On the Light Massive Flavor Dependence of the Top Quark Mass

Christopher Lepenik

University of Vienna, Austria

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Doktoratskolleg Particles and Interactions

Der Wissenschaftsfonds.

See [arXiv:1706.08526] for recent work in collaboration with Andrè Hoang and Moritz Preisser

Outline

Introduction

- Well known property of pole mass of a heavy quark Q:
  - Linearly sensitive to small momenta $\rightarrow$ sensitivity to regime where QCD non-perturbative.
  - Defined to absorb all on-shell self energy (OS-SE) corrections (including scales < 1 GeV).
  - Low momentum sensitivity grows rapidly with loop order.
- Leads to "$\mathcal{O}(\Lambda_{\rm QCD})$ renormalon":
  - Bad perturbative behavior
    $\rightarrow$ Factorially diverging asymptotic high order behavior.
  - Intrinsic $\mathcal{O}(\Lambda_{\rm QCD})$ ambiguity of pole mass value.

Introduction

- On the other hand: "Short-distance" mass schemes ($\overline{\mathrm{MS}}$, 1S, ...)
  $\rightarrow$ No linear low momentum sensitivity, no $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ renormalon.
  - $\overline{\mathrm{MS}}$ mass $\overline{m}_Q(\mu)$: Contains OS-SE corrections above scale $\mu \gtrsim m_Q$.

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  - High order renormalon behavior of pole mass can be studied in perturbative series describing relation to short-distance mass:

$$m_Q^{\mathrm{pole}} - \overline{m}_Q = \overline{m}_Q \sum_{n=1}^{\infty} a_n(n_Q, 1) \left(\frac{\alpha_s^{(n_Q+1)}(\overline{m}_Q)}{4\pi} \right)^n$$

$$\underset{\text{high orders}}{\overset{\beta_0/\mathrm{LL}}{\sim}} \mu \sum_{n=0}^{\infty} \frac{16}{3} (2\beta_0^{(n_\ell)})^n \, n! \left(\frac{\alpha_s^{(n_\ell)}(\mu)}{4\pi} \right)^{n+1}$$

($\overline{m}_Q = \overline{m}_Q(\overline{m}_Q)$, n_Q quarks lighter than quark Q, $n_\ell = n_Q$ massless quarks)

  - Independent of particle mass! Choice of μ!

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  - Independent of particle mass! Choice of μ!

- Asymptotic behavior can be relevant already at low orders:

$$m_c^{\mathrm{pole}} - \overline{m}_c = 0.2108 + 0.1984 + 0.2725 + (0.4843 \pm 0.0005)\ \text{GeV}$$

Light Mass Dependence

- Top quark physics: In most applications bottom and charm taken to be massless.
- High order pole mass considerations: Sensitive to lighter massive quarks q $(m_q > \Lambda_{\mathrm{QCD}})$ due to linear low momentum sensitivity.
  - Big impact: Mass of virtual quark loops act as IR-cutoff.
    $\Rightarrow$ Mass effects change IR structure, effectively remove massive quark from high order behavior,
    $\Rightarrow$ by itself plagued by renormalon.

Light Mass Dependence

- Light mass effects in top pole-$\overline{\mathrm{MS}}$ relation:

$$m_t^{\mathrm{pole}} - \overline{m}_t = \overline{m}_t \sum_{n=1}^{\infty} a_n(n_t+1, 0) \left(\frac{\alpha_s^{(n_t+1)}(\overline{m}_t)}{4\pi}\right)^n + \overline{m}_t \left[\overline{\delta}_t^{(t,b,c)}(1, r_{bt}, r_{ct})\right].$$

$(n_t = 5,\ r_{qQ} = \overline{m}_q/\overline{m}_Q)$

- Mass corrections $\overline{\delta}_t^{(t,b,c)}$: Account for masses of virtual top, bottom, charm loops.

