Symanzik approach in modeling of bound states of Dirac particle in singular background

Yury M.Pismak

Department of Theoretical Physics,
State University Saint-Petersburg

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Introduction

The proposed by Symanzik approach for modeling of interaction of a macroscopic material body with quantum fields is considered. Its application in quantum electrodynamics enables one to establish the most general form of the action functional describing the interaction of 2-dimensional material surface with photon and fermion fields. The models making it possible to calculate the Casimir energy and Casimir-Polder potential for non-ideal conducting material are presented. Applications of the models to descriptions of of interaction of the spinor field with a material plane are considered.

References

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Formulation of model

The proposed by Symanzik action functional describing the interaction of the quantum field with material body has the form:

$$S(\varphi) = S_V(\varphi) + S_{def}(\varphi)$$

where

$$S_V(\varphi) = \int L(\varphi(x))d^D x, \;\; S_{def}(\varphi) = \int_\Gamma L_{def}(\varphi(x))d^{D'} x,$$

and Γ is a subspace of dimension $D' \leq D$ in D-dimensional space.

Formulation of model

From the basic principles of QED (gauge invariance, locality, renormalizability) it follows that for thin film without charges and currents, which shape is defined by equation $\Phi(x) = 0$, $x = (x_0, x_1, x_2, x_3)$, the action describing its interaction with photon field $A_\mu(x)$ reads

$$S_{def}(\varphi) = S_\Phi(A) + S_\Phi(\bar{\psi}, \psi).$$

The action $S_\Phi(A)$ is a surface Chern-Simon action

$$S_\Phi(A) = \frac{a}{2} \int \varepsilon^{\lambda\mu\nu\rho} \partial_\lambda \Phi(x) A_\mu(x) F_{\nu\rho}(x) \delta(\Phi(x)) dx$$

where $F_{\nu\rho}(x) = \partial_\nu A_\rho - \partial_\rho A_\nu$, $\varepsilon^{\lambda\mu\nu\rho}$ denotes totally antisymmetric tensor ($\varepsilon^{0123} = 1$), a is a constant dimensionless parameter.

Formulation of model

The fermion defect action can be written as

$$S_\Phi(\bar\psi,\psi)=\int\bar\psi(x)[\lambda+u^\mu\gamma_\mu+\gamma_5(\tau+v^\mu\gamma_\mu)+\omega^{\mu\nu}\sigma_{\mu\nu}]\psi(x)\delta(\Phi(x))dx$$

Here, γ_μ, $\mu=0,1,2,3$, are the Dirac matrices, $\gamma_5=i\gamma_0\gamma_1\gamma_3\gamma_3$, $\sigma_{\mu\nu}=i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)/2$, and λ, τ, u_μ,v_μ, $\omega^{\mu\nu}=-\omega^{\nu\mu}$, $\mu,\nu=0,1,2,3$ are 16 dimensionless parameters.

Formulation of model

It is the most general form of gauge invariant action concentrated on the defect surface being invariant in respect to reparametrization of one and not having any parameters with negative dimensions.

Formulation of model

The full action of the model, which satisfies the requirement of locality, gauge invariance and renormalizability, has the form

$$S(\bar{\psi}, \psi, A) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\hat{\partial} - m + e\hat{A})\psi + S_{def}(A) + S_{def}(\bar{\psi}, \psi).$$

Due to the requirements of renormalizability the fields interaction is described by standard contribution $e\bar{\psi}\hat{A}\psi$ to the QED action.

