Next-to-leading order QCD corrections to paired B_c production in e^+e^- annihilation.

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What is interesting in multiple heavy quark production?

It allows to check:

- perturbative QCD;
- DPS (double parton scattering);
- hadronization models for doubly heavy systems (color octet vs color singlet, internal motion of quarks inside quarkonium);
- k_T factorization model (virtual initial gluons)

Some experimental data from LHCb

- hadronic B_c production (it looks like LO pQCD underestimates cross-section up to several times);
- double charm production (full cross-section is in good agreement with DPS, p_T distributions are in disagreement with DPS) [Aaij et al.(2012b)];
- $J/\psi + c\bar{c}$ (full cross-section is in good agreement with DPS, p_T distributions are in disagreement with DPS) [Aaij et al.(2012b)];
- paired J/ψ production (SPS +CS?)[Aaij et al.(2012a)].
- $\Upsilon + c\bar{c}$ (full cross-section is in good agreement with DPS).

To separate DPS and SPS contributions we need loop corrections to SPS.

Begin with simple NLO problems

1. $e^+e^- \xrightarrow{\gamma} J/\psi\eta_c$
2. $e^+e^- \xrightarrow{\gamma, Z_0} J/\psi\eta_c$
3. $e^+e^- \xrightarrow{\gamma} B_c^{(*)}B_c^{(*)}$
4. $e^+e^- \xrightarrow{\gamma, Z_0} B_c^{(*)}B_c^{(*)}$

- Not too large number of diagrams
- Absence of infrared divergences (no gluon radiation)
- First process is already calculated at LO and NLO, there is a possibility for checks [Feng(2014)].
- Third process is known at LO [Kiselev(1995)].

Approximation used

$$A^{SJj_z} = \int T^{Ss_z}_{b\bar{b}c\bar{c}}(p_i, k(\vec{q})) \cdot \left(\Psi^{Ll_z}_{\bar{b}c}(\vec{q})\right)^* \cdot C^{Jj_z}_{s_z l_z} \frac{d^3\vec{q}}{(2\pi)^3}$$

J and j_z — total meson angular momentum and its projection on z axis in B_c rest frame
L and l_z — orbital angular momentum and its projection
S и s_z — spin and its projection on the same axis
$C^{Jj_z}_{s_z l_z}$ — Clebsch-Gordon coefficients
p_i — momenta of B_c - meson and b, $\bar{c}$ quarks
$\vec{q}$ — 3d momentum of $\bar{b}$-quark in the rest frame of B_c-meson,
$(0, \vec{q}) \to k(\vec{q})$

$$A \sim \int d^3q\, \Psi^*(\vec{q}) \left\{ T(p_i, \vec{q})\big|_{\vec{q}=0} + \vec{q}\frac{\partial}{\partial \vec{q}} T(p_i, \vec{q})\big|_{\vec{q}=0} + \cdots \right\}$$

For S-wave state:

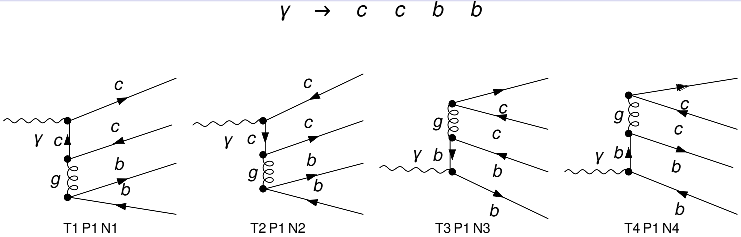
$$A \sim R_s(0) \cdot T_{b\bar{b}c\bar{c}}(p_i)\big|_{\vec{q}=0}$$

Approximation used

$$v_{\bar{b}_i}\bar{u}_{c_j} \quad \Longrightarrow \quad \left(\frac{\frac{m_b}{M}\hat{P}_{B_c} - m_b}{2m_b}\right)[\,\gamma^5 \text{ or } \hat{\epsilon}\,]\left(\frac{\frac{m_c}{M}\hat{P}_{B_c} + m_c}{2m_c}\right) \cdot \frac{\delta_{ij}}{\sqrt{3}}$$

- Color singlet.
- Relative velocity of quark motion inside quarkonium is neglected (heavy quark velocities are equal).
- At NLO we first put relative quark velocity to zero and then take integrals and perform renormalization.

