Beta (Urca) processes in moderate degenerated matter with arbitrary magnetic field

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Beta (Urca) processes in Astrophysics

Beta processes are significant in following objects:

- ***Massive neutron star***

matter: relativistic, strongly degenerate, transparent for $\nu_e, \bar{\nu}_e,$

processes: $p + e^- \to n + \nu_e, \quad n \to p + e^- + \bar{\nu}_e,$

lead to anomalously fast cooling of massive neutron stars.

- ***Accretion disk (merger in binary system, collapsar)***

matter: relativistic, weakly degenerate, transparent for $\nu_e, \bar{\nu}_e,$

processes: $p + e^- \to n + \nu_e, \quad n + e^+ \to p + \bar{\nu}_e,$

neutrino production could be energy source of gamma-ray bursts.

- ***Supernova with core collapse***

matter: relativistic, moderately degenerate, partially transparent

processes: $p + e^- \rightleftarrows n + \nu_e, \quad n + e^+ \rightleftarrows p + \bar{\nu}_e,$

could be significant component of supernova explosion mechanism.

Beta (Urca) processes in Astrophysics

Full set of beta processes with free nucleon participation are:

$$p + e^- \rightleftarrows n + \nu_e,$$
$$n + e^+ \rightleftarrows p + \bar{\nu}_e,$$
$$n \rightleftarrows p + e^- + \bar{\nu}_e.$$

Neutron stars, supernovae and accretion disks could have a strong magnetic field with strength $B{\sim}10^{15}$G (observation of SGR and AXP, MHD simulations of SN and AD).
So, beta processes could be strongly modified in these objects.

Research of magnetic field influence on beta processes has a long history: Korovina 1964, Fassio-Canuto 1969, Matese et al. 1969 …

Progress is achieved in two general ways – numerical computations and analytical calculations with different strong limitations.

Calculated quantities

Important quantities for astrophysical applications are reaction rate:

$$\Gamma = \frac{1}{V} \int \sum_{i,\, f} \frac{|S_{if}|^2}{\mathcal{T}} \prod_{i,\, f} F_i \, (1 - F_f) \, dn_i \, dn_f ,$$

neutrino energy deposition and momentum transferred to the medium (plus sign – for absorption, minus – for emission):

$$Q^{\mu} = \left(Q, \vec{\mathcal{F}}\right) = \pm \frac{1}{V} \int \sum_{i,\, f} q^{\mu} \, \frac{|S_{if}|^2}{\mathcal{T}} \prod_{i,\, f} F_i \, (1 - F_f) \, dn_i \, dn_f .$$

It is convenient to use emissivity or absorbability ($c = \hbar = k_B = 1$):

$$\mathcal{K}^{(\mathrm{em})}(q) = \sum_{i,\, f \neq \nu} \int \frac{|S_{if}|^2}{\mathcal{T}} \prod_{i,\, f \neq \nu} F_i \, (1 - F_f) \, dn_i \, dn_f .$$

$$\mathcal{K}^{(\mathrm{abs})}(q) = e^{(\omega \mp \delta\mu)/T} \, \mathcal{K}^{(\mathrm{em})}(q), \quad \delta\mu = \mu_e + \mu_p - \mu_n .$$

Assumptions

The result was obtained under following estimates:

$$eB,\, \mu_e^2,\, \overline{\omega}_{\nu,\bar{\nu}}^2 \ll m_N T \ll m_N^2, \quad \mu_p - m_p \lesssim T.$$

- ***Matter must be warm:*** $T \gg \overline{\omega}_{\nu,\bar{\nu}}^2 / m_N \sim 0.1\ \mathrm{Mev}.$

It is violated in external part of Supernova and accretion disk.
Rates of neutrino reactions are very small at such temperature.

- ***Magnetic field is not very strong:***

  $B \ll B_0 T\, m_N / m_e^2 \quad (B_0 = eB/m_e^2 \approx 4.41 \times 10^{13}\ \mathrm{G}).$

At low temperature $T \sim 0.1$ *MeV, this condition corresponds to:*

  $B \ll 10^{16}\ \mathrm{G}.$

- ***Protons must be nondegenerate:*** $\mu_p - m_p \lesssim T.$
- ***Electrons can be moderately degenerate:*** $\mu_e \ll \sqrt{m_N T}.$

It is violated in internal part of Supernova and accretion disk.