Statement of problem

We will consider the material plane $x_3 = 0$ as a defect. In this case, in the Dirac part of the action

$$S(\overline{\psi}, \psi) = \int \overline{\psi}(x)(i\hat{\partial} - m + \Omega(x_3))\psi(x)dx,$$

the interaction of the spinor field with the plane is described with matrix $\Omega(x_3) = Q\delta(x_3)$. Since $\Omega(x_3)$ and $\delta(x_3)$ have the dimension of mass, the matrix Q is dimensionless. For homogeneous isotropic material plane in more general case, the matrix Q could be presented in the form:

$$Q = r_1 I + ir_2\gamma_5 + r_3\gamma_3 + r_4\gamma_5\gamma_3 +$$
$$+r_5\gamma_0 + r_6\gamma_5\gamma_0 + ir_7\gamma_0\gamma_3 + ir_8\gamma_1\gamma_2$$

with I - identity 4x4 matrix, γ_3, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ are Dirac matrices.

Modified Dirac equations

Movement of spinor particle in the field of defect $\Omega(x_3)$ is described by the Dirac equation

$$(i\hat{\partial} - m + \Omega(x_3))\psi(x) = 0.$$

It is one of the Euler-Lagrange equations, which is obtained by variational differentiating of the action over $\overline{\psi}(x)$. Taking the derivative over $\psi(x)$ we obtain the second equation

$$(\partial_\mu \overline{\psi}(x))\gamma^\mu + \overline{\psi}(x)(m - \Omega(x_3)) = 0.$$

The condition $\bar{\psi}(x) = \psi^*(x)\gamma_0$ fulfils if $\gamma_0\Omega^+(x) = \Omega(x)\gamma_0$. It is the case for real values of parameters $r_j,\ j = 1, ..., 8$.

Modified Dirac equations

Analysis of possible singularities of spinor $\psi(x)$ in the point $x_3 = 0$ leads to the following: for $x_3 \neq 0$ the field $\psi(x)$ satisfies Dirac equation

$$(i\hat{\partial} - m)\psi(x) = 0,$$

and for $x_3 = 0$ following relations hold

$$i\gamma_3\psi_a(\bar{x}) + Q/2\psi_s(\bar{x}) = 0,$$

where

$$\psi_s(x) = \frac{1}{2}(\psi(\bar{x}, x_3) + \psi(\bar{x}, -x_3)),\ \psi_a(x) = \frac{1}{2}(\psi(\bar{x}, x_3) - \psi(\bar{x}, -x_3)).$$

Solution of modified Dirac equations

For $x_3 \neq 0$, the field $\psi(x)$ fulfills the Dirac equation

$$(i\hat{\partial} - m)\psi(x) = 0,$$

Its general solution by $x_3 > 0$ can be presented as

$$\psi(x) = \frac{1}{(2\pi)^3} \int e^{i\bar{p}\bar{x}} \psi_+(\bar{p}, x_3) d\bar{p}.$$

Here, $\psi_+(\bar{p}, x_3)$ obeys the equation

$$(i\gamma_3\partial_3 + \hat{\bar{p}} + m)\psi_+(\bar{p}, x_3) = 0.$$

which general solution is written in the form

$$\psi_+(\bar{p}, x_3) = U(\bar{p}, x_3)\chi_+(\bar{p}), \;\; U(\bar{p}, x_3) = e^{-i\gamma_3(\hat{\bar{p}}+m)x_3}.$$

with arbitrary spinor $\chi_+(\bar{p})$ depending on $\bar{p}$ only.

Solution of modified Dirac equations

Analogously, by $x_3 < 0$

$$\psi(x) = \frac{1}{(2\pi)^3}\int e^{i\bar{p}\bar{x}}\psi_-(\bar{p},x_3)d\bar{p},\ \ \psi_-(\bar{p},x_3) = e^{-i\gamma_3(\hat{\bar{p}}+m)x_3}\chi_-(\bar{p}).$$