Aims

- Aim: Study contributions from light massive flavors in coherent, systematic manner by using a renormalization group (RG) framework:
  - Study large order behavior and structure of mass corrections,
  - study overall large order behavior,
  - resum logs of quark mass ratios in resulting multi-scale problem,
  - show consequences of heavy quark symmetry (HQS).

- $\overline{\mathrm{MS}}$ mass not adequate to describe scales $< m_Q$.
- Tool for RG setup → MSR mass $m_Q^{\mathrm{MSR}}(R)$:

$$\begin{aligned} m_t^{\mathrm{pole}} &= \overline{m}_t + \overline{m}_t \sum_{n=1}^{\infty} a_n(n_t+1,0) \left(\frac{\alpha_s^{(n_t+1)}(\overline{m}_t)}{4\pi}\right)^n \\ &\quad + \overline{m}_t \left[\overline{\delta}_t^{(t,b,c)}(1, r_{bt}, r_{ct})\right] \\ &= m_t^{\mathrm{MSR}}(R) + R \sum_{n=1}^{\infty} a_n(n_t,0) \left(\frac{\alpha_s^{(n_t)}(R)}{4\pi}\right)^n \\ &\quad + \overline{m}_t \left[\delta_t^{(b,c)}(r_{bt}, r_{ct})\right] \end{aligned}$$

$(n_t = 5)$

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- Heavy particle integrated out, linear momentum scaling.
- High order behavior independent of heavy particle mass
  $\Rightarrow$ pole-MSR mass series captures renormalon just as $\overline{\mathrm{MS}}$.
- R-RGE: $R\,\frac{\mathrm{d}m_Q^{\mathrm{MSR}}(R)}{\mathrm{d}R} = -\,R\,\gamma^R(\alpha_s^{(n_Q)}(R))$, renormalon free.

Heavy Quark Symmetry

- MSR mass incorporates HQS consequences:
  - $R\sum_{n=1}^{\infty} a_n(n_t,0)\left(\frac{\alpha_s^{(n_t)}(R)}{4\pi}\right)^n$ describes contribution from gluons and $n_t = 5$ massless quarks at and below scale R.
  - For $R = \overline{m}_b$: Contributions equivalent to respective bottom $\overline{\mathrm{MS}}$ mass terms ($\mu = \overline{m}_b$).

$$m_t^{\mathrm{pole}} - m_t^{\mathrm{MSR}}(\overline{m}_b) = \overline{m}_b \sum_{n=1}^{\infty} a_n(5,0)\left(\frac{\alpha_s^{(5)}(\overline{m}_b)}{4\pi}\right)^n + \overline{m}_t\left[\delta_t^{(b,c)}(r_{bt}, r_{ct})\right]$$

$$m_b^{\mathrm{pole}} - \overline{m}_b = \overline{m}_b \sum_{n=1}^{\infty} a_n(5,0)\left(\frac{\alpha_s^{(5)}(\overline{m}_b)}{4\pi}\right)^n + \overline{m}_b\left[\overline{\delta}_b^{(b,c)}(1, r_{cb})\right]$$

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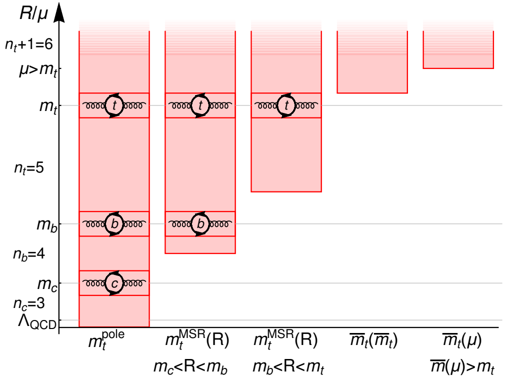
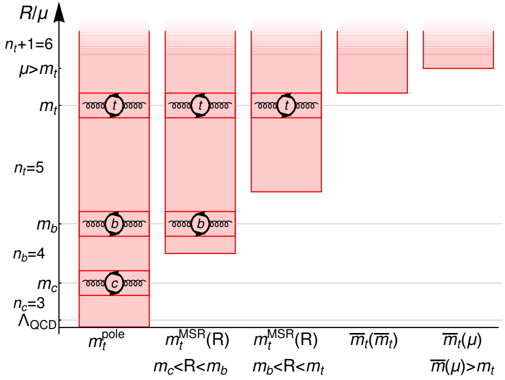
- Difference of mass contributions encodes HQS-breaking.
  - Renormalon cancels in difference!

