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High Energy Physics and Quantum Field Theory

Quantum field-theoretical description of neutrino oscillations in T2K experiment

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Outline

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Introduction

The standard S-matrix formalism is not appropriate for describing finite space-time interval processes: $t, L \to \pm\infty$.

Neutrino oscillations: the standard QM description in terms of plane waves ➔ violation of energy-momentum conservation;
the QM and QFT descriptions in terms of wave packets ➔ complicated calculations.

- C. Giunti, C.W. Kim, J.A. Lee and U.W. Lee, «Treatment of neutrino oscillations without resort to weak eigenstates,» Phys. Rev. D **48** (1993) 4310.

We put forward a modification of the perturbative formalism, which allows one to describe the processes passing at finite distances during finite time intervals.

The approach is based on the Feynman diagram technique in the coordinate representation supplemented by modified rules of passing to the momentum representation. The latter reflect the geometry of neutrino oscillation experiments and lead to a modification of the Feynman propagators of the neutrino mass eigenstates in the momentum representation.

The idea behind the approach comes from the paper

- R.P. Feynman, «Space-Time Approach to Quantum Electrodynamics,» Phys. Rev. **76** (1949), 769.

It was developed in the papers

- V.O. Egorov and I.P. Volobuev, «Neutrino oscillation processes in a quantum field-theoretical approach,» Phys. Rev. D **97** (2018) no.9, 093002,
- V.O. Egorov and I.P. Volobuev, «Quantum field theory description of processes passing at finite space and time intervals,»
Theor. Math. Phys. **199** (2019) no.1, 562,
- V.O. Egorov and I.P. Volobuev, «Coherence length of neutrino oscillations in a quantum field-theoretical approach,»
Phys. Rev. D **100** (2019) no.3, 033004.

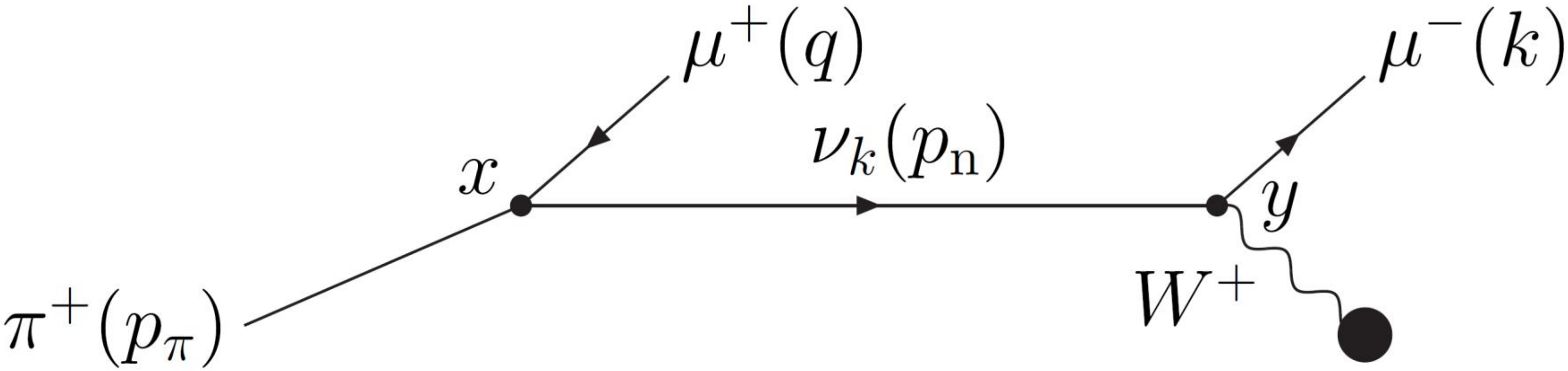
Oscillations with neutrino production in pion (kaon) decay and detection in charged-current weak interaction with nucleus

We work in the framework of the minimal extension of the SM by the right neutrino singlets. The charged-current interaction Lagrangian of the leptons:

$$L_{\mathrm{cc}} = -\frac{g}{2\sqrt{2}}\left(\sum_{i,k=1}^{3} \bar{l}_i \gamma^{\mu}\left(1-\gamma^{5}\right) U_{ik}\, \nu_k\, W_{\mu}^{-} + \mathrm{H.c.}\right).$$

The field of the charged lepton of the i-th generation	The neutrino mixing matrix (PMNS matrix)	The field of the neutrino mass eigenstate

The process is described in the lowest order by the diagram:



The points of production x and detection y are supposed to be separated by a fixed macroscopic interval. The diagram must be summed over all three neutrino mass eigenstates $k = 1, 2, 3$.

The amplitude in the coordinate representation → in accordance with the usual Feynman rules. If we integrate with respect to x and y, we lose the information about the space-time interval between them. In order to save it we introduce the delta function $\delta\left(y^0 - x^0 - T\right)$ into the integral. It is equivalent to the replacement:

$$S_k^{\mathrm{c}}\left(y - x\right) \rightarrow S_k^{\mathrm{c}}\left(y - x\right)\delta\left(y^0 - x^0 - T\right).$$

We arrive at the time-dependent propagator of k-th neutrino mass eigenstate in the momentum representation:

$$S_k^{\mathrm{c}}\left(p, T\right) \equiv \int \mathrm{d}^4 z \; e^{ipz} \, S_k^{\mathrm{c}}\left(z\right) \, \delta\left(z^0 - T\right).$$

The integral can be evaluated exactly:

$$S_k^{\mathrm{c}}\left(p,T\right)=i\,\frac{\hat{p}-\gamma_0\left(p^0-\sqrt{\left(p^0\right)^2+m_k^2-p^2}\right)+m_k}{2\sqrt{\left(p^0\right)^2+m_k^2-p^2-i\varepsilon}}\,e^{i\left(p^0-\sqrt{\left(p^0\right)^2+m_k^2-p^2}\right)T}.$$

This propagator makes sense only for macroscopic time interval T.

Particles which propagate over large macro-scopic space-time intervals are close to mass shell (GS theorem), i.e. $\left|p^2-m_k^2\right|/\left(p^0\right)^2<<1$. The time-dependent propagator takes the simple form

$$S_k^{\mathrm{c}}\left(p,T\right)=i\,\frac{\hat{p}+m_k}{2p^0}\,e^{-i\frac{m_k^2-p^2}{2p^0}T}.$$

The amplitude of the process in the momentum representation when $y^0 - x^0 = T$, neglecting neutrino masses everywhere except for the exp:

$$M = -i\,\frac{G_{\mathrm{F}}^2}{2p_{\mathrm{n}}^0}\cos\theta_{\mathrm{C}}\; f_\pi\,\varphi_\pi\, m_{(\mu)}\left(\sum_{k=1}^{3}\left|U_{2k}\right|^2 e^{-i\frac{m_k^2 - p_{\mathrm{n}}^2}{2p_{\mathrm{n}}^0}T}\right) J_\rho\left(P,P'\right)\overline{u}(k)\,\gamma^\rho\,\hat{p}_{\mathrm{n}}\left(1+\gamma^5\right)\upsilon(q)\,.$$

It contains three terms which interfere with each other leading to oscillations.

