One-loop correction to the photon velocity in LV QED
based on arXiv:1705.07796

Petr Satunin

INR
Institute for Nuclear Research
of the Russian Academy of Sciences

02.07.2017, Yaroslavl

Intro..

Physical dispersion relation for a particle — pole of the propagator.

$iG^{\mu\nu} =$ 

Sum over 1-particle reduced diagrams for photon propagator.
No loop corrections to dispersion relation — in Lorentz-invariant (LI) theories.

LI may be violated by external classical field — magnetic or gravitational.
Non-trivial photon dispersion!

Shabad 1975

book Mikheev Kuznetsov 2003-2014

Hollowood 2009

The similar situtation if LI is violated at fundamental level.

Lorentz Invariance violation

- Approaches to quantum gravity
  - Discrete spacetime, loop quantum gravity, non-commutative geometry e.t.c.
  - Modifications of eneral relativity with large space derivatives (Horava-Lifshitz e.t.c)
- Phenomenologically in non-gravity sector
  - Special type of LV (preserving other symmetries, motivations to concrete QG approaches)

    For example, $E^2 = p^2 + m^2 + \frac{p^4}{M_{LV}^2} + \ldots$
  - The most general type — SME *Kostelecky, Colladay 1998*

    Lots of parameters $\rightarrow$ complicated calculations

SME, QED sector

Kostlecky, Colladay 1998

QED sector SME:

$$\mathcal{L}_{SME} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\,\overline{\psi}\,\Gamma^{\mu}D_{\mu}\,\psi - \overline{\psi}\,M\,\psi - \\ -\frac{1}{4}(k_F)_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + (k_{AF})^{\mu}A^{\nu}\tilde{F}_{\mu\nu}\,, \quad (1)$$

$(k_F)_{\mu\nu\rho\sigma}$, $(k_{AF})^{\mu}$ – LV parameters in photon sector; Γ и M:

$$\Gamma^{\mu} = \gamma^{\mu} + c^{\mu\nu}\gamma_{\nu} + d^{\mu\nu}\gamma_5\gamma_{\nu} + if^{\mu} + \frac{1}{2}g^{\lambda\nu\mu}\sigma_{\lambda\nu} + e^{\mu}\,,$$

$$M = m + a^{\mu}\gamma_{\mu} + b^{\mu}\gamma_5\gamma_{\mu} + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}\,. \quad (2)$$

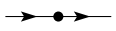
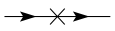
Renormalized in 1-loop

Kostelecky, Lane, Pickering 2001

SME, QED sector

$$
\begin{aligned}
\mathcal{L}_{SME} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\,\overline{\psi}\,\Gamma^{\mu} D_{\mu}\,\psi - \overline{\psi}\,M\,\psi - \\
&\quad -\frac{1}{4} (k_F)_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + (k_{AF})^{\mu} A^{\nu} \tilde{F}_{\mu\nu} \ .
\end{aligned}
$$

Electron and photon propagators are the same as in LI case, LV parameters are considered perturbatively:

 $= -i(M - m),$  $= i(\Gamma^{\mu} - \gamma^{\mu}) p_{\mu},$

μ  $\nu = -2ip^{\alpha} p^{\beta} (k_F)_{\alpha\mu\beta\nu},$

μ  $\nu = 2(k_{AF})^{\alpha} \epsilon_{\alpha\mu\beta\nu} p^{\beta}.$

Our simplified model

$$(k_F)_{\mu\nu\rho\sigma} = c_\gamma \cdot \delta^i_\mu \delta^j_\nu \delta^k_\rho \delta^l_\sigma \left(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}\right), \qquad c_{\mu\nu} = c_e \cdot \delta^i_\mu \delta^j_\nu \, \delta_{ij}.$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi - \tag{3}$$
$$-\frac{c_\gamma}{2} F_{ij} F^{ij} - i c_e \bar{\psi}\gamma^i D_i \psi.$$

