The splitting of a photon in a strong magnetic field with taking account of the positronium in the polarization operator.

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The amplitude of the process $\gamma \to \gamma + \gamma$ in a strong magnetic field

$$\gamma \to \gamma + \gamma$$

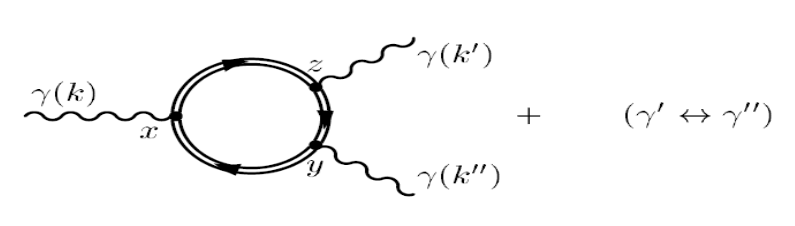


Figure: Photon decay

The invariant amplitude for the process $\gamma \to \gamma + \gamma$ has the form

$$M = e^3 \int d^4X d^4Y \mathrm{Sp}[\hat{\varepsilon}(k)S(Y)\hat{\varepsilon}(k'')S(-X-Y)\hat{\varepsilon}(k')S(X)]$$
$$* \, e^{-ie(XFY)/2} e^{i(k'X - k''Y)} + (\gamma' \to \gamma'')$$

where $X = z - x$, $Y = x - y$, $S(X)$ — the exact electronic propagator.

$$S(X) = \frac{i\beta}{2\pi} e^{-\frac{\beta X_\perp^2}{4}} \int \frac{d^2 q_\parallel}{(2\pi)^2} \frac{(q\gamma)_\parallel + m_e}{q_\parallel^2 - m_e^2} \Pi_- e^{-i(qX)_\parallel},$$

$$\Pi_- = \frac{1}{2}\Big(1 - i\gamma_1\gamma_2\Big)$$

Further calculations are conveniently carried out by passing to partial amplitudes:

$$M_{\lambda\lambda'\lambda''} = M(\varepsilon^{(\lambda)}(k), \varepsilon^{(\lambda')}(k'), \varepsilon^{(\lambda'')}(k'')), \qquad \lambda, \lambda', \lambda'' = 1, 2.$$

The spitting probability of a photon in a strong magnetic field

The splitting probability of a photon in a strong magnetic field has the form:

$$W_{\lambda\lambda'\lambda''}(\gamma \to \gamma' + \gamma'') = \frac{g}{32\pi^2\omega} \int |M_{\lambda\lambda'\lambda''}|^2 Z_\lambda Z_{\lambda'} Z_{\lambda'}$$

$$\delta\Big(\omega_\lambda(k) - \omega_{\lambda'}(k') - \omega_{\lambda''}(k - k')\Big) \frac{d^3k'}{\omega_{\lambda'}\omega_{\lambda''}}$$

where $g = 1 - \frac{1}{2}\delta_{\lambda'\lambda''}$, $Z_1 = \frac{1}{1 - \frac{\partial \Pi}{\partial q_\perp^2}}$, $Z_2 = \frac{1}{1 - \frac{\partial \Pi}{\partial q_\parallel^2}}$, Π — photon polarization operator.

Photon polarization operator with the positronium contribution taken into account

1) The probability of a photon decay into a free e^+e^--pair

$$\mathrm{Im}(\Pi_{e^+e^-}) = -\omega W_{\gamma\to e^+e^-} = -\frac{4\alpha(eB)m_e^2 e^{-\frac{q_\perp^2}{2eB}}}{\sqrt{q_\parallel^2(q_\parallel^2 - 4m_e^2)}}\theta(q_\parallel^2 - 4m_e^2)$$

2) The probability of a photon decay into the bounded e^+e^--pair (positronium)

$$\mathrm{Im}(\Pi_{Ps}) = -\omega W_{\gamma\to Ps} = -4\pi\alpha M(eB)|\chi(0)|^2 e^{-\frac{q_\perp^2}{2eB}}\delta(q_\parallel^2 - M^2)$$

$M = 2m_e - \epsilon$ is the mass of the positronium, ϵ is the binding energy, $\chi(z)$ is the wave function of a positronium in a strong magnetic field.

