Optimized determination of the polarized Bjorken sum rule

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QFTHEP'19
Sochi, Russia, September 22–29, 2019

Abstract

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Phys. Rev. D 100, 056007 (2019)

We determine theoretically the polarized Bjorken sum rule Γ_1^{p-n} with use of the renormalization group for the optimization of the truncated perturbation series.

Presented approach is universal and can be applied to any of the DIS sum rules.

Introduction

Understanding the behavior of $\alpha_s(Q^2)$ with the scale of the virtual momenta Q^2 allows one to describe hadronic interactions at both short and long distances.

The running of the coupling $\alpha_s(\mu^2)$ is defined by the renormalization group (RG) equation:

$$\mu^2 \frac{da_s(\mu^2)}{d\mu^2} = \beta(a_s) = -a_s^2(\mu^2) \sum_{i \geq 1} \beta_{i-1} a_s^{i-1}(\mu^2)$$

where

$$a_s \equiv \frac{\alpha_s}{4\pi}$$

Introduction

The calculation of the one-loop β-function in pQCD enabled the discovery of asymptotic freedom 46 years ago:

$$a_s(\mu^2) = \frac{1}{\beta_0\,\ell} \qquad \ell = \ln\frac{\mu^2}{\Lambda^2_{qcd}}$$

β_0: Vanyashin, Terenrev (1965); Khriplovich (1970); T'Hooft (1972); Gross, Wilczek; Politzer (1973)

β_1 (two-loop): Caswell, Jones (1974); Egorian, Tarasov (1979)

β_2 (three-loop): Tarasov, Vladimirov, Zharkov (1980); Larin Vermaseren (1993)

β_3 (four-loop): van Ritbergen, Vermaseren, Larin (1997); Czakon (2005)

β_4 (five-loop): Baikov, Chetyrkin, Kühn (2016); Herzog, Ruijl, Ueda, Vermaseren, Vogt (2017)

Introduction

The Bjorken sum rule (BSR) for the polarized deep inelastic lepton-hadron scattering (DIS) provides fundamental spin predictions of the nucleon.

The Q^2 dependence of BSR is given by

$$\Gamma_1^{\rm th}(Q^2) = \Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \left|\frac{g_A}{6}\right| C_{\rm Bjp}(a_s) + \sum_{i=2}^{\infty} \frac{\mu_{2i}^{p-n}}{Q^{2i-2}} \,,$$

where $C_{\rm Bjp}(a_s)$ is the leading-twist nonsinglet coefficient function (c.f.) including radiative QCD corrections obtained within the $\overline{\rm MS}$ scheme and is known to 4 loops:

α_s: J. Kodaira, S. Matsuda, T. Muta, K. Sasaki, T. Uematsu (1979)
α_s^2 (two-loop): S. G. Gorishnii and S. A. Larin (1986)
α_s^3 (three-loop): S.A. Larin, J. A. M. Vermaseren (1991)
α_s^4 (four-loop): P. A. Baikov, K. G. Chetyrkin, and J. H. Kühn (2010)

Optimization

The perturbation expansion for the c.f. $C_{\rm Bjp}(a_s)$ reads

$$C_{\rm Bjp}\left(\frac{Q^2}{\mu^2}, a_s(\mu^2)\right) = 1 + c_1\left(a_s(\mu^2) + c_2\ a_s^2(\mu^2) + c_3\ a_s^3(\mu^2) + c_4\ a_s^4(\mu^2) + \ldots\right)$$

$$c_{i>1} = c_i(Q^2/\mu^2)$$

In order to optimize the perturbative series in α_s of a physical observable, various methods can be used:

- Brodsky-Lepage-Mackenzie (BLM) approach
(S.J. Brodsky, G.P. Lepage, and P.B. Mackenzie, 1982),
later on developed in two different approaches:
- The sequential BLM (S. V. Mikhailov 2007, A.L. Kataev and S.V. Mikhailov 2015)
- The Principle of Maximal Conformality (PMC) (A. Deur, J.-M. Shen, X.-G. Wu, S.J. Brodsky, and G.F. de Teramond, 2017)

Optimization

We present another method of optimization:

We **DO NOT** discuss the structure of the $\{\beta\}$-expansion and **DO NOT** consider the intrinsic structure of the coeff. c_i.