The squared modulus of the amplitude factorizes as follows:

$$\left\langle\left|M\right|^2\right\rangle = \underbrace{\left\langle\left|M_1\right|^2\right\rangle}_{\text{Production process}}\underbrace{\left\langle\left|M_2\right|^2\right\rangle}_{\text{Detection process}}\frac{1}{4\left(p_{\mathrm{n}}^0\right)^2}\left[1-4\sum_{\substack{i,k=1\\ i<k}}^{3}\left|U_{2i}\right|^2\left|U_{2k}\right|^2\sin^2\left(\frac{\Delta m_{ik}^2}{4p_{\mathrm{n}}^0}T\right)\right].$$

Let us find the probability of the process. The ⟨|amplitude|^2⟩ is multiplied by the delta function of energy-momentum conservation $(2\pi)^4\delta(p_\pi + P - q - k - P')$ and integrated with respect to the phase volume of the final particles.

The integration would result in variation of the virtual neutrino momentum direction, which contradicts the experimental setting since the neutrino propagates from the production point to the detection point. Thus, one must calculate the differential probability of the process where p_n is fixed.

It can be achieved by additional multiplying the ⟨|amplitude|^2⟩ by $2\pi\delta(p_\mathrm{n} - p)$, or, equivalently, substituting p instead of p_n there and multiplying the result by $2\pi\delta(p_\pi - q - p)$.

We find:

$$\frac{d^3W^{(\pi,\mu)}}{d^3p} = \frac{d^3W_1^{(\pi)}}{d^3p} W_2^{(\mu)} \underbrace{\left[1-4\sum_{\substack{i,k=1\\ i<k}}^{3}|U_{2i}|^2|U_{2k}|^2 \sin^2\left(\frac{\Delta m_{ik}^2}{4|\vec{p}|}L\right)\right]}_{P_{\mu\mu}(|\vec{p}|,L)\text{ – the standard oscillating factor}}.$$

Neutrino is almost on-shell $\rightarrow T = Lp^0/|\vec{p}|$

The additional delta function fixes not only the direction of neutrino momentum, but also its length, thus we must integrate the result above with respect to $|\vec{p}|$. The final probability of detecting an electron in the considered process:

$$\frac{dW^{(\pi,\mu)}}{d\Omega} = \int \underbrace{\frac{d^3W^{(\pi,\mu)}}{d^3p}}_{\text{Singular}} |\vec{p}|^2 d|\vec{p}| = \frac{dW_1^{(\pi)}}{d\Omega} W_2^{(\mu)}\Big|_{|\vec{p}|=|\vec{p}|^*} P_{\mu\mu}\left(|\vec{p}|^*, L\right).$$

Is determined by energy-momentum conservation in the production vertex

The differential probability of neutrino production in the pion decay:

$$\frac{\mathrm{d}W_1^{(\pi)}}{\mathrm{d}\Omega}=\frac{G_\mathrm{F}^2\cos^2\theta_\mathrm{C}\,f_\pi^2}{8(2\pi)^2}\,\frac{m_{(\mu)}^2\left(m_\pi^2-m_{(\mu)}^2\right)^2}{p_\pi^0\left(p_\pi^0-|\vec{p}_\pi|\cos\theta\right)^2}.$$

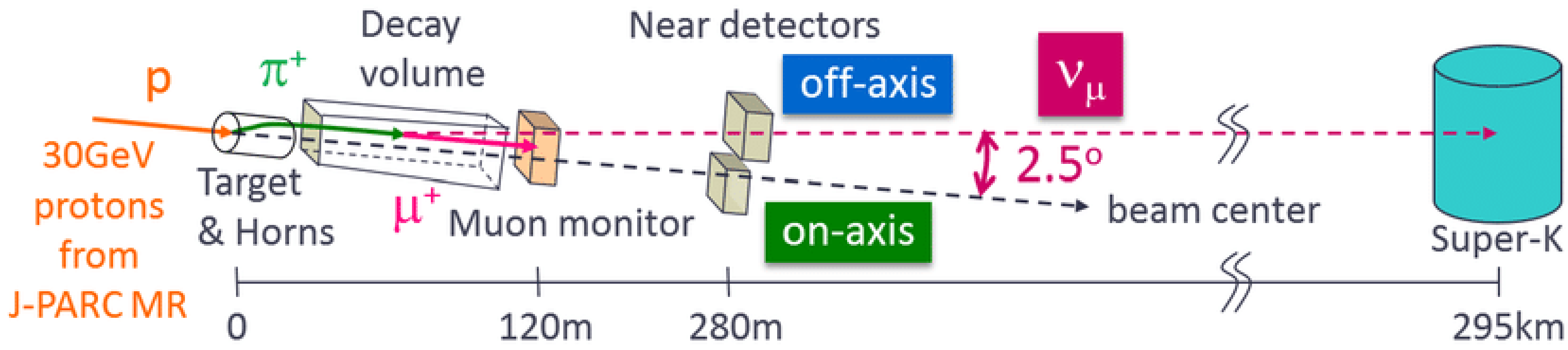
The differential probability of neutrino production in the kaon decay $K^+\to\nu_i+\mu^+$ is given by the same formula with the replacements $\cos\theta_\mathrm{C}\to\sin\theta_\mathrm{C},\quad f_\pi\to f_K,\quad m_\pi\to m_K$:

$$\frac{\mathrm{d}W_1^{(K)}}{\mathrm{d}\Omega}=\frac{G_\mathrm{F}^2\sin^2\theta_\mathrm{C}\,f_K^2}{8(2\pi)^2}\,\frac{m_{(\mu)}^2\left(m_K^2-m_{(\mu)}^2\right)^2}{p_K^0\left(p_K^0-|\vec{p}_K|\cos\theta\right)^2}.$$

The whole probability of finding an electron in the detection process differs from the obtained one by the replacements $W_2^{(\mu)}\to W_2^{(e)},\quad P_{\mu\mu}\to P_{\mu e}$, i.e.:

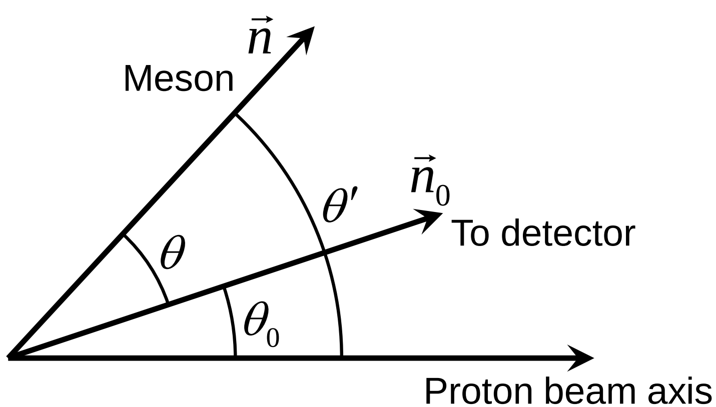
$$\frac{\mathrm{d}W^{(\pi,e)}}{\mathrm{d}\Omega}=\int\frac{\mathrm{d}^3W^{(\pi,e)}}{\mathrm{d}^3p}|\vec{p}|^2\,\mathrm{d}|\vec{p}|=\frac{\mathrm{d}W_1^{(\pi)}}{\mathrm{d}\Omega}\,W_2^{(e)}\Big|_{|\vec{p}|=|\vec{p}|^*}\underbrace{P_{\mu e}\left(|\vec{p}|^*,L\right)}_{\sum\limits_{\substack{i,k=1\\ i<k}}^{3}\left[-4\,\mathrm{Re}\left(U_{1i}U_{1k}^*U_{2i}^*U_{2k}\right)\sin^2\left(\Delta m_{ik}^2L/4|\vec{p}|^*\right)+2\,\mathrm{Im}\left(U_{1i}U_{1k}^*U_{2i}^*U_{2k}\right)\sin\left(\Delta m_{ik}^2L/2|\vec{p}|^*\right)\right]}.$$

Scheme of the T2K experiment:



Main detection channels: $\nu_i + n \to e^- + p, \quad \nu_i + n \to \mu^- + p.$

We should average the probability of the process with detection of the lepton l (muon or electron) with respect to the momenta of the initial mesons.



$$\vec{n}_0 = (\sin\theta_0, 0, \cos\theta_0),$$

$$\vec{n} = (\sin\theta'\cos\varphi', \sin\theta'\sin\varphi', \cos\theta'),$$

$$(\vec{n}_0\vec{n}) = |\vec{n}_0||\vec{n}|\cos\theta = \cos\theta =$$

$$= \sin\theta_0 \sin\theta'\cos\varphi' + \cos\theta_0\cos\theta'.$$

For pion:

Pion momentum distribution

$$\overline{\frac{\mathrm{d}W^{(\pi,l)}}{\mathrm{d}\Omega}} = \int \mathrm{d}\varphi' \sin\theta' \,\mathrm{d}\theta' |\vec{p}_\pi|^2 \mathrm{d}|\vec{p}_\pi| \rho_\pi(|\vec{p}_\pi|,\theta') \frac{\mathrm{d}W_1^{(\pi)}}{\mathrm{d}\Omega}(|\vec{p}_\pi|,\theta(\theta',\varphi')) P_{\mu l}(|\vec{p}|^*, L) W_2^{(l)}(|\vec{p}|^*) =$$

$$= \int d|\vec{p}|^* \rho_{(\nu)}^{\pi}(|\vec{p}|^*) P_{\mu l}(|\vec{p}|^*, L) W_2^{(l)}(|\vec{p}|^*),$$

$$|\vec{p}|^* = \frac{m_\pi^2 - m_{(\mu)}^2}{2(p_\pi^0 - |\vec{p}_\pi|\cos\theta)}$$

where the pion-produced neutrino momentum distribution

$$\rho_{(\nu)}^{\pi}(|\vec{p}|^*) = \int \mathrm{d}\varphi' \sin\theta' \,\mathrm{d}\theta' |\vec{p}_\pi|^2 \frac{D(\varphi',\theta',|\vec{p}_\pi|)}{D(\varphi',\theta',|\vec{p}|^*)} \rho_\pi(|\vec{p}_\pi|,\theta') \frac{\mathrm{d}W_1^{(\pi)}}{\mathrm{d}\Omega}(|\vec{p}_\pi|,\theta(\theta',\varphi')).$$

The probability from both the π- and K-meson decays:

$$\overline{\frac{\mathrm{d}W^{(l)}}{\mathrm{d}\Omega}} = \left(a_\pi \overline{\frac{\mathrm{d}W^{(\pi,l)}}{\mathrm{d}\Omega}} + a_K \overline{\frac{\mathrm{d}W^{(K,l)}}{\mathrm{d}\Omega}} \right) = \int \rho_{(\nu)}(|\vec{p}|) W_2^{(l)}(|\vec{p}|) P_{\mu l}(|\vec{p}|, L) \mathrm{d}|\vec{p}|.$$

where $\frac{a_\pi}{a_K} \approx 10.$

Total neutrino momentum distribution:

$$\rho_{(\nu)} \equiv a_\pi \rho_{(\nu)}^{\pi} + a_{\mathrm{K}} \rho_{(\nu)}^{K}$$

Redesignate for short:

$$|\vec{p}|^* \to |\vec{p}|$$

Based on the work K. Abe *et al.*, Nuclear Instruments and Methods in Physics Research Section A **659** №1 (2011), we fit the neutrino momentum distribution for $\theta_0 = 2.5°$ by:

$$\rho_{(\nu)}\left(|\vec{p}|\right)=\begin{cases} a|\vec{p}|^m\left(\varepsilon+|\vec{p}|^n\right)e^{-\left(|\vec{p}|-b\right)^2/c}, & |\vec{p}|<0.65\ \text{GeV},\\ y_0+Ae^{-|\vec{p}|/z}, & |\vec{p}|>0.65\ \text{GeV}. \end{cases}$$