Framework



- Incorporate lighter massive quarks in RG-setup as usual:
  - RG-evolution down to mass threshold of next lighter quark,

Framework



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    $\rightarrow$ renormalon free matching contribution,

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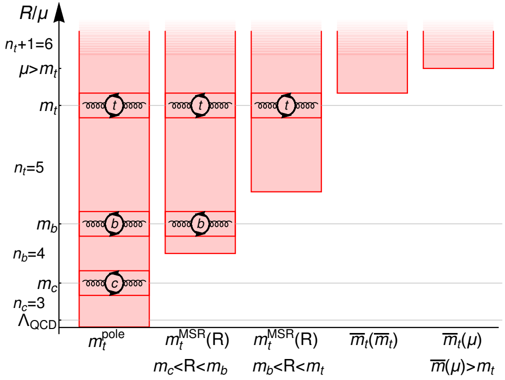
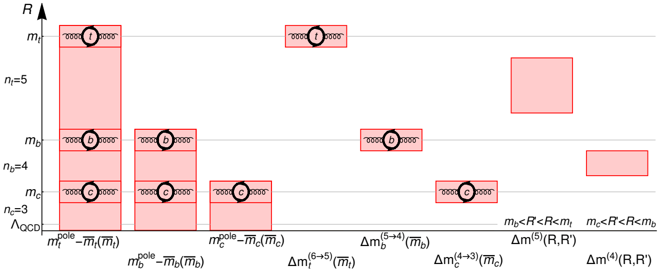


- Incorporate lighter massive quarks in RG-setup as usual:
  - RG-evolution down to mass threshold of next lighter quark,
  - integrate out lighter flavor
    $\rightarrow$ renormalon free matching contribution,
  - continue evolution down with reduced flavor number.

Framework

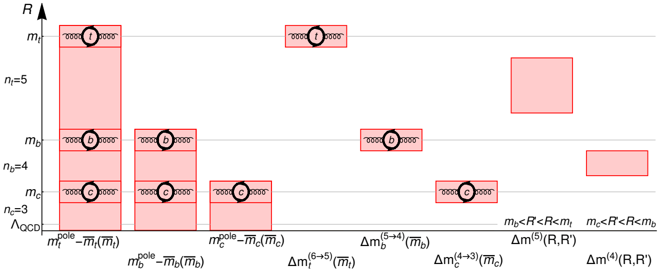


- Disentangle different momentum regions and resum logs by evolving down to scales $R < \overline{m}_b$ $(m_c = 0)$:

$$m_t^{\text{pole}} - \overline{m}_t = \Delta m_t^{(6\to5)}(\overline{m}_t) + \Delta m^{(5)}(\overline{m}_t, \overline{m}_b) + \delta m_{b,c}^{(t\to b)}(\overline{m}_b, 0)$$
$$+ \Delta m_b^{(5\to4)}(\overline{m}_b) + \Delta m^{(4)}(\overline{m}_b, R) + R\sum_{n=1}^{\infty} a_n(n_\ell = 4, 0)\left(\frac{\alpha_s^{(4)}(R)}{4\pi}\right)^n$$

- Matching from integrating out virtual massive flavor loops,
- R-evolution between mass scales → sums logs to all orders,

Framework



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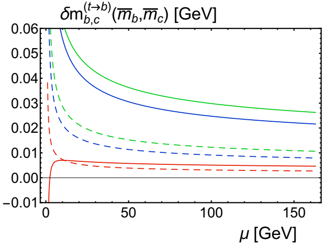
- Mass matching $\overline{m}_t\delta_t^{(b)} - \overline{m}_b\overline{\delta}_b^{(b)}$ (HQS-breaking term),
- Series containing the renormalon (scales $< R$).