Instead of that, we optimize numerically the truncated QCD PT series for the $C_{Bjp}(\alpha_s)$ making the radiative corrections RC minimal:

$$C_{\rm Bjp}(1, a_s(\mu^2)) = 1 - RC$$

$$C_{Bjp}(a_s) = 1 - 4\Big[a_s + a_s^2\left(\frac{55}{3} - \frac{4}{3}n_f\right) + a_s^3\left(663.04 - 121.72\,n_f + 2.84\,n_f^2\right) + a_s^4\left(30684.6 - 7897.05\,n_f + 482.64\,n_f^2 - 6.64\,n_f^3\right)\Big]$$

We consider the transformation of the coefficients c_i of the RGI quantity $C_{\rm Bjp}(a_s)$ under the change of the normalization scale $\mu \to \mu'$.

Optimization $\mu \to \mu'$

$$\ell' \equiv \ell - \Delta \qquad \ell = \ln\left(\frac{\mu^2}{\Lambda_{qcd}^2}\right) \qquad \Delta = \ln\frac{\mu^2}{\mu'^2} \qquad a_s' \equiv a_s(\ell') = a_s\left(\ell - \Delta(a_s')\right)$$

Reexpanding the running coupling $a_s(\ell) = a_s(\Delta, a_s')$ we have

$$a_s = \exp\left[-\Delta\beta(a_s)\partial_{a_s}\right] a_s\Big|_{a_s = a_s'} = a_s' - \beta(a_s')\frac{\Delta}{1!} + \beta(a_s')\partial_{a_s'}\beta(a_s')\frac{\Delta^2}{2!} + \ \ldots$$

where

$$\Delta \equiv \ell - \ell' = \Delta(a_s') = \Delta_0 + a_s'\beta_0\ \Delta_1 + (a_s'\beta_0)^2\ \Delta_2 + \ \ldots$$

Reexpansion a_s in terms of a_s' and Δ_i leads to rearrangement of the perturbation series for c.f. $C_{\mathrm{Bjp}}(a_s)$:

$$C_{\mathrm{Bjp}}(a_s) = \sum_i a_s^i c_i \to \sum_i (a_s')^i c_i'$$

Optimization

The new coefficients c'_i can be expressed as $c'_i = B_{ij} c_j$:

$$C_{\rm Bjp}(a_s) = \sum_{i\geqslant 0} a_s^i c_i \to \sum_{i\geqslant 0} (a'_s)^i c'_i = 1 + \sum_{i,j\geqslant 1} (a'_s)^i B_{ij} c_j \,,$$

$\{B_{ij}\}$ is a triangular matrix:

$\mathbf{1}$	0	0	0
$-\beta_0\Delta_0$	$\mathbf{1}$	0	0
$-\beta_0^2\Delta_1 \; -\beta_1\Delta_0 \; +\beta_0^2\Delta_0^2$	$-2\beta_0\Delta_0$	$\mathbf{1}$	0
$-\beta_0^3\Delta_2 \; -\beta_2\Delta_0 - \beta_0\beta_1\Delta_1$ $+\frac{3}{2}\beta_0\beta_1\Delta_0 + 2\beta_0^3\Delta_0\Delta_1$	$-2\beta_0^2\Delta_1$ $-2\beta_1\Delta_0 \; +3\beta_0^2\Delta_0^2$	$-3\beta_0\Delta_0$	$\mathbf{1}$

Criteria of the PT series optimization

Our goal is to fit the parameters $\{\Delta_0, \Delta_1, \Delta_2, \ldots\} \equiv \{\boldsymbol{\Delta}\}$ numerically following some criteria of the PT series optimization.