Assumptions

Medium is electrically neutral: $N_p = N_{e-} - N_{e+}$
and chemical potentials of charged particles are not independent.

Condition on electron degeneration is violated first of all:

$$\mu_e \ll \sqrt{m_N T}.$$

In field-free case It is equivalent the following assumption:

$$\rho \ll \rho_0 \approx \frac{m_p}{3\pi^2} \frac{1+Y}{Y} \left(m_p T\right)^{3/2} \approx 2.11 \times 10^{11} \frac{1+Y}{Y} \left(\frac{T}{1\,\mathrm{MeV}}\right)^{3/2} \mathrm{g/cm^3},$$

$\rho_0 \sim 10^{13}$ g/cm³ ($T \approx 5$ MeV and $Y = N_p/N_n \approx 1/4$) is of the same order as neutrino trap density.

So, the general assumption is violated in the area where neutrino are in equilibrium and don't change medium parameters.

Assumptions are satisfied in SN envelope and AD!

Neutrino and antineutrino emissivity

Surprisingly, the assumptions enable to obtain analytical result!

$$\mathcal{K}_\nu(q) = K_0\,\Phi(b, x_\nu, \beta_\nu)\,\frac{e^{(\delta\mu+\Delta)/T-a_e}}{e^{x_\nu/t-a_e}+1}, \qquad \mathcal{K}_{\bar\nu}(q) = K_0\,\Phi(b, x_{\bar\nu}, \beta_{\bar\nu})\,\frac{1}{e^{x_{\bar\nu}/t+a_e}+1},$$

$$\Phi(x, b, \beta) = 2\,x\,\sqrt{x^2-1}\;\theta\big(x-\sqrt{1+2b}\big) + \frac{b\,x}{\sqrt{x^2-1}}\Big[\theta\big(x-1\big)-\theta\big(x-\sqrt{1+2b}\big)\Big] - g_{va}\,\frac{b\,x\,\cos\beta}{\sqrt{x^2-1}}\,\theta\big(x-1\big),$$

$$x_{\nu,\bar\nu} = (\omega\pm\Delta)/m_e, \quad t = T/m_e, \quad a_e = \mu_e/T, \quad b = eB/m_e^2 = B/B_0.$$

- ***Weak magnetic field:*** $b \ll 1$ ($B \ll 4\times10^{13}$G)

Changes of Γ and Q are very small $\sim b^2$.

- ***Strong magnetic field:*** $b \sim x^2$ ($B \sim 10^{16}$G).

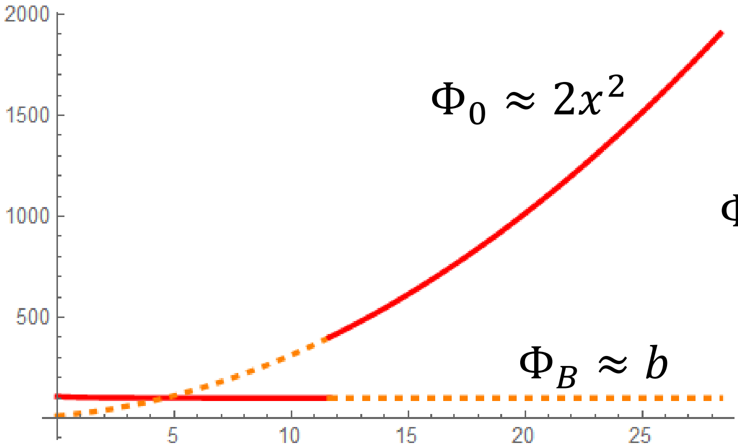
All integrated quantities significantly changed

- ***Arbitrary strength of magnetic field***

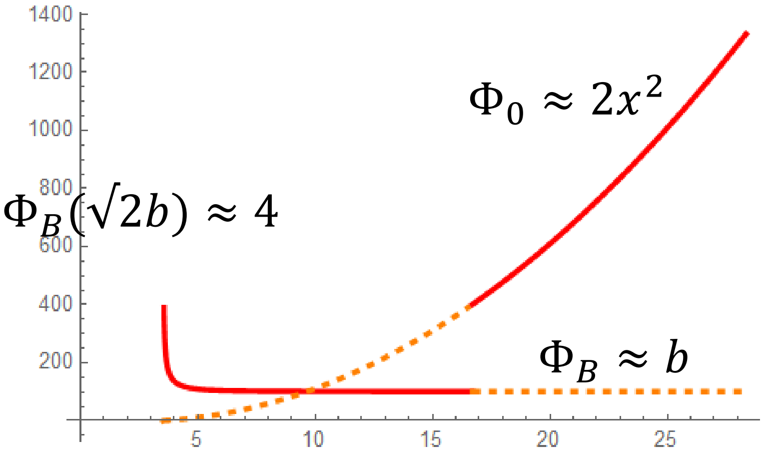
Momentum transferred to the medium $\sim B$ and could be significant.

