meson exchange

New direction in the study of the energy spectrum (μp) is connected with processes of two-photon interaction leading to effective one meson exchange



Two-photon interaction leads to one-meson (pseudoscalar (P), axial-vector (AV), scalar (S), tensor (T)) exchange potential.

One meson production vertex



The general parametrization of meson - two photon vertex function for different mesons in the leading order.

Scalar mesons:

$$T_S^{\mu\nu}=e^2\Big\{A(t^2,k_1^2,k_2^2)(g^{\mu\nu}(k_1\cdot k_2)-k_1^\nu k_2^\mu)+B(t^2,k_1^2,k_2^2)(k_2^\mu k_1^2-k_1^\mu(k_1\cdot k_2))(k_1^\nu k_2^2-k_2^\nu(k_1\cdot k_2))\Big\}. \qquad (1)$$

Pseudoscalar mesons:

$$T_P^{\mu\nu}=i\varepsilon^{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}\frac{\alpha}{\pi F_\pi}F_{\pi^0\gamma^*\gamma^*}(k_1^2,k_2^2), \qquad (2)$$

Axial-vector mesons:

$$T_{AV}^{\mu\nu\alpha}=4\pi i\alpha\varepsilon_{\mu\nu\alpha\tau}(k_1^\tau k_2^2-k_2^\tau k_1^2)F_{AV\gamma^*\gamma^*}(k_1^2,k_2^2), \qquad (3)$$

Tensor mesons:

$$T_T^{\mu\nu\alpha\beta}=e^2\frac{k_1k_2}{M}\mathcal{M}_{\mu\nu\alpha\beta}(k_1,k_2)\mathcal{F}_{T\gamma^*\gamma^*}(k_1^2,k_2^2), \qquad (4)$$

Our recent results

We have already investigated the contributions of scalar, pseudoscalar and axial-vector mesons to the energy spectrum of muonic hydrogen.

- A.E. Dorokhov, N.I. Kochelev, A.P. Martynenko, F.A. Martynenko, R.N. Faustov, The contribution of pseudoscalar mesons to hyperfine structure of muonic hydrogen, Phys.Part.Nucl.Lett. 14 (2017) no.6, 857-864
- A.E. Dorokhov, N.I. Kochelev, A.P. Martynenko, F.A. Martynenko, A.E. Radzhabov, The contribution of axial-vector mesons to hyperfine structure of muonic hydrogen , Phys.Lett. B776 (2018) 105-110
- A.E. Dorokhov, A.P. Martynenko, F.A. Martynenko, A.E. Radzhabov, The sigma-meson exchange contribution to the muonic hydrogen Lamb shift, EPJ Web Conf. 212 (2019) 07003

Muon proton interaction amplitude

Now we are interested in the interaction via tensor meson exchange. Following paper

V.Pauk, M. Vanderhaeghen, Single meson contribution to the muon's anomalius magnetic moment, EPJ C (2014), **74**:3008.

the relevant part of amplitude for the proces $\gamma^*\gamma^* \to T$ can be parameterized by

$$T^T_{\mu\nu\alpha\beta}(k_1,k_2) = e^2\frac{\mathbf{k_1k_2}}{M}\mathcal{M}_{\mu\nu\alpha\beta}(k_1,k_2)\mathcal{F}_{T\gamma^*\gamma^*}(k_1^2,k_2^2), \tag{5}$$

where $\mathcal{F}_{T\gamma^*\gamma^*}(k_1^2,k_2^2)$ is a transition form factor,

$$\mathcal{M}_{\mu\nu\alpha\beta}(k_1,k_2) = \Big\{R_{\mu\alpha}(k_1,k_2)R_{\nu\beta}(k_1,k_2) + \frac{1}{8(k_1+k_2)^2\left[(k_1k_2)^2-k_1^2k_2^2\right]}R_{\mu\nu}(k_1,k_2)\times \tag{6}$$

$$\left[(k_1+k_2)^2(k_1-k_2)_\alpha-(k_1^2-k_2^2)(k_1+k_2)_\alpha\right]\times\left[(k_1+k_2)^2(k_1-k_2)_\beta-(k_1^2-k_2^2)(k_1+k_2)_\beta\right]\Big\},$$

$$R_{\mu\nu}(k_1,k_2) = -g_{\mu\nu}+\frac{1}{X}\left[(k_1k_2)(k_1^\mu k_2^\nu+k_2^\mu k_1^n u)-k_1^2k_2^\mu k_2^\nu-k_2^2k_1^\mu k_1^\nu\right],\ X=(k_1k_2)^2-k_1^2k_2^2.$$