Diagrams for $e^+e^- \xrightarrow{\gamma} B_c^{(*)} B_c^{(*)}$ at LO



Diagrams:
FeynArts [Hahn(2001)]

Amplitudes:
Form [Kuipers et al.(2013)Kuipers, Ueda, Vermaseren, and Vollinga],
FeynCalc [Shtabovenko et al.(2016)Shtabovenko, Mertig, and Orellana],
Redberry [Poslavsky and Bolotin(2015)]

LO cross-sections for $e^+e^- \xrightarrow{\gamma} B_c^{(*)} B_c^{(*)}$ production

$$r = \frac{m_c}{m_c + m_b}, \qquad m = m_c + m_b, \qquad \tilde{s} = s/m^2$$

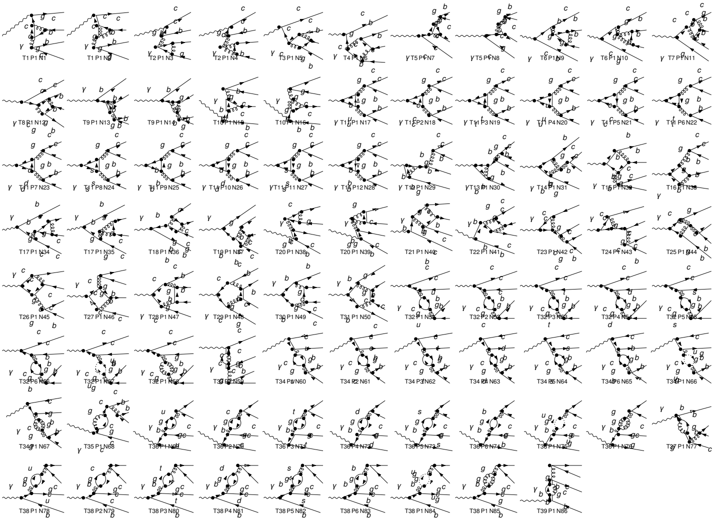
$$\sigma(B_c B_c) = \frac{64\pi\alpha^2\alpha_s^2 R_S^4 (\tilde{s}-4)^{3/2}\left(-3r^4(\tilde{s}+2) + r^3(5\tilde{s}+8) - 3r^2(\tilde{s}+4) + r(\tilde{s}+8) - 2\right)^2}{243 m^8 (r-1)^6 r^6 \tilde{s}^{13/2}}$$

$$\sigma(B_c B_c^*) = \frac{128\pi\alpha^2\alpha_s^2 \left(-3r^3 + 3r^2 - 3r + 1\right)^2 R_S^4 (\tilde{s}-4)^{3/2}}{243 m^8 (r-1)^6 r^6 \tilde{s}^{11/2}}$$

$$\sigma(B_c^* B_c^*) = \frac{64\pi\alpha^2\alpha_s^2 R_s^4 (\tilde{s}-4)^{3/2}}{243 m^8 (r-1)^6 r^6 \tilde{s}^{13/2}} \Big(9r^8(\tilde{s}^2 - 4\tilde{s} + 12) - 6r^7(5\tilde{s}^2 - 18\tilde{s} + 48) + r^6(43\tilde{s}^2 - 184\tilde{s} + 624)$$
$$- 36r^5(\tilde{s}^2 - 7\tilde{s} + 24) + r^4(19\tilde{s}^2 - 168s + 888)$$
$$+ r^3(-6\tilde{s}^2 + 28\tilde{s} - 672) + r^2(\tilde{s}^2 + 32\tilde{s} + 336) - 4r(5\tilde{s} + 24) + 4(\tilde{s} + 3)\Big)$$

See also [Kiselev(1995)]