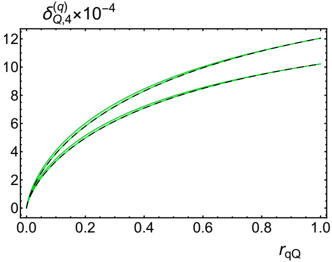
Applications: Higher Order Light Mass Corrections



$\mathcal{O}(\alpha_s^n)$	$\delta m_{b,c}^{(t\to b)}(\overline{m}_b, \overline{m}_c)$	$\delta m_c^{(b\to c)}(\overline{m}_c)$
2	0.007 ± 0.004	0.004 ± 0.002
3	0.006 ± 0.001	0.004 ± 0.001

- Observation: Mass matching (HQS-breaking) renormalon free and extremely well convergent.
    - 3-loop correction: Amounts to only 1 MeV, cancellation between individual terms already $\gtrsim 90\%$!
    - Cancellation at higher orders expected due to increased IR-sensitivity and HQS.

Applications: Higher Order Light Mass Corrections



- Virtual lighter mass correction known up to $\mathcal{O}(\alpha_s^3)$.
- Virtual mass contributions from same flavor encoded in a_n.
  - Known explicitly up to $\mathcal{O}(\alpha_s^4)$ (Beyond: High order behavior).
- Cancellation property: Predict 4-loop virtual lighter mass correction by demanding full cancellation (precision better than 1 MeV):

$$\delta^{(q)}_{Q,4}(r_{qQ}) \approx r_{qQ}\Bigg[\delta^{(q)}_{q,4}(1) - \Big(6\,\beta_0^{(n_Q)}\delta^{(q)}_{q,3}(1) + 4\,\beta_1^{(n_Q)}\delta_2(1)\Big)\ln(r_{qQ}) + 12\,\delta_2(1)\Big(\beta_0^{(n_Q)}\ln(r_{qQ})\Big)^2\Bigg], \qquad (\mu = \overline{m}_Q)$$

Applications: Pole Mass Differences

- Renormalon depends only on number of massless quarks.
  $\Rightarrow$ Difference of heavy quark pole masses short-distance quantity.
- Use RG-setup to determine pole mass differences.
- Example: Top and bottom:

$$m_t^{\rm pole} - m_b^{\rm pole} = [\overline{m}_t - \overline{m}_b] + \Delta m_t^{(6\to 5)}(\overline{m}_t) + \Delta m^{(5)}(\overline{m}_t, \overline{m}_b) + \delta m_{b,c}^{(t\to b)}(\overline{m}_b, \overline{m}_c)$$

- Some numbers:

$$\begin{aligned} m_t^{\rm pole} - m_b^{\rm pole} &= 168.169 \pm 0.016 \text{ GeV}\,, \\ m_b^{\rm pole} - m_c^{\rm pole} &= \quad 3.331 \pm 0.017 \text{ GeV}\,, \\ m_t^{\rm pole} - m_c^{\rm pole} &= 171.500 \pm 0.024 \text{ GeV}\,. \end{aligned}$$

Summary

- Relation between pole and short-distance mass renormalization schemes involve light massive flavor dependence.
  - Inclusion sometimes crucial due to modification of IR structure.

- Introduced RG-framework which allows to
  - sum logs of quark mass ratios,
  - disentangle contributions from different momentum regions,
  - study structure of light massive flavor corrections,
  - make HQS consequences explicit.

- Applications:
  - Prediction of $\mathcal{O}(\alpha_s^4)$ light massive flavor contribution,
  - precise determination of pole mass differences.
  - Not covered here: Determination of the renormalon ambiguity of heavy quark masses $\rightarrow$ 250 MeV ($\approx \Lambda_{\mathrm{QCD}}^{(n_\ell=3)}$).