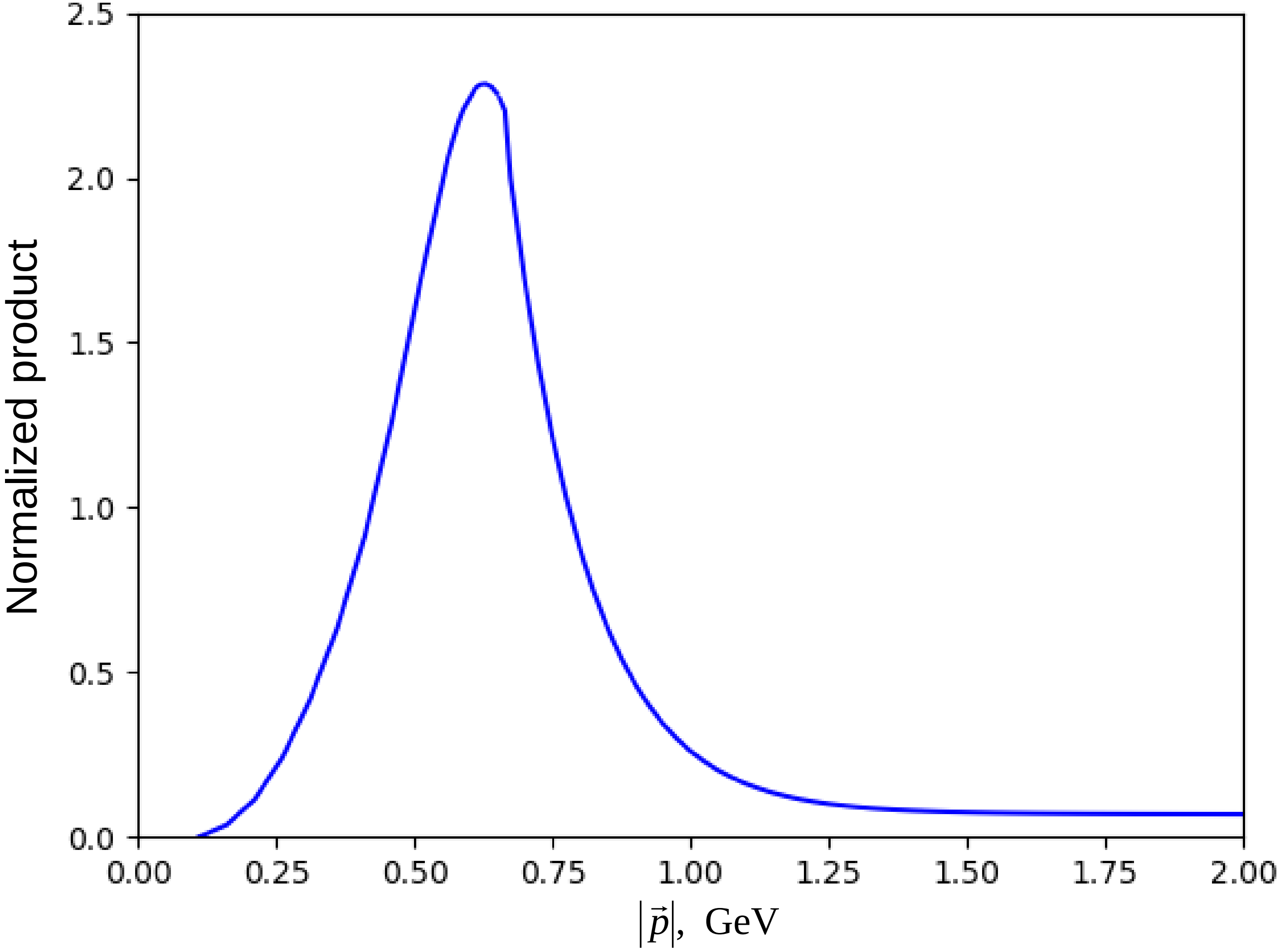
$a=87,\ m=0.16,\ \varepsilon=0.006,$
$n=6.5,\ b=0.38,\ c=0.048,$
$y_0=0.03,\ A=151,\ z=0.14.$

The probability of neutrino quasielastic scattering with production of the lepton l has the form:

$$W_2^{(l)}=\frac{M^2G_F^2\cos^2\theta_C}{8\pi|\vec{p}|^2}\int\limits_{Q^2_{\min}}^{Q^2_{\max}}\left(A_{(l)}\left(Q^2\right)-\frac{D}{M^2}B_{(l)}\left(Q^2\right)+\frac{D^2}{M^4}C_{(l)}\left(Q^2\right)\right)\mathrm{d}Q^2.$$

where $A_{(l)}$, $B_{(l)}$, $C_{(l)}$ are extremely bulky and can be found along with M and D in the book T. J. Leitner, "Neutrino Interactions with Nucleons and Nuclei," Justus-Liebig University, Germany (2005).

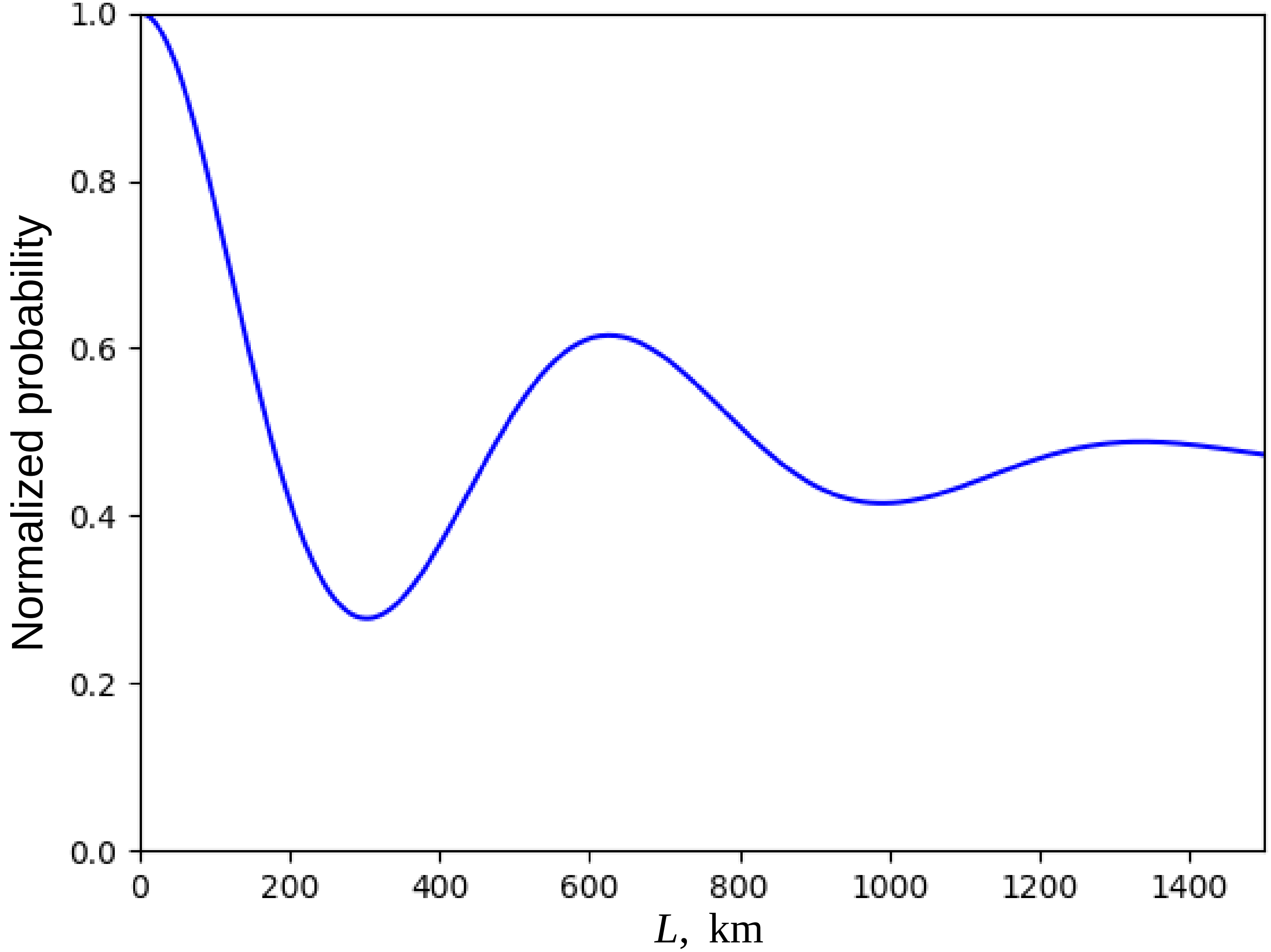
Let us consider the muon detection first. The product of the neutrino momentum distribution and the detection probability looks like:

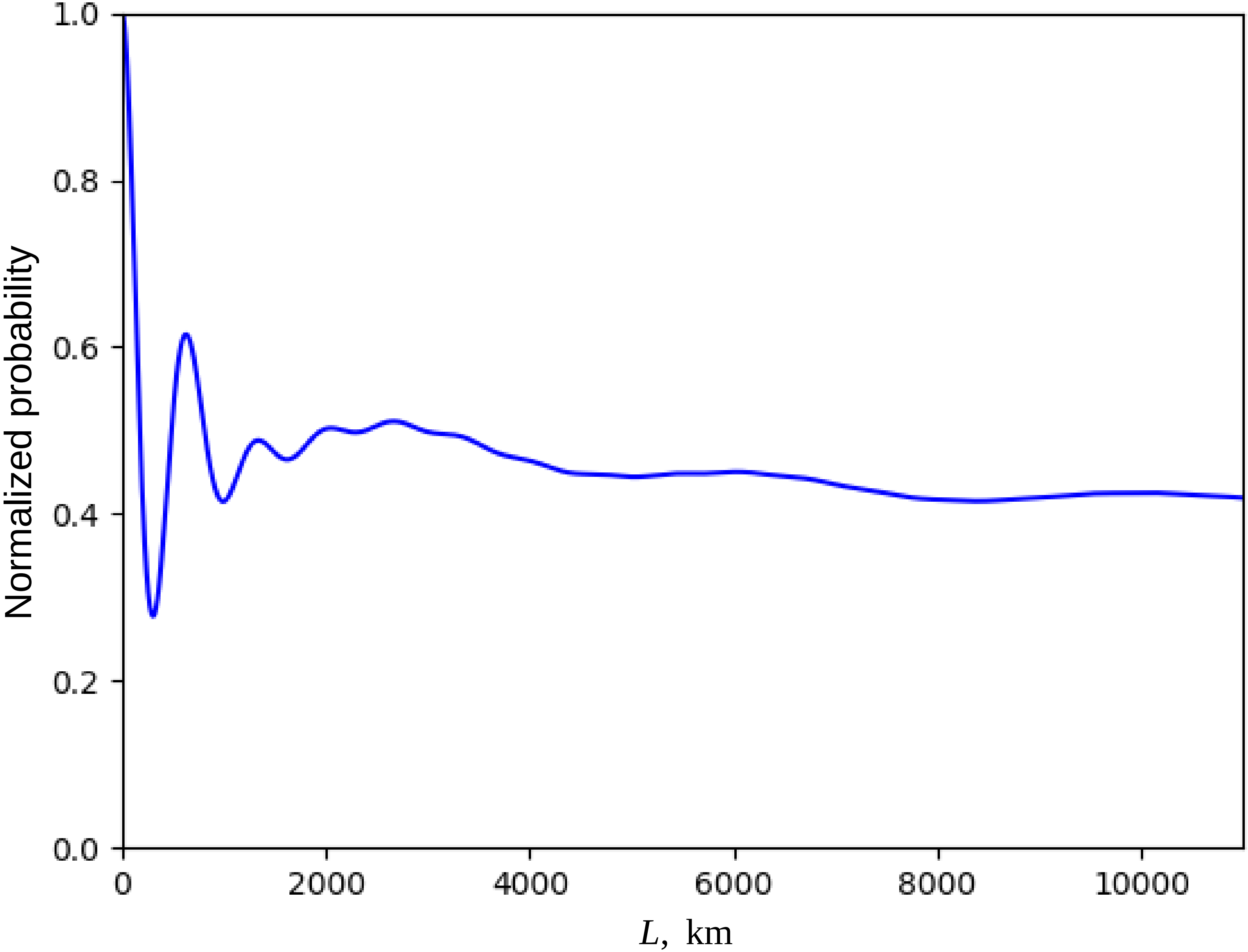


The results of numerical integration with the parameters

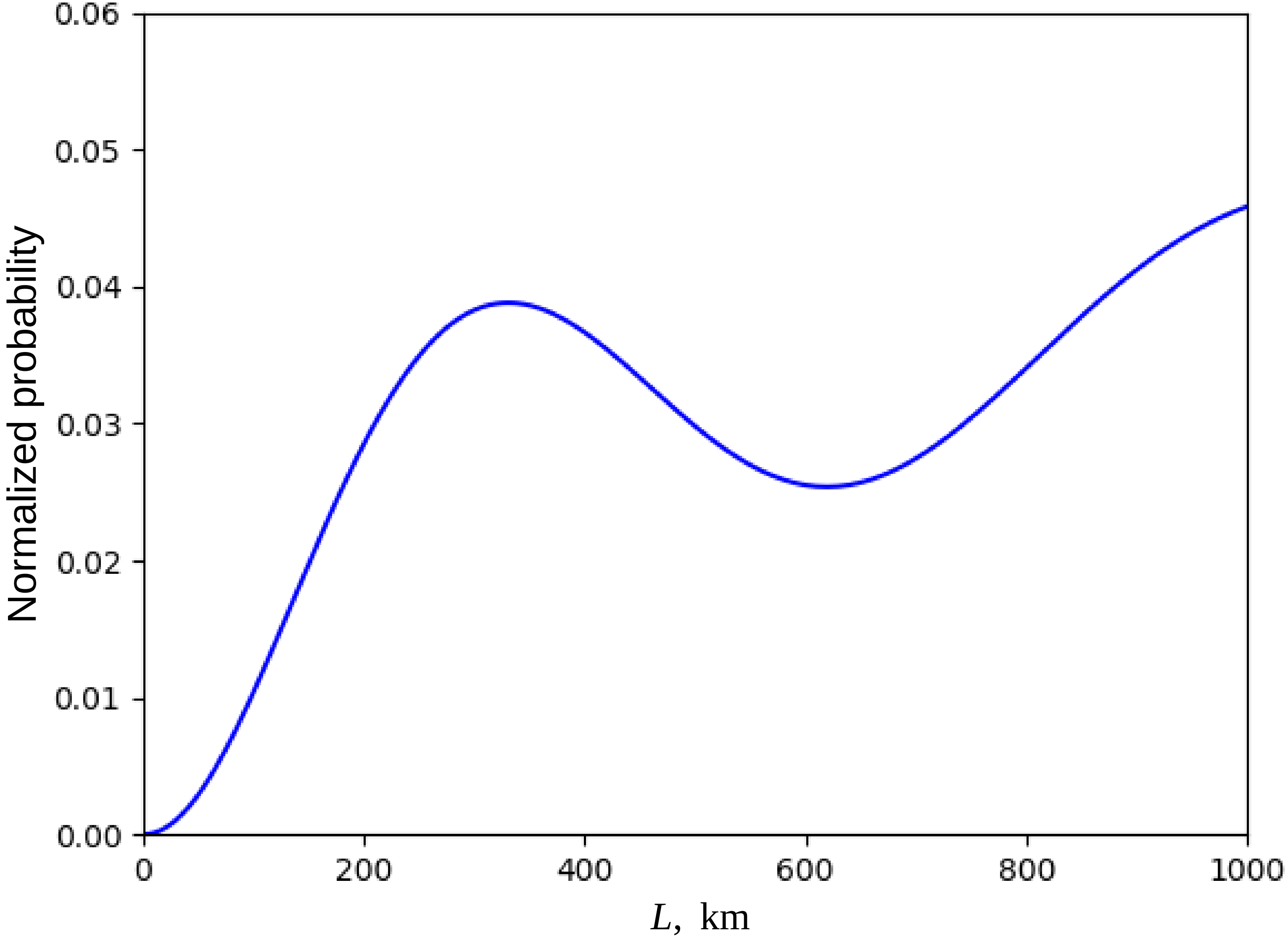
$$\Delta m_{21}^2 = 7.39\cdot 10^{-5}\ \mathrm{eV}^2,\quad \Delta m_{32}^2 = 2.53\cdot 10^{-3}\ \mathrm{eV}^2,$$