NLO diagrams for $\gamma \to B_c^{(*)} B_c^{(*)}$



Renormalization scheme

On-shell scheme used:

$$
\begin{aligned}
Z_m^{OS} &= 1 - \frac{3g_s^2}{16\pi^2} C_F C_\epsilon \left[\frac{1}{\epsilon_{UV}} + \frac{4}{3}\right] \\
Z_2^{OS} &= 1 - \frac{g_s^2}{16\pi^2} C_F C_\epsilon \left[\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} + 4\right] \\
Z_g^{\overline{MS}} &= 1 + \frac{g_s^2}{16\pi^2} \left(-\frac{11}{6} C_A + \frac{1}{3} n_f\right) \left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln(4\pi)\right]
\end{aligned}
$$

$C_F = \frac{N_c^2 - 1}{2N_c}$, $C_A = N_c$, $C_\epsilon = \left(\frac{4\pi\mu^2}{m^2} e^{-\gamma_E}\right)^\epsilon$

m is the heavy quark pole mass

n_f is a number of fermions taken into account in gluon self-energies

γ_E is Euler's gamma constant

γ^5 prescription:

West prescription [West(1993)] or Larin prescription [Larin(1993)].

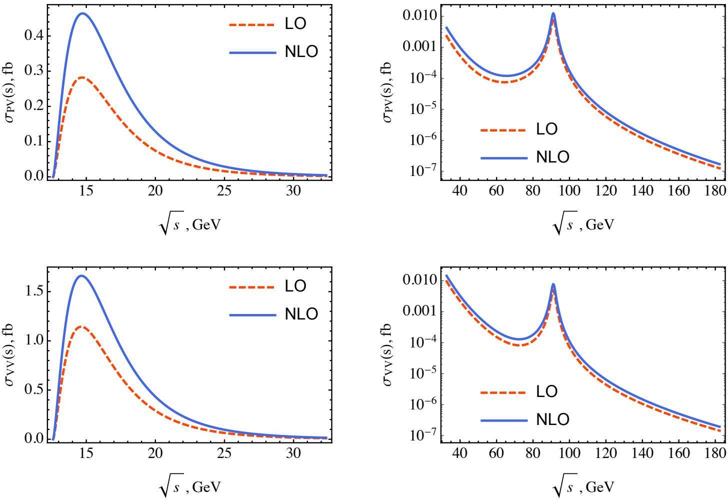
Calculation techniques

The calculations were done within several workflows:

- FeynArts $\to$ FeynCalc ($\to$ FeynCalcFormLink) $\to$ Apart $\to$ FIRE $\to$ Package-X;
- FeynArts $\to$ Redberry $\to$ Apart $\to$ FIRE $\to$ Package-X;
- FeynArts $\to$ FORM $\to$ prototyping and simplification within the original framework $\to$ FIRE $\to$ Package-X.

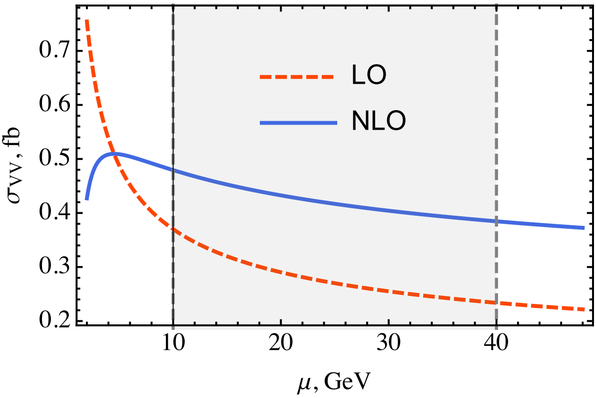
All applied methods lead to same results.

Energy dependence



$$\mu = \sqrt{s}$$

Scale dependence



$\sqrt{s} = 20$ GeV $\qquad \sqrt{s}/2 \le \mu \le 2\sqrt{s}$

Summary

- Computed NLO cross-section for paired B_c-meson production in e^+e^--annihilation using several different techniques.
- One-loop corrections are sizable at all energies.
- The dependence on the renormalization scale μ stabilizes with the account of NLO corrections.
- Developed code will be used to calculate cross-sections for other processes of multiple heavy quark production at NLO.
- The results are published in Nucl.Phys.B [Berezhnoy et al.(2017)Berezhnoy, Likhoded, Onishchenko, and Poslavsky].