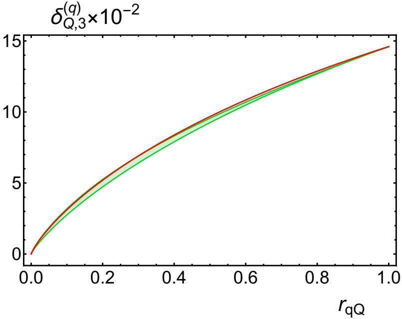
Thank you for your attention!

Some references:

- This Work:
  - A. Hoang, C. Lepenik, M. Preisser [arXiv:1706.08526]
- Renormalons Review:
  - M. Beneke [arXiv:hep-ph/9807443]
- 4-Loop Pole-$\overline{\mathrm{MS}}$ Relation:
  - P. Marquard, A. V. Smirnov, V. A. Smirnov, M. Steinhauser, D. Wellmann [arXiv:1606.06754]
- 3-Loop Lighter Mass Effects:
  - S. Bekavac, A. Grozin, D. Seidel, M. Steinhauser [arXiv:0708.1729]
- MSR-Mass, R-Evolution:
  - A. Hoang, A. Jain, I. Scimemi, I. W. Stewart [arXiv:0803.4214]
  - A. Hoang, A. Jain, C. Lepenik, V. Mateu, M. Preisser, I. Scimemi [arXiv:1704.01580]

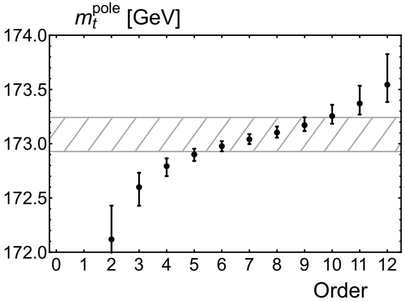
Backup

Applications: Higher Order Light Mass Corrections



- Comparison of prediction and exact result at 3-loop.

Best Possible Estimate of the Top Quark Pole Mass



- Our method:
  - Take the series which contains the renormalon,
  - determine size of smallest term and numerically close ones,
  - include uncertainty/scale variation of these terms,
  - take whole covered region as the inevitable uncertainty.

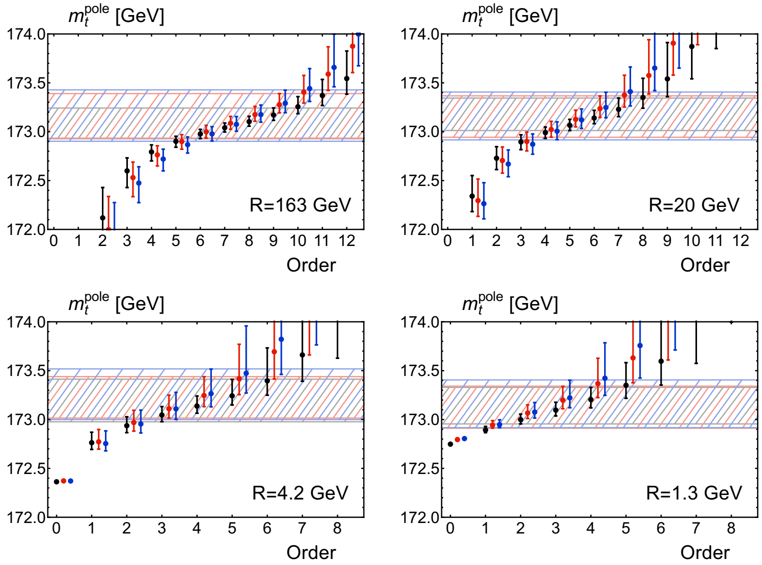
Best Possible Estimate of the Top Quark Pole Mass

- We considered different scenarios with massless or massive charm and bottom.
- Outcome of analysis:
  - Ambiguity increases if number of massless quarks decreases. Expected, since $\Lambda_{\rm QCD}^{(n_\ell)} < \Lambda_{\rm QCD}^{(n_\ell - 1)}$.
  - Largest ambiguity reached for massive bottom and charm: ~ 250 MeV.
  - This value should be taken as the intrinsic ambiguity if the pole mass is considered as a global concept.
  - 250 MeV already relevant at HL-LHC!!