The positronium contribution to the photon polarization operator:

$$\Pi^{Ps} = -\frac{2\alpha(eB)|\chi(0)|^2 z}{m_e(1-\frac{\epsilon}{m_e}-z)} \mathrm{e}^{-\frac{q_\perp^2}{2eB}}$$

The expression for the polarization operator can be written as:

$$\Pi = -2\alpha(eB)\mathrm{e}^{-\frac{q_\perp^2}{2eB}} \left\{ \frac{1}{\pi} H(z) + \frac{\lambda z}{1-\lambda^2-z} \right\}$$

$$H(z) = \int\limits_0^1 \frac{\mathrm{d}x}{1-z(1-x^2)-\mathrm{i}0} - 1$$

$z = \frac{q_\parallel^2}{4m_e^2}$, $\lambda = \frac{\alpha}{2\nu}$

The binding energy and wave function of a positronium in a strong magnetic field

The wave function $\chi(z)$ satisfies the Schrödinger equation:

$$\Big(-\frac{1}{2\mu}\frac{\partial^2}{\partial z^2}+U_{eff}(z)+\epsilon\Big)\chi(z)=0$$

$\mu = m_e/2$ is the reduced mass
The "effective" potential energy U_{eff} has the form:

$$U_{eff}(z)=-\frac{\alpha}{2\pi}\int\limits_{-\infty}^{\infty}\frac{\mathrm{e}^{-\frac{x^2+y^2}{2}}\mathrm{d}x\mathrm{d}y}{\sqrt{\frac{x^2}{eB}+\left(y+\frac{P_\perp}{\sqrt{eB}}\right)^2\frac{1}{eB}+z^2}}$$

$P_\perp$ is the component of the positronium momentum transversal to the magnetic field.

The solution of the Schrödinger equation has the form:

$$\chi(z) = \chi(0)\frac{W_{\nu,\frac{1}{2}}(\frac{\alpha m_e}{\nu}|z|)}{W_{\nu,\frac{1}{2}}(0)},$$

where $W_{\nu,\frac{1}{2}}(x)$ is the Whittaker function. The binding energy has the form:

$$\epsilon = \frac{\alpha^2 m_e}{4}\frac{1}{\nu^2}$$

The ν parameter can be found from the equation:

$$\frac{1}{\nu} - 2\ln\nu + 2\psi(1-\nu) = \ln\frac{2b}{\alpha^2} - \ln\rho + E_i(-\rho) - 4\gamma_E$$

$b = \frac{B}{B_e}$, $\rho = \frac{P_\perp^2}{2eB}$, $B_e \approx 4,41 \cdot 10^{13}$G is the critical magnetic field.
$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$ is the logarithmic derivative of the gamma function,
$\gamma_E = 0.5772...$ is the Euler constant,
$E_i(-x) = \int\limits_{-\infty}^{-x} \frac{\mathrm{e}^t}{t}\mathrm{d}t$ is the integral exponent. The value $|\chi(0)|^2$ can be found from the normalization condition:

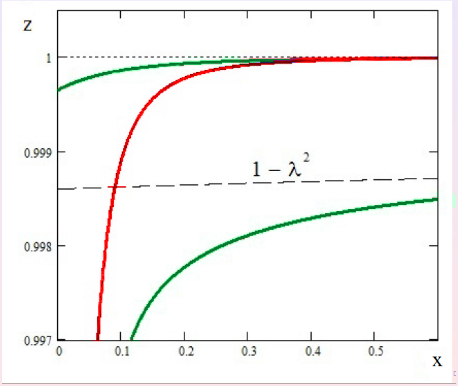
$$\int\limits_{-\infty}^{\infty} \mathrm{d}z|\chi(z)|^2 = 1$$

In the approximation $\nu \ll 1$ we obtain:

$$|\chi(0)|^2 \approx \frac{\alpha m_e}{2\nu}(1 - 2\nu + ...)$$

Photon dispersion with taking account of the positronium influence

$$q_{\parallel}^2 - q_{\perp}^2 = \Pi(q_{\parallel}, q_{\perp})$$



$$x = \frac{q_{\perp}^2}{4m_e^2}, \quad z = \frac{q_{\parallel}^2}{4m_e^2}$$

Conclusions

- We have calculated the decay probability of a photon to the free and the bounded e^+e^--pair.
- We have found the energy levels and the wave functions of a positronium in a strong magnetic field.
- We have obtained the expression for the photon polarization operator with taking account of the positronium.
- We have investigated the photon dispersion in a strong magnetic field with taking account of the positronium.