Thank you for attention!

Backup slides

Dimensional regularization of NLO amplitudes

- $D = 4 - 2\epsilon$
- $\overline{\text{MS}}$ scheme: $\frac{1}{\epsilon} - \gamma_E + \log(4\pi)$
- $\{\gamma^\mu, \gamma^\nu\} = 0 \quad g^{\mu\nu} g_{\mu\nu} = D$
  ['t Hooft and Veltman(1972)]

γ^5 problem

For $D \neq 4$ equalities $\{\gamma^5, \gamma^\mu\} = 0$ and $\mathrm{Tr}\{\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\} \neq 0$ cannot be simultaneously satisfied. We have used to prescriptions for γ^5.

West (applied within Feyncalc, Feyncalc+Formlink, Form):

$$\mathrm{Tr}\{\gamma^5\gamma^{\alpha_1}\ldots\gamma^{\alpha_n}\} = \frac{2}{n-4}\sum_{i=2}^{n}\sum_{j=1}^{i-1}(-1)^{i+j+1} g_{\alpha_i\alpha_j}\mathrm{Tr}\{\gamma^5 \prod_{\substack{k=1\\k\neq i,j}}^{n}\gamma^{\alpha_k}\}\ (n>4)$$

Larin scheme (applied within RedBerry [Poslavsky and Bolotin(2015)] or FeynCalc):

γ^5 anticommute to the right

$$\gamma^5\gamma^\mu \to -\frac{i}{6}\varepsilon_{\mu\alpha\beta\sigma}\gamma^\alpha\gamma^\beta\gamma^\sigma$$

Integration over loop momentum

- Passarino-Veltman reduction procedure (removing k^μ and $k^\mu \varepsilon_{\mu\ldots}$) [Passarino and Veltman(1979), 't Hooft and Veltman(1979)]: realized within FeynCalc and within the original framework.
- simplification of the integrals: within FeynCalc + $Apart [Feng(2012), Feng(2014)]; or prototyping and a simplification within the original framework.
- reduction to master integrals: FIRE [Smirnov and Smirnov(2013)].
- the analytical expressions for master integrals: Package-X [Patel(2015)].

Coefficients $\sim 1/(4-D)$

After FIRE in the amplitude $\gamma \to B_c^* B_c$ we obtain the term

$$-\frac{1}{3m^5(r-1)^5 r^5 (s-4) s^3 (4-D)} 4ie(s-2) C_F g_s^4 \epsilon^{\gamma\varepsilon(B_c^*)p(B_c^*)p(B_c)}$$
$$(r(m^2(r-1)((r-1)^3(r^2(s-4)-r(s-4)-1)B_0(m^2(r^2(-(s-4))+r(s-4)+1); mr, m-mrt)$$
$$+(5r^3-3r^2+3r-1)(r^2(s-4)-r(s-4)-1)B_0(m^2(r^2(-(s-4))+r(s-4)+1); m-mr, mr)$$
$$-r(3r^4-6r^3+6r^2-4r+1)s(B_0(m^2r^2s; mr, mr)+B_0(m^2(r-1)^2 s; m-mr, m-mr)))$$
$$+2(6r^4-9r^3+9r^2-5r+1)A_0(m-mr))+2(6r^5-15r^4+18r^3-14r^2+6r-1)A_0(mr))$$

$$A_0(m) \sim \int \frac{dk}{k^2-m^2} \qquad B_0(p^2; m_1, m_2) \sim \int \frac{dk}{(k^2-m_1^2)((k+p)^2-m_2^2)}$$

Expansion of A_0 and B_0 up to $O(1)$ (as in package-X) is not enough!

We used the expansion of A_0 and B_0 up to $O(\epsilon)$ derived from [Davydychev and Kalmykov(2001)].

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