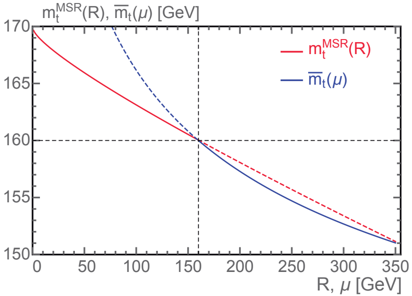
Best Possible Estimate of the Top Quark Pole Mass

$\overline{m}_t = 163\,\text{GeV}, \quad \overline{m}_b = \overline{m}_c = 0\,\text{GeV}$	
R	m_t^{pole}
163	173.086(157)
20	173.178(166)
4.2	173.195(218)
1.3	173.142(187)
$\overline{m}_t = 163\,\text{GeV}, \quad \overline{m}_b = 4.2\,\text{GeV}, \quad \overline{m}_c = 0\,\text{GeV}$	
R	m_t^{pole}
4.2	173.227(212)
1.3	173.126(215)
$\overline{m}_t = 163\,\text{GeV}, \quad \overline{m}_b = 4.2\,\text{GeV}, \quad \overline{m}_c = 1.3\,\text{GeV}$	
R	m_t^{pole}
1.3	173.159(244)

Best Possible Estimate of the Top Quark Pole Mass

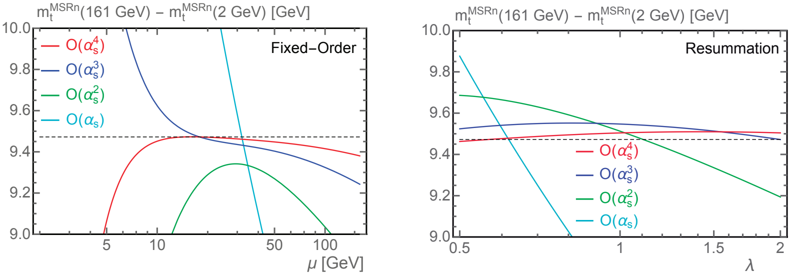


MSR Mass



- The MSR mass is the natural generalization of the $\overline{\text{MS}}$ mass for scales $R < m_Q$.

R-evolution

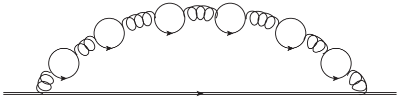


- R-evolution vs. FOPT for widely separated scales:
  - Significant μ dependence in FOPT - large logs spoil convergence!

Renormalons

- QFTs of phenomenological relevance:
  - Not possible to construct non-perturbatively from perturbative expansions and analyticity properties of Green's function.
  - Non-trivial, non-perturbative structure of vacuum and its excitations.
  - Non-perturbative power corrections.

Renormalons



- Explore contributions to $a_n^{\overline{\mathrm{MS}}}$ from class of diagrams with massless quark bubble insertions to all orders.
- Leads to ($\mu = \overline{m}_Q$, $\alpha_s \equiv \alpha_s(\overline{m}_Q)$, $\hat{q}^2 \equiv q^2/\overline{m}_Q^2$)

$$A(\alpha_s) \sim \sum_{n=0}^{\infty} \frac{\alpha_s^n n_\ell^n}{(6\pi)^n} \int_0^\infty \mathrm{d}\hat{q}^2 \underbrace{F\left(\hat{q}^2\right)}_{\text{"Gluon momentum distribtion"}} \log^n\left(\hat{q}^2\,\mathrm{e}^{-5/3}\right) \,.$$

- Large logarithmic enhancement for $\hat{q}^2 \ll 1$!