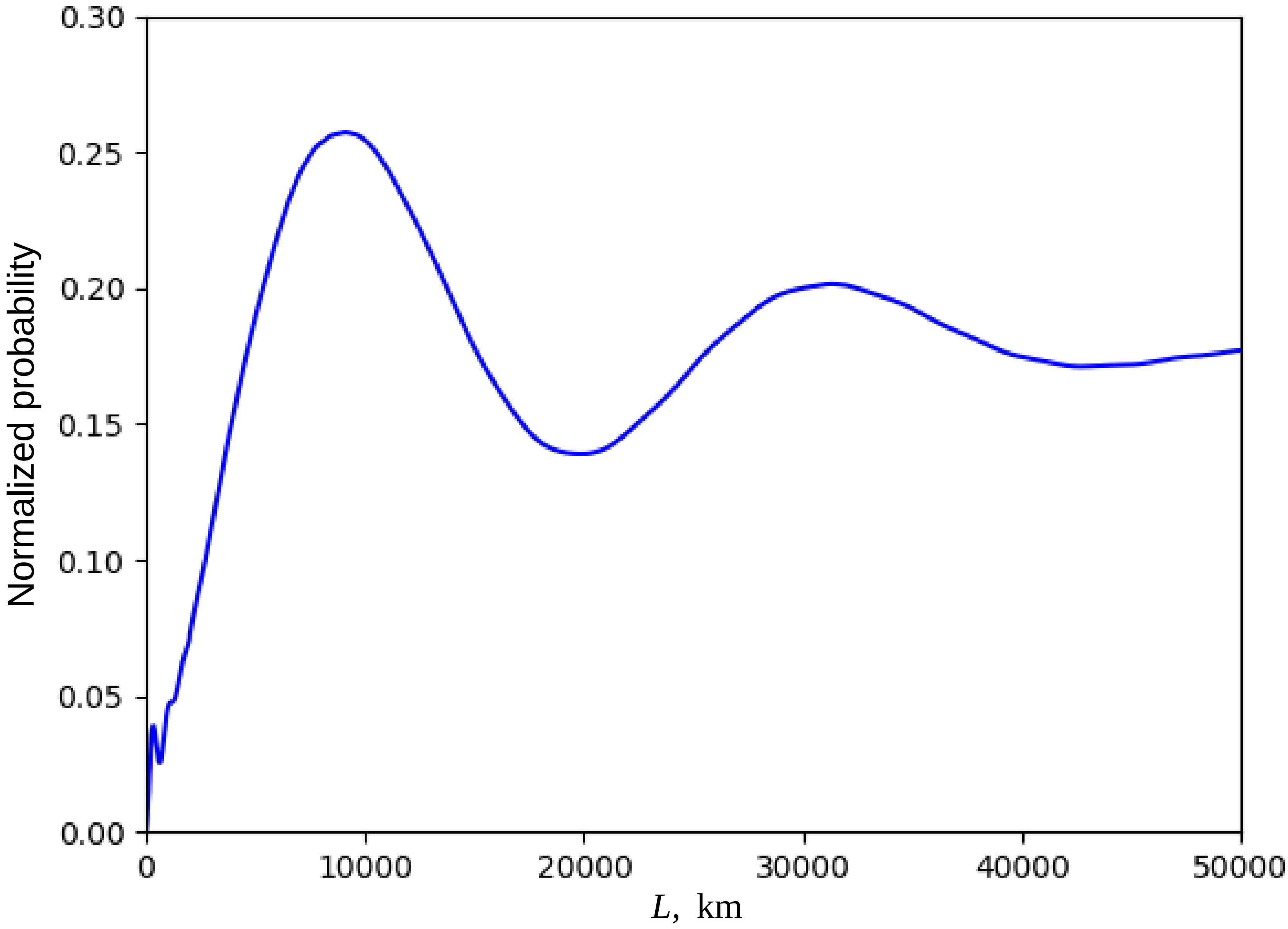
$$\theta_{12} = 0.5903,\quad \theta_{23} = 0.8657,\quad \theta_{13} = 0.1503,\quad \delta_{\mathrm{CP}} = 3.753.$$