Neutrino and antineutrino emissivity

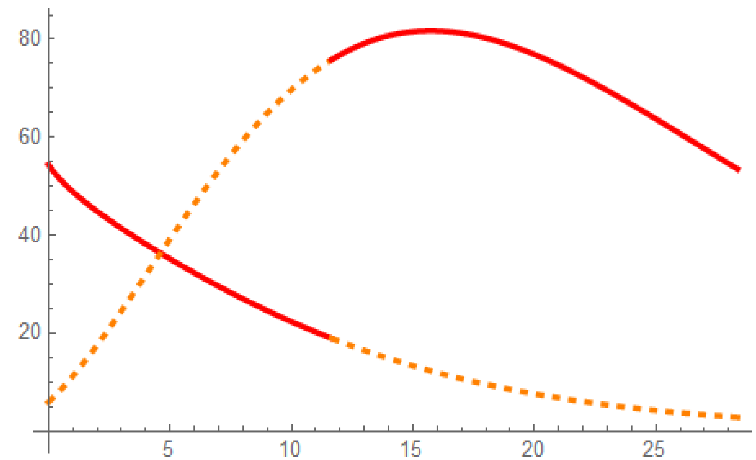
Φ for neutrino and antineutrino as a function of ω/m_e $(b = 100)$:

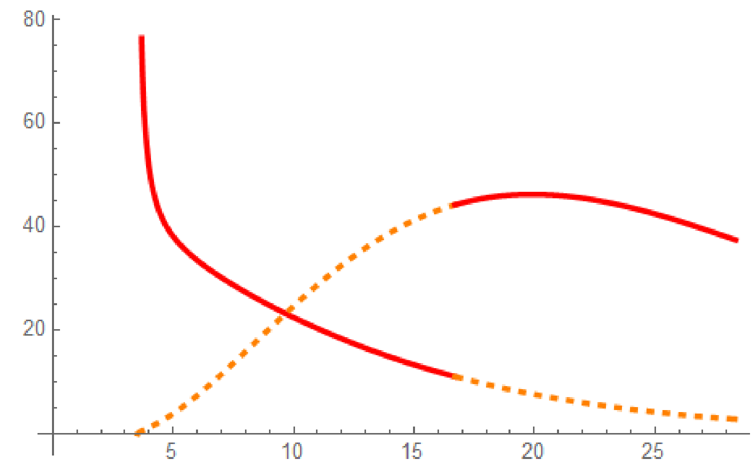


$\Phi_0(\sqrt{2}b)/\Phi_B(\sqrt{2}b) \approx 4$



$\Phi \times (e^{x/8} + 1)^{-1}$ as a function of ω/m_e $(b = 100,\ T \approx 4$ MeV):





Optically transparent medium

Asymmetric term in emissivity leads to ***macroscopic momentum transferred to the medium*** along magnetic field direction.

- ***This effect is absent in field-free case.***
- ***This effect is larger in the electron degenerate matter.***
- ***Contribution of antineutrino emission is very small in this case.***