The boundary conditions for $\psi(x)$ by $x_0 = 0$ is rewritten for the spinors $\chi_\pm(\bar{p})$ as

$$\chi_- = S\chi_+, \quad S = (i\gamma_3 + Q)^{-1}(i\gamma_3 - Q).$$

Using the notations

$$\kappa(\bar{p}) = \sqrt{\bar{p}^2 - m^2},\ \ P^\pm(\bar{p}) = \frac{1}{2}\left(1 \pm \frac{\gamma_3(\hat{\bar{p}}+m)}{\kappa(\bar{p})}\right).$$

we can present $U(\bar{p},x_3)$ as follows

$$U(\bar{p},x_3) = e^{-i\gamma_3(\hat{\bar{p}}+m)x_3} = \cos(\kappa(\bar{p})x_3) - i\sin(\kappa(\bar{p})x_3)\frac{\gamma_3(\hat{\bar{p}}+m)}{\kappa(\bar{p})} =$$

$$= e^{-i\kappa(\bar{p})x_3}P^+(\bar{p}) + e^{i\kappa(\bar{p})x_3}P^-(\bar{p}).$$

Solution of modified Dirac equations

The matrices $P^{\pm}(\bar{p})$ are the projectors:

$$P^{\pm}(\bar{p})P^{\pm}(\bar{p}) = P^{\pm}(\bar{p}),\ \ P^{+}(\bar{p}) + P^{-}(\bar{p}) = 1,$$
$$P^{+}(\bar{p})P^{-}(\bar{p}) = P^{-}(\bar{p})P^{+}(\bar{p}) = 0.$$

They have 2-dimensional eigen spaces L^{+} è L^{-}, which are the linear combinations of two eigenspinors $v_1^{\pm}, v_2^{\pm}$:

$$L^{\pm}(a_1^{\pm}, a_2^{\pm}) = a_1^{\pm}v_1^{\pm} + a_2^{\pm}v_2^{\pm},$$
$$P^{\pm}(\bar{p})L^{\pm}(a_1^{\pm}, a_2^{\pm}) = L^{\pm}(a_1^{\pm}, a_2^{\pm}),\ \ P^{\pm}(\bar{p})L^{\mp}(a_1^{\mp}, a_2^{\mp}) = 0.$$

Solution of modified Dirac equations

Here, $a_i^{\pm}$, $i = 1, 2$ - are complex parameters, and $v_i^{\pm}$, $i = 1, 2$ are the spinors, which we choose in the form

$$v_1^+ = \left(-h, -hg^-, 1, g^-\right), \ v_2^+ = \left(h, hg^+, 1, g^+\right),$$
$$v_1^- = \left(-h, hg^+, 1, -g^+\right), \ v_2^- = \left(h, -hg^-, 1, -g^-\right),$$
$$h = \frac{\sqrt{p_0^2 - m^2}}{m + p_0}, \ g^{\pm} = \frac{\kappa(\bar{p}) \pm \sqrt{p_0^2 - m^2}}{p_1 - ip_2}.$$

They are the eigen spinors for the helicity operator:

$$\sigma(\vec{p}) = \frac{i}{2|\vec{p}|}(\vec{p}\,\vec{s}), \ \vec{s} = (\gamma_2\gamma_3, -\gamma_1\gamma_3, \gamma_1\gamma_2)$$

with

$$\sigma(\vec{p})\Big|_{p_3 = \mp\kappa(\bar{p})} v_1^{\pm} = -\frac{1}{2} v_1^{\pm}, \ \sigma(\vec{p})\Big|_{p_3 = \mp\kappa(\bar{p})} v_2^{\pm} = \frac{1}{2} v_2^{\pm}.$$

Solution of modified Dirac equations

Presenting the spinors $\chi_\pm(\overline{p})$ in the form

$$\chi_\pm(\overline{p}) = b^1_\pm(\overline{p})v_1^-(\overline{p}) + b^2_\pm(\overline{p})v_2^-(\overline{p}) + c^1_\pm(\overline{p})v_1^+(\overline{p}) + c^2_\pm(\overline{p})v_2^+(\overline{p}).$$

we obtain by $\bar{p}^2 > m^2$

$$I_3^\pm = \chi^*_\pm(\overline{p})\gamma_0\gamma_3\chi_\pm(\overline{p}) = \overline{\chi}_\pm(\overline{p})\gamma_3\chi_\pm(\overline{p}) = \frac{\kappa(\overline{p})}{p_0}(|c^1_\pm|^2 + |c^2_\pm|^2 - |b^1_\pm|^2 - |b^2_\pm|^2).$$