1.0
0.8
0.6
0.4
0.2
0.0
Normalized probability
0
2000
4000
6000
8000
10000
L, km

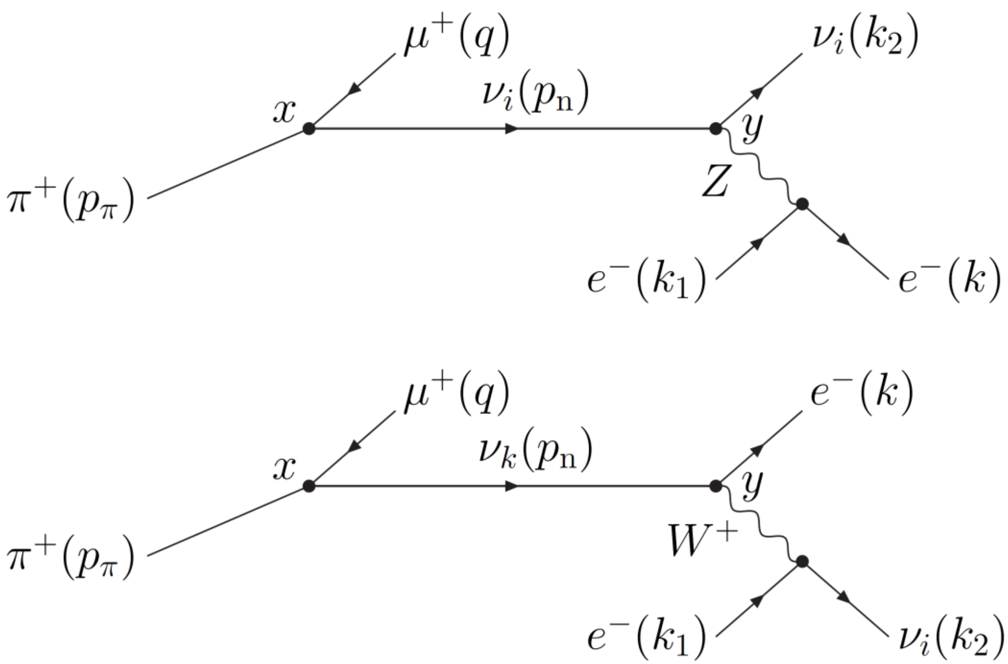
Similarly one can consider the process where an electron is produced in the neutrino scattering by a neutron:




Normalized probability
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0
10000
20000
30000
40000
50000
L, km

Neutrino detection in charged- and neutral-current weak interactions with electron

Let us consider the process which gives a contribution in the T2K experiment, even though it is small:



$\leftarrow \sum_k$ (Amplitude)

$\sum_i$ (Probability)

The amplitudes in the momentum representation, corresponding to the diagrams, when $y^0 - x^0 = T$:

$$M_{\rm nc}^{(i)} = -\frac{G_{\rm F}^2}{2p_{\rm n}^0}\cos\theta_{\rm C}\, f_\pi\, \varphi_\pi\, m_{(\mu)}\, U_{2i}^*\, e^{-i\frac{m_i^2-p_{\rm n}^2}{2p_{\rm n}^0}T}\, \overline{\nu_i}\,(k_2)\,\gamma^\mu\,\hat{p}_{\rm n}\left(1+\gamma^5\right)v\,(q)\times$$
$$\times\left[\left(-\frac{1}{2}+\sin^2\theta_{\rm W}\right)\overline{u}\,(k)\,\gamma_\mu\left(1-\gamma^5\right)u\,(k_1)+\sin^2\theta_{\rm W}\,\overline{u}\,(k)\,\gamma_\mu\left(1+\gamma^5\right)u\,(k_1)\right],$$

$$M_{\rm cc}^{(i)} = \frac{G_{\rm F}^2}{2p_{\rm n}^0}\cos\theta_{\rm C}\, f_\pi\, \varphi_\pi\, m_{(\mu)}\, U_{1i}^*\left(\sum_{k=1}^{3}U_{1k}U_{2k}^*\, e^{-i\frac{m_k^2-p_{\rm n}^2}{2p_{\rm n}^0}T}\right)\times$$
$$\times\,\overline{\nu_i}\,(k_2)\,\gamma_\mu\left(1-\gamma^5\right)u\,(k_1)\,\overline{u}\,(k)\,\gamma^\mu\,\hat{p}_{\rm n}\left(1+\gamma^5\right)v\,(q)\,.$$

The final probability:

$$\frac{dW^{(\pi\text{ or }K)}}{d\Omega} = \int\frac{d^3W^{(\pi\text{ or }K)}}{d^3p}\left|\vec{p}\right|^2 d\left|\vec{p}\right| = \frac{dW_1^{(\pi\text{ or }K)}}{d\Omega}\,W_2\Big|_{|\vec{p}|=|\vec{p}|^*}\,.$$

The same differential probability of neutrino production

The probability of detection contains the oscillating factor

The detection probability reads

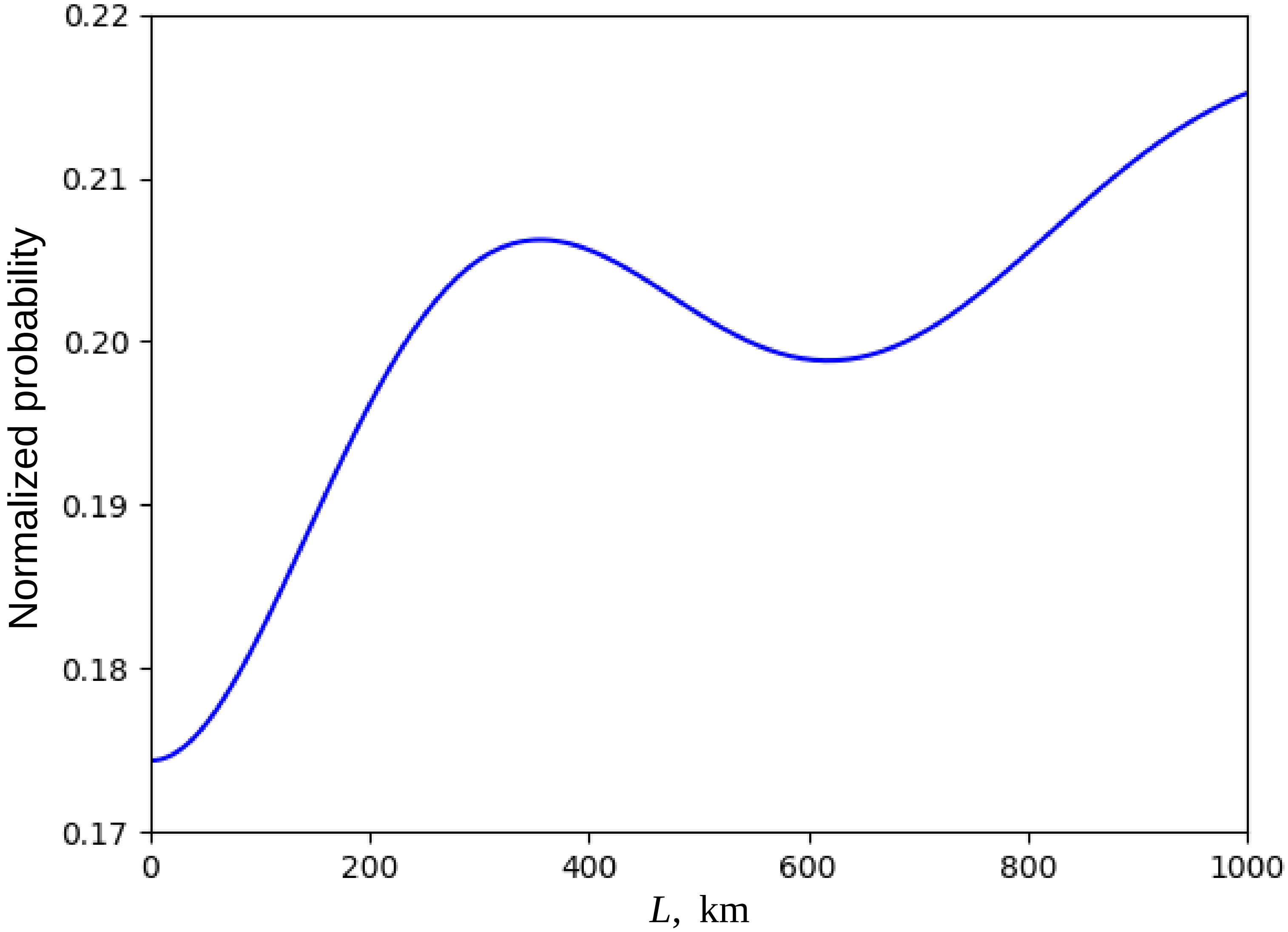
$$W_2\left(|\vec{p}|\right) = \frac{G_{\mathrm{F}}^2 m_e}{2\pi}\frac{2(|\vec{p}|)^2}{2\,|\vec{p}| + m_e}\left[1 + 4\sin^4\theta_{\mathrm{W}}\left(1 + \frac{1}{3}\left(\frac{2\,|\vec{p}|}{2\,|\vec{p}| + m_e}\right)^2\right) + \right.$$
$$\left. + 2\sin^2\theta_{\mathrm{W}}\left(1 + \frac{2\,|\vec{p}|}{2\,|\vec{p}| + m_e}\right)\left(2P_{\mu e}\left(|\vec{p}|\,, L\right) - 1\right)\right]$$