Here c_γ, c_e — maximal velocities for photon and electron.
Dispersion relations:

$$\begin{aligned} \gamma: \qquad & k_0^2 = (1+c_\gamma)^2 \vec{k}^2 \simeq (1+2c_\gamma)\vec{k}^2, \\ e^\pm: \qquad & E^2 = (1+c_e)^2 p^2 + m^2 \simeq (1+2c_e)\vec{k}^2 + m^2. \end{aligned}$$

Electron propagator

$$S(p) = \frac{\gamma^\mu \hat{p}_\mu - m}{\hat{p}^2 - m^2},$$

here $\hat{p}_\mu = (p_0, (1 + c_e)\, p_i)$, $\hat{p}^2 = \hat{p}_\mu \hat{p}^\mu$.

Vertex:

$$\Gamma_\mu = (\gamma_0, (1 + c_e)\gamma_i)\,.$$

Photon propagator

Pseudo-Lorentz gauge

Gauge fixing term

$$\mathcal{L}_{gf} = -\frac{1-2c_\gamma}{2}\left(\partial_0 A_0 - (1+2c_\gamma)\partial_i A_i\right)^2 .$$

Photon propagator

$$D^{\mu\nu}(k) = \frac{\mathrm{diag}\left((1+2c_\gamma),\ -1,\ -1,\ -1\right)}{k_0^2 - (1+2c_\gamma)\vec{k}^2}.$$

Coulomb gauge

$$D^{00}(k) = -\frac{1}{(1+2c_\gamma)\vec{k}^2}, \quad D^{0i}(k) = 0, \quad D^{ij}(k) = -\frac{\delta^{ij} - \frac{k^i k^j}{\vec{k}^2}}{k_0^2 - (1+2c_\gamma)\vec{k}^2}.$$

Photon polarization operator

LI case (dimensional regularisation)

$$\Pi^{LI}_{\mu\nu}(k) = \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) k^2 \Pi(k^2),$$

$$\Pi(k^2) = -\frac{q^2}{2\pi^2}\int_0^1 dx\, x(1-x)\left[\frac{1}{\epsilon} + \ln 4\pi - \gamma_E - \ln\frac{m^2 - x(1-x)k^2}{\mu^2}\right].$$

In our model $\Pi_{\mu\nu}$ may be calculated exactly on c_e.

$$\Pi_{\mu\nu}(k) = q^2 \int \frac{d^4 k_{loop}}{(2\pi)^4} \mathrm{Tr}\left[\Gamma_\mu S(k + k_{loop}) \Gamma_\nu S(k_{loop})\right],$$

Components of $\Pi_{\mu\nu}$ may be represented as components of LI polarization operator

Notation: $\hat{k} = (k_0, (1 + c_e) k_i)$:

Photon polarization operator

$$\Pi_{\mu\nu}(k) = \left[(1-c_e)k^2(P_1)_{\mu\nu} - 2c_e\vec{k}^2(P_2)_{\mu\nu}\right]\Pi(\hat{k}^2),$$

projectors:

$$P_1^{\mu\nu} = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}, \qquad P_2^{\mu\nu} = -\delta_i^\mu \delta_j^\nu \left(\delta^{ij} - \frac{k^i k^j}{\vec{k}^2}\right),$$

with properties $P^\mu_{1\nu} P^\nu_{1\lambda} = P^\mu_{1\lambda}, \quad P^\mu_{2\nu} P^\nu_{2\lambda} = P^\mu_{2\lambda}, \quad P^\mu_{1\nu} P^\nu_{2\lambda} = P^\mu_{2\lambda},$

$$\Pi(\hat{k}^2) = \frac{q^2}{2\pi^2}\int_0^1 dx\, x(1-x)\ln\left(1 - x(1-x)\frac{\hat{k}^2}{m^2}\right) + \Pi_0,$$