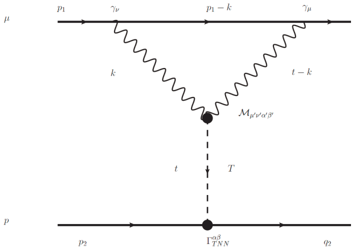
We also need to know tensor meson propogator. We can take it in the form:

F. Jegerlehner, The Anomalous Magnetic Moment of the Muon, Springer Tracts in Modern Physics 274 (ISBN 978-3-319-63577-4)

$$\mathcal{D}^{\mu\nu\alpha\beta}_T(t) = \frac{1}{t^2-M_T^2+i\varepsilon}\Big\{\frac{1}{2}(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}-g_{\mu\nu}g_{\alpha\beta})+ \tag{7}$$

$$\frac{1}{2}\left(g_{\mu\alpha}\frac{t^\nu t^\beta}{M_T^2}+g_{\nu\beta}\frac{t^\mu t^\alpha}{M_T^2}+g_{\mu\beta}\frac{t^\nu t^\alpha}{M_T^2}+g_{\nu\alpha}\frac{t^\mu t^\beta}{M_T^2}\right)+\frac{2}{3}\left(\frac{1}{2}g_{\mu\nu}+\frac{t^\mu t^\nu}{M_T^2}\right)\left(\frac{1}{2}g_{\alpha\beta}+\frac{t^\alpha t^\beta}{M_T^2}\right)\Big\}$$

Muon proton interaction amplitude



Then the interaction amplitude can be presented in the form:

$$i\mathcal{M} = 4\pi Z\alpha \frac{1}{16m_1^2 m_2^2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(p_1-k)^2-m_1^2}\mathcal{D}_{\mu\mu'}(t-k)\mathcal{D}_{\nu\nu'}(k)\mathcal{D}_T^{\alpha'\beta'\alpha\beta}(t)\mathcal{M}_{\mu'\nu'\alpha'\beta'}(k_1,k_2) \qquad (8)$$

$$[\bar{u}(0)(\hat{q}_1+m_1)\gamma_\mu(\hat{p}_1-\hat{k}+m_1)\gamma_\nu(\hat{p}_1+m_1)u(0)][\bar{v}(0)(\hat{p}_2-m_2)\Gamma^{\alpha\beta}_{TNN}(\hat{q}_2-m_2)v(0)],$$

where the interaction vertex of tensor meson with nucleon has the form:

$$\Gamma^{\alpha\beta}_{TNN}(p_2,q_2) = \frac{G_{TNN}}{m_2}\left[(q_2+p_2)_\alpha\gamma_\beta+(q_2+p_2)_\beta\gamma_\alpha\right]+\frac{F_{TNN}}{m_2^2}(q_2+p_2)_\alpha(q_2+p_2)\beta,\;\; G_{TNN}\gg F_{TNN}.$$

Y. Oh, T.-S. H. Lee, ρ meson production at low energies, Phys. Rev. C (2004), **69**, 025201.

Projection operators

To obtain the contribution in muon-proton interaction potential we use the projection operators formalism on the states with $F = 0, 1$. We construct that operators from free wave functions of muon and proton. In the case of S-states we introduce projection operators on the states with $F = 0, 1$:

$$\hat{\Pi}_{F=0[1]} = u(0)\bar{v}(0)|_{F=0[1]} = \frac{1}{2\sqrt{2}}(1+\gamma_0)\gamma_5[\hat{\varepsilon}] \quad (9)$$

$$\hat{\Pi}^*_{F=0[1]} = v(0)\bar{u}(0)|_{F=0[1]} = \frac{1}{2\sqrt{2}}\gamma_5[\hat{\varepsilon}^*](1+\gamma_0).$$

In the case of P-states we also need to introduce projection operators on the states with total angular momentum of muon $J = 1/2$. We need them when we add together $S_\mu = \frac{1}{2}$ and $L = 1$.