We have to *control simultaneously* both the expansion for Δ:

$$(I) \qquad |\Delta_0| \geqslant |A'\Delta_1| \geqslant |A'^2\Delta_2|\,, \qquad A' \equiv \beta_0 a'_s$$

and for the coefficients c'_i:

$$(II) \qquad 1 \geqslant \left|A'\frac{c'_2}{\beta_0}\right| \geqslant \left|A'^2\frac{c'_3}{\beta_0^2}\right| \geqslant \left|A'^3\frac{c'_4}{\beta_0^3}\right|$$

Additionally, we fix the PT domain:

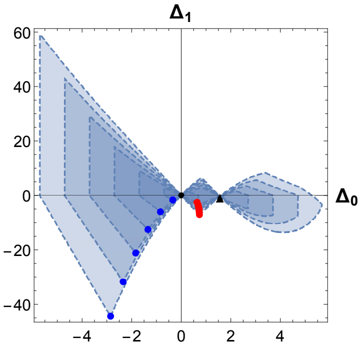
$$(III) \qquad \ell, \ell' \geqslant \ell_{\mu_0} \Rightarrow \ell - 2.3 \geqslant \Delta(\ell') = \Delta_0 + A'\Delta_1 + A'^2\Delta_2$$

where $\mu_0^2 \simeq 1$ GeV^2, $\ell_{\mu_0} = \ln\left(\mu_0^2/\Lambda_{qcd}^2\right) \simeq 2.3$ at $\Lambda_{qcd} = 0.318\,$GeV.

The admissible domains of parameters $\{\Delta\}$

We scan ℓ in the practically interesting interval $2.3 < \ell \leqslant 8$ ($1 < \mu^2 \leqslant 301$ GeV^2) and localize at each ℓ the region of the parameters $\{\Delta_0, \Delta_1, \Delta_2\}$, where the constraint conditions I-III are fulfilled simultaneously.

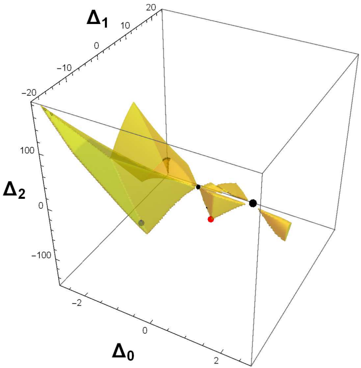
2D optimization



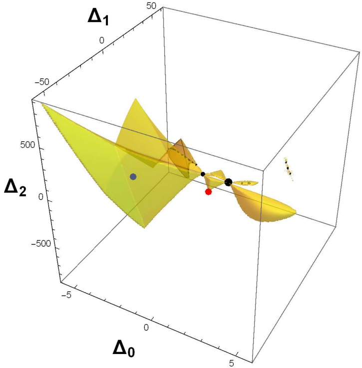
$\ell = 3$ (dark), ..., $\ell = 8$ (light)
The black triangle: $c_2' = c_3' = 0$

The minima of the radiative corrections:
Blue points: global minima
Red points: local minima at BLM inspired condition $\Delta_0 > 0$

3D optimization



$\ell = 5$



$\ell = 8$

The results of the optimization for BSR

We compare optimization results with the initial non-optimized ones

Summarizing our tests among the considered cases (2D, 3D, $\Delta_0 > 0$), only the 2D result with the global minimum ($\Delta_0 < 0$) provides satisfactory convergence of the PT series.

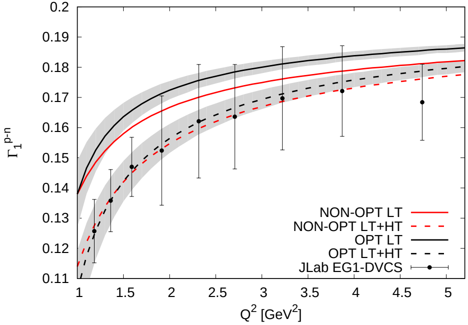
Results for the scale m_τ^2 ($3.2\ \mathrm{GeV}^2$, $\ell \approx 3.4$):

$$\begin{aligned} & \{\Delta_0 = \Delta_1 = \ldots = 0\} \\ C_{\mathrm{Bjp}}^{\mathrm{non-opt}} &= 1 - 4\,(0.0264 + 0.0090 + 0.0041 + 0.0032 + \ldots) \\ &= 1 - 4(\ \ 0.0428\ \ ) = 1 - \mathbf{0.171} \end{aligned}$$

$$\begin{aligned} & \{\Delta_0 = -0.571, \Delta_1 = -3.35, \Delta_2 = 0\} \\ C_{\mathrm{Bjp}}^{\mathrm{opt}} &= 1 - 4\,(0.0205 + 0.0074 + 0.0054 + 0.0038 + \ldots) \\ &= 1 - 4\,(\ \ 0.0371\ \ ) = 1 - \mathbf{0.149} \end{aligned}$$