Modified propagator (Coulomb gauge)

Sum over one-particle reduced diagrams

$$D^{00}_{1-loop}(k) = -\frac{1}{(1-c_e)\vec{k}^2}, \qquad D^{0i}_{1-loop}(k) = 0,$$

$$D^{ij}_{1-loop}(k) = -\frac{1}{1-\Pi(\hat{k}^2)(1-c_e)} \cdot \frac{\delta^{ij} - \frac{k^i k^j}{\vec{k}^2}}{k_0^2 - \vec{k}^2(1+2c_\gamma + 2(c_\gamma - c_e)\Pi(\hat{k}^2))}.$$

Denominator determines modified dispersion relation.
To solve equation

$$k_0^2 - \vec{k}^2(1+2c_\gamma + 2(c_\gamma - c_e)\Pi(\hat{k}^2)) = 0,$$

Modified dispersion relation

In the 1st order on α_{em}, dispersion relation

$$k_0^2 = \vec{k}^2 \left(1 + 2c_\gamma + 2(c_\gamma - c_e)\Pi_\epsilon(\vec{k})\right),$$

$$\Pi_\epsilon(\vec{k}) = \frac{q^2}{2\pi^2}\int_0^1 dx\, x(1-x)\ln\left(1 - 2\,(c_\gamma - c_e)x(1-x)\frac{\vec{k}^2}{m^2}\right) + \Pi_0. \quad (4)$$

Gauge fixing: $\Pi_\epsilon(0) = 0$. Notation: $y \equiv (c_e - c_\gamma)\,\frac{\vec{k}^2}{m^2}$, integrate (4) analitically. In the case $y > -2$:

$$\Pi_\epsilon(y) = \frac{q^2}{2\pi^2}\left[\frac{y-1}{3y}\sqrt{\frac{y+2}{y}}\mathrm{arcth}\sqrt{\frac{y}{y+2}} + \frac{1}{3y} - \frac{5}{18}\right];$$

Otherwise ($y < -2$) the polarization operator acquires imaginary part. Following the Optical theorem, photon decays $\gamma \to e^+e^-$. ($y = -2$) — threshold.

Small LV limit

Setting $y > -2$ (no photon decay), consider 2 limiting cases

$$|y| = |c_e - c_\gamma| \frac{\vec{k}^2}{m^2} \ll 1$$

$$\Pi_\epsilon(y) = \frac{q^2}{30\pi^2} y$$

Photon dispersion relation acquires quartic term

$$k_0^2 = \vec{k}^2 (1 + 2c_\gamma) + \frac{\vec{k}^4}{M_{LV}^2},$$

effective LV scale M_{LV}:

$$M_{LV} = \frac{\sqrt{15}\pi}{q} \cdot \frac{m}{|c_e - c_\gamma|}.$$

Large LV limit

In the limit $y \gg 1 \quad \left((c_e - c_\gamma) \frac{\vec{k}^2}{m^2} \gg 1 \right)$ we obtain

$$\Pi_\epsilon(y) = \frac{q^2}{12\pi^2} \left(\ln(2y) - \frac{5}{3} \right).$$

The correction to the dispersion relation is logarithmic:

$$k_0^2 = \vec{k}^2 \left[1 + 2c_\gamma + \frac{q^2}{6\pi^2}(c_\gamma - c_e) \cdot \left[\ln \left(2\,(c_e - c_\gamma)\,\frac{\vec{k}^2}{m^2} \right) - \frac{5}{3} \right] \right].$$

Group photon velocity $c_{ph} = \frac{\partial k_0}{\partial k}$:

$$c_\gamma^{ph} = c_\gamma - \frac{q^2}{6\pi^2} \cdot (c_e - c_\gamma) \cdot \ln \left(2(c_e - c_\gamma) \frac{\vec{k}^2}{m^2} \right).$$