$$\hat{\Pi}_\tau = u(0)\varepsilon^*_\tau(0)|_{J=1/2} = \frac{1}{\sqrt{3}}\gamma_5(\gamma_\tau - v_\tau)\psi(0), \quad (10)$$

$$\hat{\Pi}^*_\tau = \varepsilon_\tau(0)\bar{u}(0)|_{J=1/2} = \frac{1}{\sqrt{3}}\bar{\psi}(0)(\gamma_\tau - v_\tau)\gamma_5,$$

Trace calculation

The introduction of projection operators allows us to simplify our calculation and calculate trace from all gamma-factors in numerator of amplitude. For example in the case of S-state with $F = 1$ we obtain:

$$\mathcal{T}(S, F = 1) = Tr\Big[\gamma_{\varepsilon_1}\frac{1+\gamma_0}{2\sqrt{2}}(\hat{q}_1 + m_1)\gamma_\mu(\hat{p}_1 - \hat{k} + m_1)\gamma_\nu(\hat{p}_1 + m_1)\frac{1+\gamma_0}{2\sqrt{2}}\gamma_{\varepsilon_2}(\hat{p}_2 - m_2)\Gamma^{\alpha\beta}_{TNN}(\hat{q}_2 - m_2)\Big]\times \quad (11)$$

$$\frac{1}{3}(-g_{\varepsilon_1\varepsilon_2} + v_{\varepsilon_1}v_{\varepsilon_2})$$

Doing trace calculation in package FORM we obtain in leading order (does't contain t)

$$\mathcal{T}_{S,F=1} = \mathcal{T}_{S,F=0} = \frac{G_{TNN}}{M_T}4m_1k^4\left(1 + \frac{k_0^4t^4}{\left[(kt)^2 - k^2t^2\right]^2} + 2\frac{k_0^2t^2}{\left[(kt)^2 - k^2t^2\right]}\right), \quad t = p - q \sim \alpha. \quad (12)$$

We also obtain contribution to the hyperfine splitting ($\mathcal{T}_{F=1} - \mathcal{T}_{F=0}$) (proportional to t^2):

$$\mathcal{T}_{S,hfs} = -\frac{4}{3}\frac{G_{TNN}}{M_Tm_2}t^2k^4\left(1 - \frac{t^4k_0^4}{\left[(kt)^2 - k^2t^2\right]^2}\right) \quad (13)$$

Transition form factor parametrization

For some kinds of mesons we have experimental data on transition form factors:

1. For pseudoscalar mesons experimental data were obtained by CLEO collaboration
   - J. Gronberg et al.(CLEO collaboration), Phys. Rew. D **57** 33 (1998)
2. For axial-vector mesons experimental data were obtained by L3 Collaboration
   - L3 Collaboration (P. Achard et al.), f(1)(1285) formation in two photon collisions at LEP, Phys.Lett. B526 269-277 (2002)

All these data were successfully described using monopole parametrization on each variable $k_1,\ k_2$:

$$F(k_1,k_2)=F(0,0)\frac{\Lambda^2}{k_1^2+\Lambda^2}\frac{\Lambda^2}{k_2^2+\Lambda^2},\ \Lambda\ \approx\ 1\ GeV.$$

For the transition form factor into tensor meson we haven't experimental data, but we also use that monopole form:

$$\mathcal{F}_{T\gamma^*\gamma^*}(k_1^2,k_2^2)=\mathcal{F}_{T\gamma^*\gamma^*}(0,0)\frac{\Lambda^4}{(k^2+\Lambda^2)^2},\ k_1=k,\ k_2=-k \tag{14}$$

Loop-momentum integration

We use transition to the Euclidean space to calculate integral over loop momentum k:

$$\begin{cases} k^2 \to -(k^E)^2 \\ k_0^2 \to -(k_0^E)^2 \\ k_0 \to ik_0^E \end{cases} , \quad \begin{cases} k_0^E \to kCos(\phi) \\ |\mathbf{k}^E| \to kSin(\phi) \end{cases}$$

$$\mathcal{I} = \int_0^\infty \frac{d^4k}{k^4}\frac{1}{k^4-4k_0^2m_1^2}\mathcal{F}_{T\gamma^*\gamma^*}(0,0)\frac{\Lambda^4}{(k^2+\Lambda^2)^2} \times k^4\left(1+\frac{k_0^4t^4}{\left[(kt)^2-k^2t^2\right]^2}+2\frac{k_0^2t^2}{\left[(kt)^2-k^2t^2\right]}\right) =$$

After some simplifications we obtain:

$$= \frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\alpha\sqrt{\pi M_T}}\int_0^\infty \frac{kdk}{(1+k^2)^2}\int_0^\pi \frac{Sin^2\psi d\psi}{k^2+a_1^2Cos^2\psi}\int_0^\pi Sin\theta d\theta \frac{Sin^4\theta Sin^4\psi}{\left[1-Sin^2\psi Cos^2\theta\right]^2},$$

where $a_1 = \frac{2m_1}{\Lambda}$, $\mathcal{F}_{T\gamma^*\gamma^*}(0,0) = \frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\alpha\sqrt{\pi M_T}}$. We can analytically integrate over θ and k.