Renormalons

- Evaluate integral for small momenta:

$$F(\hat{q}^2) = \frac{2}{\sqrt{\hat{q}^2}} + \mathcal{O}\left(\sqrt{\hat{q}^2}\right)$$

$$\Rightarrow \quad A(\alpha_s) \sim \sum_{n=0}^{\infty} \alpha_s^n \left(\frac{-2n_\ell}{6\pi}\right)^n n! + \ldots$$

- Infrared renormalon behavior!
- How can we deal with the result?
- Can we assign a number to the series?

Renormalons

- How to deal with divergent series?
  - Generalize summation process - has to give right answer for convergent series.
  - Particularly useful for summing divergent asymptotic series is Borel summation:

A series $s(x) = \sum_{n=0}^{\infty} c_n x^n$ is called Borel-summable if the Borel transform

$$B[s](t) = \sum_{n=0}^{\infty} \frac{c_n t^n}{n!}$$

is convergent for $t > 0$ and if the integral

$$S(x) = \int_0^{\infty} \mathrm{d}t \; \mathrm{e}^{-t} B[s](tx)$$

exists. $S(x)$ is the value of the series.

Renormalons

- Assumption: Perturbative series is asymptotic in the sense

$$\left| \mathcal{O}(\alpha) - \sum_{i=0}^{n} c_i \alpha^i \right| < K_{n+1} \alpha^{n+1} \, .$$

  - Can not be proven but is reasonable since phenomenology using perturbation theory works very well.
- Ordinary summation: Best approximation typically given when truncating at smallest term $\rightarrow$ irreducible error.
- Note: While a divergent perturbative series implies non-analyticity at $\alpha = 0$, non-analyticity does not imply divergence. A convergent series can still differ from $\mathcal{O}$ by exponentially small terms $\exp(-1/\alpha)$.

Renormalons

$$A(\alpha_s) \sim \sum_{n=0}^{\infty} \alpha_s^n \left(\frac{\beta_0}{2\pi}\right)^n n! + \dots$$

- Before applying tools to our example: Use a "dirty trick" - naive non-Abelianization $n_\ell \to -3/2(11 - 2/3 n_\ell) = -3/2\beta_0$.
  - Can be justified diagrammatically - includes some non-Abelian corrections. Profound consequences!
- Apply Borel summation technique ($u \equiv t\beta_0/4\pi$)

$$B[A](u) \sim \frac{1}{1 - \frac{t\beta_0}{2\pi}} + \dots = \frac{1}{2}\,\frac{1}{1/2 - u} + \dots .$$

- Pole of the Borel transform at $u = 1/2$.

Renormalons

$$B[A](u) \sim \frac{1}{1/2 - u} + \dots$$

- Meaning of the pole? Let's Borel sum the series:

$$A(\alpha_s) = \int_0^\infty \mathrm{d}u \; \mathrm{e}^{-\frac{4\pi u}{\beta_0 \alpha_s}} B[A](u)\,.$$

- Integral exists, but one has to choose path in complex plain to avoid singularity → Ambiguity of the Borel summation!

Renormalons

$$B[A](u) \sim \frac{1}{1/2 - u} + \dots$$

- Size of ambiguity

$$\Delta \left[\int_0^\infty \mathrm{d}u \, \mathrm{e}^{-\frac{4\pi u}{\beta_0 \alpha_s(\overline{m}_Q)}} \frac{1}{u-k} \right] \sim \left(\frac{\Lambda_{\mathrm{QCD}}^2}{\overline{m}_Q^2} \right)^k .$$

- Gives rise to non-perturbative power corrections.
- Pole-$\overline{\mathrm{MS}}$ relation: $k = 1/2$, multiplied with $\overline{m}_Q \Rightarrow \mathcal{O}(\Lambda_{\mathrm{QCD}})$ ambiguity!
- More informations about renormalons: [Beneke, hep-ph/9807443]