For $p_0 > 0, \kappa(\overline{p}) > 0$, the spinors $\chi_\pm(\overline{p})$ describe the scattering of Dirac particles on the plane $x_3 = 0$. These processes are studied in D.Yu. Pismak, Yu. M. Pismak, Theor. Math. Phys. **184**, 3, 1329-1341 (2015).

Bound states

The localized in the area $x_3 = 0$ states arise if $\bar{p}^2 - m^2 < 0$ and $\kappa(\bar{p}) = i|\kappa(\bar{p})|$ is imaginary. In this case the following conditions must be fulfilled

$$P^+(\bar{p})\chi_+(\bar{p}) = 0, \;\; P^-(\bar{p})\chi_-(\bar{p}) = 0.$$

Hence, we can present $\chi_+(\bar{p})$ in the form

$$\chi_+(\bar{p}) = a(v_1^-(\bar{p}) + \alpha\, v_2^-(\bar{p}))$$

with coefficients a, α. It follows from the boundary conditions that

$$P^-(\bar{p})S\chi_+(\bar{p}) = 0.$$

This homogeneous equation for a, α has nontrivial solution if a solvability condition (dispersion relation) is fulfilled. In this case α is found and a remains to be an arbitrary constant.

Bound states

The components of the electrical current $\vec{I} = (I_1, I_2, I_3)$ have the form

$$I_3 = \bar{\psi}\gamma_3\psi = 0,$$

$$I_1 = \bar{\psi}\gamma_1\psi \sim p_1 - p_2\frac{(1-|\alpha|^2)\sqrt{p_1^2+p_2^2+m^2-p_0^2}}{(1+|\alpha|^2)\sqrt{p_0^2-m^2}},$$

$$I_2 = \bar{\psi}\gamma_2\psi \sim p_2 + p_1\frac{(1-|\alpha|^2)\sqrt{p_2^2+p_1^2+m^2-p_0^2}}{(1+|\alpha|^2)\sqrt{p_0^2-m^2}}.$$

Thus, $\vec{I}$ is parallel to the vector $(p_1, p_2, 0)$ if $|\alpha| = 1$

Bound states

If $r_5 = r_6 = r_7 = r_8 = 0$, then the dispersion relation for the bound state has the form $\overline{p}^2 = \xi^{\pm} m^2$, where

$$\xi^{\pm} = 1 - \left(\frac{\varsigma_1 \varsigma_3 \pm \sqrt{\varsigma_2^2 (1 + \varsigma_4^2)}}{\varsigma_1^2 + \varsigma_2^2} \right)^2 .$$

Here

$$\varsigma_1 = -1 + \frac{2}{1 - r_1^2 + r_2^2 + r_4^2}, \ \varsigma_2 = -\frac{2r_4}{1 - r_1^2 + r_2^2 + r_4^2},$$
$$\varsigma_3 = -\frac{2r_1}{1 - r_1^2 + r_2^2 + r_4^2}, \ \varsigma_4 = \frac{2ir_2}{1 - r_1^2 + r_2^2 + r_4^2}.$$

The coefficient $\xi^{\pm}$ at the corresponding values of the parameters ς_i can be positive or zero. For example, $\xi^{+} = 0$ at $\varsigma_1 = \varsigma_3$, $\varsigma_2 = 1$, $\varsigma_4 = 0$, and $\xi^{-} = 0$ at $\varsigma_1 = -\varsigma_3$, $\varsigma_2 = 1$, $\varsigma_4 = 0$. If $\varsigma_2 = 0$, $|\varsigma_1| > \sqrt{1 + \varsigma_3^2}$, and $\varsigma_4 = \pm\sqrt{\varsigma_1^2 - 1 - \varsigma_3^2}$, then $\xi^{\pm} > 0$.