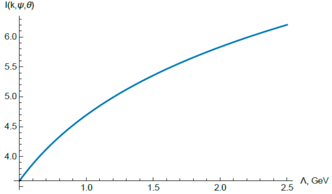
$$\mathcal{I} = -\int_0^\pi \frac{Sin\psi d\psi}{4\left(a_1^2Cos\psi+a_1^2-2\right)^2}\left(a_1^2Cos2\psi-2\log\left(a_1^2Cos^2\psi\right)+a_1^2-2\right)\times$$

$$\left(Sin^3\psi-3Sin\psi\left(Cos^2\psi+3\right)+Cos^2\psi(Cos2\psi-7)\log\left(\frac{2}{Sin\psi+1}-1\right)\right)$$

Interaction potentials

After that we can write interaction potential contributing to the lamb shift as follows:

$$\Delta V_T^{LS}(r) = -\frac{16\alpha^2 m_1 G_{TNN}}{M_T}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\alpha\sqrt{\pi M_T}}\mathcal{I}\frac{1}{4\pi r}e^{-M_T r}, \quad (15)$$



The dependence of the integral on Λ

Interaction potential contributing to the hyperfine structure:

$$\Delta V_T^{HFS}(r) = \frac{8Z\alpha^2 G_{TNN}}{3\pi m_2 M_T}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\alpha\sqrt{\pi M_T}}\mathcal{J}\left(\delta(\mathbf{r}) - \frac{M_T^2}{4\pi r}e^{-M_T r}\right), \quad \mathcal{J} = \int_0^\pi \frac{Sin\psi d\psi}{(-2+a_1^2+a_1^2 Cos2\psi)^2}\times \quad (16)$$

$$\left(a_1^2 Cos2\psi - 2\log\left(a_1^2 Cos^2\psi\right) + a_1^2 - 2\right)\left(Sin^3\psi + 7Sin\psi - 3Sin\psi Cos^2\psi + 2Cos^4\psi \log\left(\frac{2}{Sin\psi+1} - 1\right)\right)$$

where m_1 is muonic mass, M_T - tensor meson mass. For calculation $\mathcal{I}$ and $\mathcal{J}$ we use numerical integration.

Light tensor mesons (PDG)

There are some tensor mesons which can contribute to LS and HFS, but in our calculation we take into account only the contribution $f_2(1270)$, because for it we know all parameters, including coupling constant with nucleon.

- $f_2(1270)\ (0^+(2^{++})):\ \Gamma_{\gamma\gamma}/\Gamma = (1.42 \pm 0.24) * 10^{-5},\ \Gamma = 186.7\ MeV$

  Y. Oh, T.-S. H. Lee, ρ meson production at low energies, Phys. Rev. C (2004), **69**, 025201.

  $G^2_{TNN}/4\pi = 3.31 \pm 0.63$
- $a_2(1320)\ (1^-(2^{++})):\ \Gamma_{\gamma\gamma}/\Gamma = (9.4 \pm 0.7) * 10^{-6},\ \Gamma = 107.5\ MeV$
- $f_2^{'}(1525)\ (0^+(2^{++})):\ \Gamma_{\gamma\gamma}/\Gamma = (1.10 \pm 0.14) * 10^{-6},\ \Gamma = 73\ MeV$
- $\eta_2(1645)\ (0^+(2^{-+}))$: Decay $\eta_2(1645) \to \gamma + \gamma$ does not exist.
- $\pi_2(1670)\ (1^-(2^{-+})) : \Gamma_{\gamma\gamma}/\Gamma < (2.8) * 10^{-7},\ \Gamma = 258\ MeV$.
- $a_2(1700)\ (1^-(2^{++})) : \Gamma_{\gamma\gamma}/\Gamma = (1.16 \pm 0.27) * 10^{-6},\ \Gamma = 258\ MeV$.
- $\eta_2(1870)\ (0^+(2^{-+}))$: Decay $\eta_2(1870) \to \gamma + \gamma$ is not studied well.
- $f_2(1950)\ (0^+(2^{++}))$: Decay $f_2(1950) \to \gamma + \gamma$ is not studied well.
- $f_2(2010)\ (0^+(2^{++}))$: Decay $f_2(2010) \to \gamma + \gamma$ does not exist.
- $f_2(2300)\ (0^+(2^{++}))$: Decay $f_2(2300) \to \gamma + \gamma$ is not studied well.
- $f_2(2340)\ (0^+(2^{++}))$: Decay $f_2(2340) \to \gamma + \gamma$ does not exist.