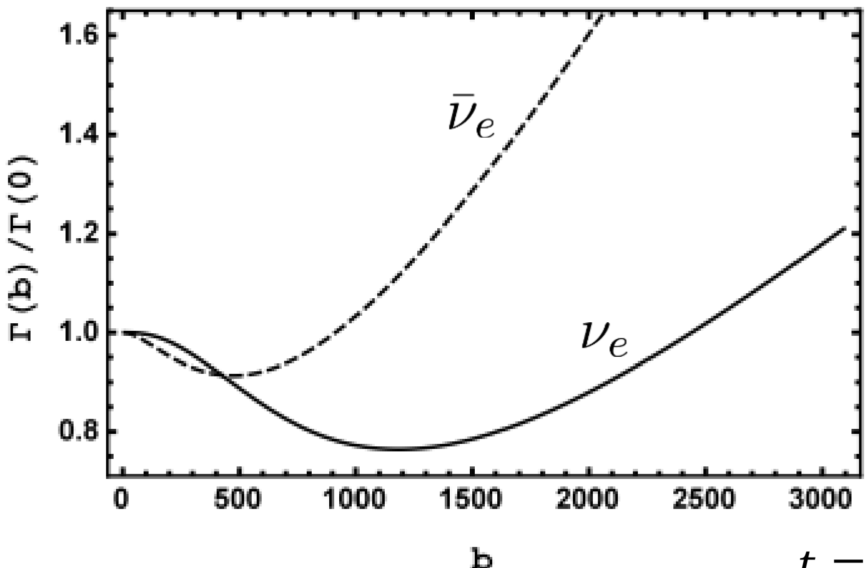
Force density along magnetic field in degenerate matter

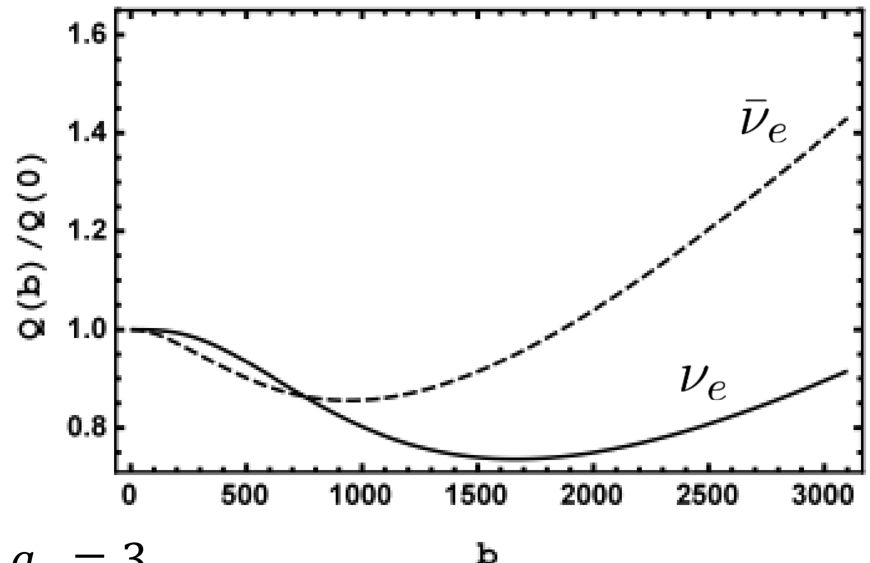
$$\mathcal{F}_0 = \frac{(g_a^2 - g_v^2)\cos^2\theta_c}{48\pi^3} \frac{Y}{1+Y} G_F^2\, eB\, \mu_e^4 \frac{\rho}{m_N}$$
$$\approx 6.15 \times 10^{20} \frac{Y}{1+Y} \left(\frac{B}{10^{15}\,\mathrm{G}}\right) \left(\frac{\rho}{10^{12}\,\mathrm{g/cm^3}}\right) \left(\frac{\mu_e}{10\,\mathrm{MeV}}\right)^4 \mathrm{dyn/cm^3}$$

Maybe this effect is significant for some astrophysical objects!

Optically transparent medium

Relative reaction rate and energy emission in beta processes:



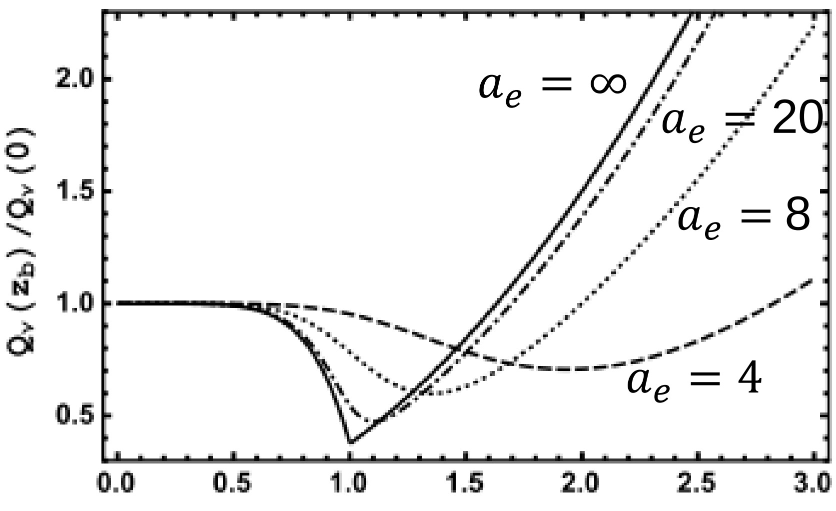


$t = 8, a_e = 3.$

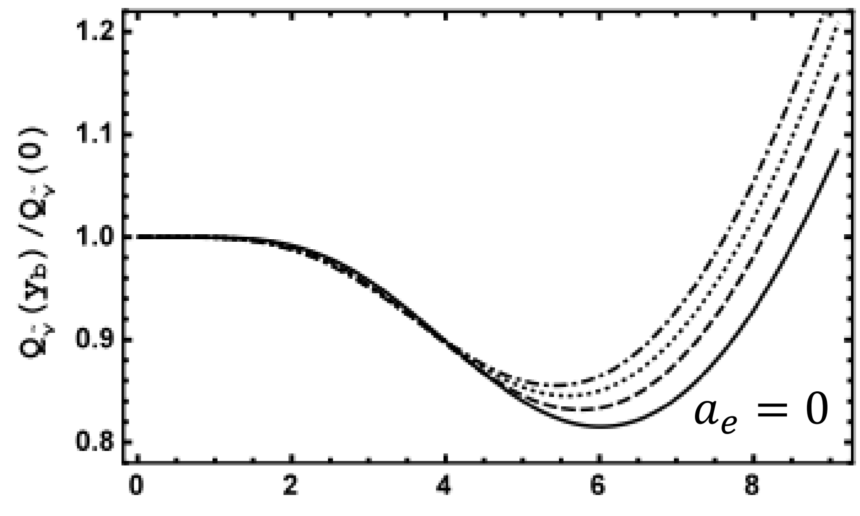
- In the strong magnetic field limit, Γ and Q grows up linearly. This increase corresponds to magnetic field strength which is unrealistic for supernova and accretion disc.
- Magnetic field suppresses beta processes at moderate strength.
- Emission of antineutrinos is suppressed weaker than for neutrino.

Optically transparent medium

Maximal suppression of neutrinos and antineutrinos emission:



$z_b = (\sqrt{2b})m_e/\mu_e$ z_b



$y_b = (\sqrt{2b})/t$ y_b

- Maximal suppression of neutrino emission can be observed in the degenerate plasma and suppression factor ≈ 2.5.
- Suppression of antineutrino emission depends slightly on plasma degeneration and maximal suppression factor ≈ 1.2 corresponds to nondegenerate plasma.

Optically transparent medium

Roadmap of magnetic field influence on beta processes.

- ***Neutrino emission:*** $p + e^- \to n + \nu_e.$

1) It leads to momentum transferred in arbitrary magnetic field strength: $F \approx 6 \times 10^{20}\, Y\, B_{15}\, \rho_{12}\, \mu_{10}^4$ dyn/cm³ (degenerate plasma).

2) Reaction rate and energy emission are:

  a) changed very small when $B \ll B_0$ ($B_0 \approx 4.4 \times 10^{13}$G)

  b) suppressed by maximal factor ≈ 2.5 in degenerate plasma for

$$B \approx \mu_e^2/(2m_e^2)\, B_0$$

- ***Antineutrino emission:*** $n + e^+ \to p + \bar{\nu}_e.$

1) It leads to momentum transferred: $F \approx 10^{19}\, B_{15}\, \rho_{12}\, T_4^4$ dyn/cm³.

2) Reaction rate and energy emission are:

  a) changed very small when $B \ll B_0$;

  b) suppressed by maximal factor ≈ 1.2 in any matter for

$$B \approx 10\, t^2\, B_0$$

Conclusions

- The simple analytical result for the emissivity in beta processes in arbitrary magnetic field was obtained. It could be used for astrophysical problems which require exact accounting of the magnetic field influence.

- The Influence of magnetic field on beta processes in optically transparent medium can be neglected if a factor ~few is not important in the problem.
- However, the momentum transferred to the matter along magnetic field direction could be significant in some astrophysical objects!

- In the case of partially transparent medium, results obtained for the transparent medium can not be affected by order of magnitude effect but a few times only.