Bound states

The dispersion law $\bar{p}^2 = \xi^{\pm} m^2$ describes free particles with the effective mass $m\sqrt{\xi^{\pm}} < m$ in the $(2+1)$- dimensional space-time with two spatial coordinates and one temporal coordinate if $\xi^{\pm} > 0$. If $\xi^{+} = 0$ or $\xi^{-} = 0$, then the corresponding particles are massless.

For the matrix Q of the model with homogenous isotropic plane

$$Q = r_1 I + ir_2\gamma_5 + r_3\gamma_3 + r_4\gamma_5\gamma_3 +$$
$$+r_5\gamma_0 + r_6\gamma_5\gamma_0 + ir_7\gamma_0\gamma_3 + ir_8\gamma_1\gamma_2$$

the dispersion law has the form

$$(p_1^2 + p_2^2)R_3 - (\lambda R_1 - 2(mr_{18}^- - p_0 r_{45}^+)(\lambda R_2 - 2(mr_{18}^+ + p_0 r_{45}^-)) = 0$$

Here, the following notations are used $r_{ij}^\pm = (r_i \pm r_j)/2$,

$$R_1 = {r_{18}^-}^2 + {r_{27}^-}^2 + {r_{36}^+}^2 - {r_{45}^+}^2 + 1,\ R_2 = {r_{18}^+}^2 + {r_{27}^+}^2 + {r_{36}^-}^2 - {r_{45}^-}^2 + 1$$
$$R_3 = r_6^2 + r_7^2 + r_8^2 - r_4^2,\ \lambda = \sqrt{m^2 + p_1^2 + p_2^2 - p_0^2}.$$

Restriction condition $\det S = 1$ on the parameters of the model reads

$$r_{36}^+ R_2 + r_{36}^- R_1 - 4r_3 = 0$$

Bound states

Choosing the parameters of model as

$$R_1 = R_2 = r_{18}^+ = r_{18}^- = 0, r_{36}^+ = -r_{36}^- = r_6,$$

we obtain the dispersion relation in the form

$$p_0^2 - v_\pm^2(p_1^2 + p_2^2) = 0.$$

It describe the propagation of massless particle in the plane with the Fermi-velocity

$$v_\pm = \sqrt{\frac{1}{2}\left(1 \pm \frac{1 + r_{27}^+ r_{27}^- - r_6^2}{\sqrt{1 + {r_{27}^+}^2 + r_6^2}\sqrt{1 + {r_{27}^-}^2 + r_6^2}}\right)}.$$

The motion of such particles explains numerous effects in graphene.

Bound state

The dispersion law can be presented as

$$p_1^2 + p_2^2 = a_0 m^2 + a_1 p_0 m + a_2 p_0^2 \pm$$

$$\pm (b_0 m + b_1 p_0)\sqrt{c_0 m^2 + c_1 p_0 m + c_2 p_0^2}.$$

Here, the dimensionless constants $a_0, a_1, a_2, b_0, b_1, c_0, c_1, c_2$ are expressed in terms of parameters $r_1, ..., r_8$

The main results

- In the framework of the Symanzik approach, we build the model of the fermionic field interaction with the material plane. The action of the model consist of the usual spinor Dirac action and extra defect contribution.The action contains parameters, that characterize the material property.
- The characteristics of the Dirac particles scattering on the defect plane are calculated in the model, also the properties of localized states near the defect plane are investigated.
- The Model and obtained on its basis results could be used for the theoretical description of the interaction of electrons, positrons and neutrons with two-dimensional materials (graphene, thin films, sputters, sharp boundaries of a solid body). Simple modifications of the model allows to take into account the effects of external electromagnetic fields.

Thank you for your attention!