M. Tanabashi et al. (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)

Numerical results

Using obtained potentials we can calculate contributions to the energy levels of muonic hydrogen:

$$\Delta E^{LS}_{1S} = -\frac{16(Z\alpha)^4\mu^3 m_1 G_{TNN}}{\pi M_T}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\sqrt{\pi M_T}}\frac{1}{(M_T+2W)^2}\mathcal{I} = -0.0528\ meV, \tag{17}$$

$$\Delta E^{LS}_{2S} = -\frac{2(Z\alpha)^4\mu^3 m_1 G_{TNN}}{\pi M_T}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\sqrt{\pi M_T}}\frac{(2M_T^2+W^2)}{2(M_T+2W)^4}\mathcal{I} = -0.0066\ meV, \tag{18}$$

$$\Delta E^{HFS}_{1S} = -\frac{8(Z\alpha)^4\mu^3 G_{TNN}}{3\pi^2 M_T m_2}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\sqrt{\pi M_T}}\left(1-\frac{M_T^2}{(M_T+2W)^2}\right)\mathcal{J} = -0.0551\ \mu eV, \tag{19}$$

$$\Delta E^{HFS}_{2S} = -\frac{(Z\alpha)^4\mu^3 G_{TNN}}{3\pi^2 M_T m_2}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\sqrt{\pi M_T}}\left(1-\frac{M_T^2(2M_T^2+W^2)}{2(M_T+W)^4}\right)\mathcal{J} = -0.0069\ \mu eV, \tag{20}$$

where $W = \mu Z\alpha$.

The main uncertainty is associated with:

1. Significant difference between values of the G_{TNN} constant: different estimations give us

$$G^2_{TNN}/4\pi = 0.38 \pm 0.04,\ 1.12,\ 2.2 \pm 0.9,\ 3.31 \pm 0.63,\ 4.0 \pm 1.0.$$

2. Lack of experimental data, which can determine the choice of Λ

Numerical results for all mesons

Таблица: Contribution of one-meson exchange to the energy level of muonic hydrogen.

Meson	$I^G(J^{PC})$	$\Delta E^{LS}(2P-2S)$ in meV	$\Delta E^{HFS}(1S)$ in meV	$\Delta E^{HFS}(2S)$ in meV
$\sigma(550)$	$0^+(0^{++})$	0.0113	—	—
$f_0(980)$	$0^+(0^{++})$	0.0009	—	—
$a_0(980)$	$1^-(0^{++})$	0.0008	—	—
$f_0(1370)$	$0^+(0^{++})$	0.0014	—	—
$\pi^0(134.9)$	$1^-(0^{-+})$	—	-0.0017	-0.0002
$f_1(1285)$	$0^+(1^{++})$	—	-0.0093	-0.0012
$a_1(1260)$	$1^-(1^{++})$	—	-0.0437	-0.0055
$f_1(1420)$	$0^+(1^{++})$	—	-0.0013	-0.0002
$f_2(1270)$	$0^+(2^{++})$	0.0066	-0.00006	-0.000007

Energy intervals of (μp) without taking into account one-meson exchange:

$$\Delta E^{LS} = 206.0668 - 5.2275 r_E^2 \ meV, \ \delta E_{LS}^{exp} \approx 1 \ \mu eV.$$

$$\Delta E^{HFS}(1S) = 182.6380 \ meV, \ FAMU \ accuracy \ 1 \ ppm \approx 0.2 \ \mu eV$$

$$\Delta E^{HFS}(2S) = 22.8148 \ meV, \ FAMU \ accuracy \ 1 \ ppm \approx 0.02 \ \mu eV$$

Thank you for attention!