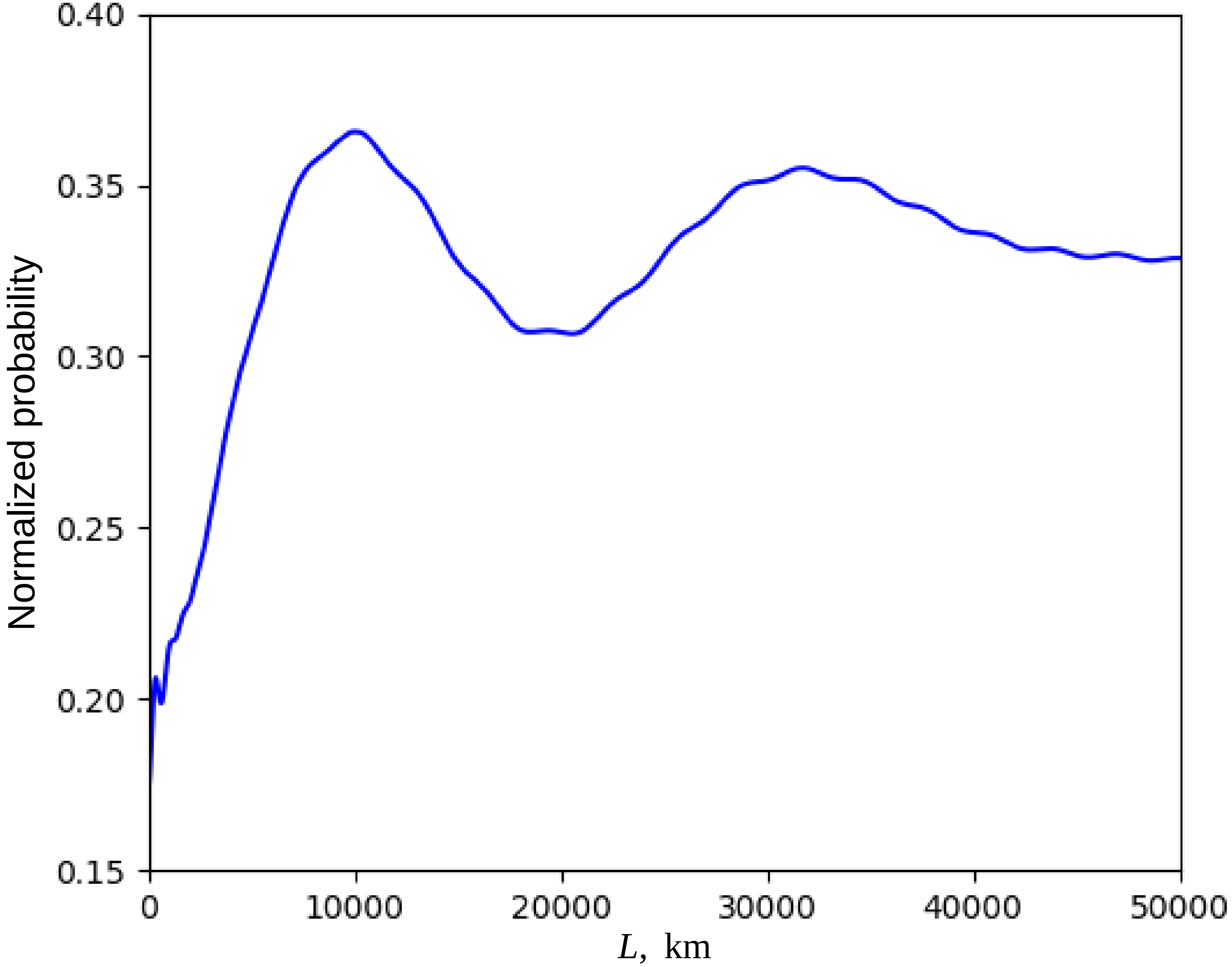
and can be written in the form

$$W_2 = P_{\mu e}\left(|\vec{p}|\,, L\right)\sigma_{\nu_e e} + \left(1 - P_{\mu e}\left(|\vec{p}|\,, L\right)\right)\sigma_{\nu_\mu e}$$

which is in accordance with the prediction of the standard approach.

The results of numerical integration with the same parameters:




Normalized probability
0.40
0.35
0.30
0.25
0.20
0.15
0
10000
20000
30000
40000
50000
L, km

Conclusion

- A novel QFT approach to the description of processes passing at finite space-time intervals is discussed. It is based on the Feynman diagram technique in the coordinate representation supplemented by the modified rules of passing to the momentum representation, which reflect the experimental situation at hand.
- It is explicitly shown that the approach allows to consistently describe neutrino oscillation processes. The main results of the standard approach are reproduced.

- The neutrino flavor states are redundant, we deal only with neutrino mass eigenstates.
- The T2K experimental setting is considered and the oscillation fading out due to momentum distribution of the initial particles is taken into account. The obtained results confirm that the far detector position corresponds to the first maximum for e^- production and the first minimum for μ^- production.
- The advantages of the approach are technical simplicity and physical transparency. Wave packets are not employed, we use only the description in terms of plane waves.

Thank you for your attention!