$C_{\mathrm{Bjp}}^{\mathrm{opt}}$ is significantly higher than the standard one.

Comparison to the JLab EG1-DVCS data 2014



At lower Q^2 the HT power corrections are needed to describe the data.

$\mu_{4(opt)}^{p-n}/M^2 = -0.034 \pm 0.007$

$\mu_{4(JLab)}^{p-n}/M^2 = -0.021 \pm 0.016$

$\mu_{4(theor)}^{p-n}/M^2 \approx -0.05 \pm 0.02$

Comparison with COMPASS 2017 data at $Q^2 = 3\,\mathrm{GeV}^2$

Approach	$\Gamma_1^{p-n}(0)$
non-opt	0.177 ± 0.003
opt	0.182 ± 0.003
$\mathrm{TMM}_{\mathrm{EXP}}$	0.191 ± 0.010
EXP	$0.192 \pm 0.007 \pm 0.015$

The optimization reduces the differences between theoretical and experimental (EXP, TMM) estimations.

Conclusions

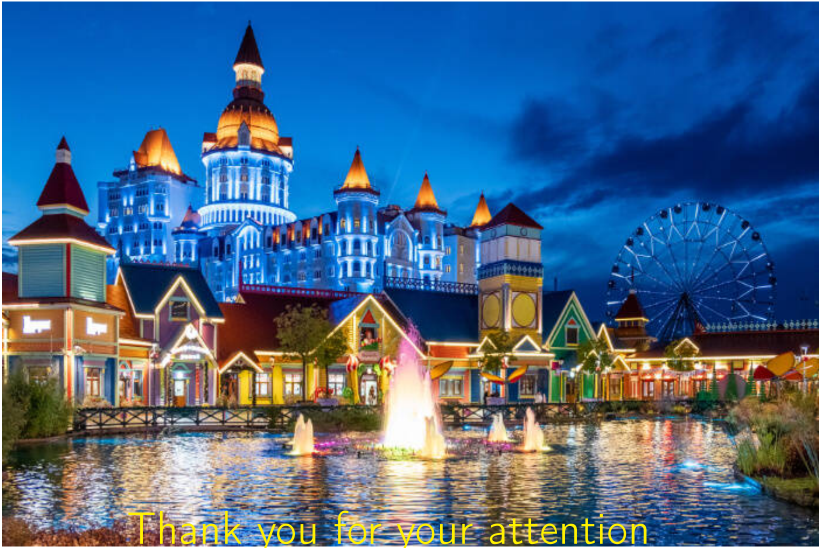
We have presented a possible improvement in theoretical determination of the polarized Bjorken sum rule Γ_1^{p-n}.

We found numerically the minimum of the radiative QCD corrections to $C_{\rm Bjp}(\alpha_s)$ based on four-loop run of $\alpha_s(\mu^2)$. This leads to the optimum values of the theoretical predictions for BSR.

This approach is universal and applicable for analysis of any renormalization group invariant quantities.

The optimized results for Γ_1^{p-n} in the order $O(\alpha_s^4)$ are systematically higher than the standard ones and the difference varies between 0.006 at $Q^2 = 2\,\mathrm{GeV}^2$ and 0.003 at $Q^2 = 10\,\mathrm{GeV}^2$.

From comparison with the EG1-DVCS precise data for $Q^2 > 1\,\mathrm{GeV}^2$ we found that the optimized approach LT+HT describes well the Q^2 evolution of $\Gamma_1^{p-n}(Q^2)$ even down to small Q values.



Thank you for your attention