Coincides with *Kostelecky, Lane, Pickering 2001*

$$(k_F)_{\mu\nu\rho\sigma} = c_\gamma \cdot \delta^i_\mu \delta^j_\nu \delta^k_\rho \delta^l_\sigma \left(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}\right), \qquad c_{\mu\nu} = c_e \cdot \delta^i_\mu \delta^j_\nu \, \delta_{ij}.$$

$$\begin{aligned} (k_F)_{\kappa\lambda\mu\nu} &= (k_F)_{0\kappa\lambda\mu\nu} + \frac{1}{6}(1 - Q^{-3}) \times \\ &\times \Big[\eta_{\mu\kappa}(c_{\nu\lambda} + c_{\lambda\nu} - (k_F)_{0\nu\alpha\lambda}{}^{\alpha}) - \eta_{\nu\kappa}(c_{\mu\lambda} + c_{\lambda\mu} + (k_F)_{0\mu\alpha\lambda}{}^{\alpha}) \\ &- \eta_{\mu\lambda}(c_{\nu\kappa} + c_{\kappa\nu} + (k_F)_{0\nu\alpha\kappa}{}^{\alpha}) + \eta_{\nu\lambda}(c_{\mu\kappa} + c_{\kappa\mu} - (k_F)_{0\mu\alpha\kappa}{}^{\alpha}) \Big]. \end{aligned}$$

$Q(\mu) \equiv 1 - \frac{q_0^2}{6\pi^2} \ln \frac{\mu}{\mu_0}$

Renormgroup

Renormgroup scale

$$\mu = \sqrt{c_\gamma - c_e} E_\gamma$$

Interpretation:
set $c_e = 0$ (redefinition)
The LV photon polarization operator, considered on-shell may be interpreted as off-shell polarization operator, calculated in LI theory with the squared photon momentum

$$q^2 \equiv E_\gamma^2 - \vec{k}^2 = 2(c_\gamma - c_e)\, E_\gamma^2 .$$

This value plays the role of the “transferred momentum” — the standard meaning of the renormalization group scale.

Constraints on LV in photon sector

Direct tests of photon dispersion

$$k_0^2 = \vec{k}^2 \pm \frac{\vec{k}^4}{M_{LV}^2}.$$

The absence of dispersion from distant GRB, AGN.

The best bound from GRB 090510

FERMI-LAT coll. 2013

$$M_{LV} \; > \; M_{LV}^{GRB} \equiv 1.3 \cdot 10^{11}\ \mathrm{GeV}, \qquad\qquad 95\%\ \mathrm{CL}$$

Constraints on LV for fermions

Not only electrons in loop, but arbitrary charged massive fermion

$$|c_e - c_\gamma| < \frac{\sqrt{15\pi}}{q} \cdot \frac{m}{M_{LV}^{GRB}} \simeq 3 \cdot 10^{-10} \cdot \left(\frac{q}{e}\right)^{-1} \cdot \left(\frac{m}{\mathrm{GeV}}\right).$$

	our bound	current bounds	
electron	$1.5 \cdot 10^{-13}$	10^{-15}	*Altschul 2010*
muon	$3 \cdot 10^{-11}$	10^{-11}	*Altschul 2006*
tau-lepton	$1.2 \cdot 10^{-9}$	10^{-8}	*Altschul 2006*
t-quark	$1.6 \cdot 10^{-7}$	10^{-2}	*D0 collab. 2012*

Conlusions

- In the absence of LI for charged fermions photon dispersion relation (and physical velocity) acquires non-trivial one-loop correction.
- Experimental non-observation of photon dispersion leads to constraints on LV for all charged fermions. These constraints are the best for heavy fermions (tau-lepton, top-quark)
- Effective Lagrangian for photons acquires high space derivative terms like $\delta\mathcal{L} \sim \frac{1}{M_{LV}^2} F_{\mu\nu} \Delta F^{\mu\nu}$.

Thank you for your attention!