Anglular correlations in $B\bar{B}$ pair production at the LHC in the Parton Reggeization Approach

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Motivation for the present study

- A study of angular correlations in pair $b\bar{b}$ production provides a test of dynamics of hard interactions, which is highly sensitive to the higher-order corrections in QCD.
- Usually calculations with additional hard jets are improved via Monte-Carlo (MC) generators by matching of the full NLO corrections in Collinear Parton Model (CPM) with the parton showers. But neither MC-calculations (PYTHIA, MC@NLO, Cascade), nor the calculations in the LO of standard k_T-factorization approach, e.g. [H. Jung, M. Kraemer, A. V. Lipatov and N. P. Zotov, Phys. Rev. D **85**, 034035 (2012)], could accurately describe the shape of angular distributions.

Introduction to the Parton Reggeization Approach (PRA)

Model process and Sudakov's decomposition

To derive the factorization formula in PRA, let's consider the following auxilliary hard subprocess:

$$g(p_1) + g(p_2) \to g(k_1) + \mathcal{Y}(P_{\mathcal{A}}) + g(k_2), \tag{1}$$

where $p_1^2 = p_2^2 = k_1^2 = k_2^2 = 0$, $M_{\mathcal{A}}^2 = P_{\mathcal{A}}^2$.
We use the Sudakov (light-cone) components of any four-momentum k:

$$k^\mu = \frac{1}{2}\left(k^+ n_-^\mu + k^- n_+^\mu\right) + k_T^\mu,$$

where $n_\pm^\mu = \left(n^\pm\right)^\mu = (1, 0, 0, \mp 1)^\mu$, $n_\pm^2 = 0$, $n_+ n^- = 2$, $k^\pm = k_\pm = (n_\pm k) = k^0 \pm k^3$, $n_\pm k_T = 0$, so that $p_1^- = p_2^+ = 0$ and $s = (p_1 + p_2)^2 = p_1^+ p_2^- > 0$. Then the dot-product of two four-vectors k and q in this notation is equal to:

$$(kq) = \frac{1}{2}\left(k^+ q_- + k^- q_+\right) - \mathbf{k}_T \mathbf{q}_T.$$

Collinear limit

Now we introduce the "t-channel" momentum transfers $q_{1,2} = p_{1,2} - k_{1,2}$, which implies that $\mathbf{q}_{T1,2} = -\mathbf{k}_{T1,2}$, $q_1^- = -k_1^-$ and $q_2^+ = -k_2^+$. Let us define $t_{1,2} = \mathbf{q}_{T1,2}^2$, and the corresponding fractions of the "large" light-cone components of momenta:

$$z_1 = \frac{q_1^+}{p_1^+}, \quad z_2 = \frac{q_2^-}{p_2^-}.$$

In the **collinear limit**, when $\mathbf{k}_{T1,2}^2 \ll \mu^2$, while $0 \leq z_{1,2} \leq 1$

$$\overline{|\mathcal{M}|^2}_{\text{C.L.}} \simeq \frac{4g_s^4}{\mathbf{k}_{T1}^2 \mathbf{k}_{T2}^2} P_{gg}(z_1) P_{gg}(z_2) \frac{\overline{|\mathcal{A}_{CPM}|^2}}{z_1 z_2}, \tag{2}$$

where $P_{gg}(z) = 2C_A\left((1-z)/z + z/(1-z) + z(1-z)\right)$ – LO gluon-gluon DGLAP splitting function and $\mathcal{A}_{CPM}$ is the amplitude of the subprocess $g(z_1 p_1) + g(z_2 p_2) \to \mathcal{Y}(P_{\mathcal{A}})$ with on-shell initial-state gluons.

Multi-Regge Kinematics (MRK)

The limit of **Multi-Regge Kinematics** (MRK) for the subprocess (1) is defined as:

$$\Delta y_1 = y(k_1) - y(P_{\mathcal{A}}) \gg 1,\ \Delta y_2 = y(P_{\mathcal{A}}) - y(k_2) \gg 1, \tag{3}$$

$$\mathbf{k}_{T1}^2 \sim \mathbf{k}_{T2}^2 \sim M_{T\mathcal{A}}^2 \sim \mu^2 \ll s, \tag{4}$$

where rapidity for the four-momentum k is equal to $y(k) = \log\left(k^+/k^-\right)/2$.
From (3) and (4) $\Rightarrow$

$$\begin{aligned} & z_{1,2} \ll 1, \\ & M_{T\mathcal{A}} \sim |\mathbf{k}_{T1}| \sim q_1^+ \sim O(z_1) \gg q_1^- \sim O(z_1^2), \\ & M_{T\mathcal{A}} \sim |\mathbf{k}_{T2}| \sim q_2^- \sim O(z_2) \gg q_2^+ \sim O(z_2^2). \end{aligned}$$

MRK limit of QCD amplitudes can be obtained using **Lipatov's EFT for MRK processes in QCD.**

The field content of the effective theory.

Light-cone derivatives:

$$x^{\pm} = n^{\pm} x = x^0 \pm x^3, \; \partial_{\pm} = 2\frac{\partial}{\partial x^{\mp}}$$

Lagrangian of the effective theory $L = L_{kin} + \sum_y \left(L_{QCD} + L_{ind} \right)$, $v_\mu = v_\mu^a t^a$, $\left[t^a, t^b \right] = f^{abc} t^c$. The rapidity space is sliced into the subintervals, corresponding to the groups of final-state particles, close in rapidity. Each subinterval in rapidity $(1 \ll \eta \ll Y)$ has it's own set of QCD fields:

$$L_{QCD} = -\frac{1}{2} tr \left[G_{\mu\nu}^2 \right], \; G_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + g \left[v_\mu, v_\nu \right].$$

Different rapidity intervals communicate via the gauge-invariant fields of Reggeized gluons ($A_{\pm} = A_{\pm}^a t^a$) with the kinetic term:

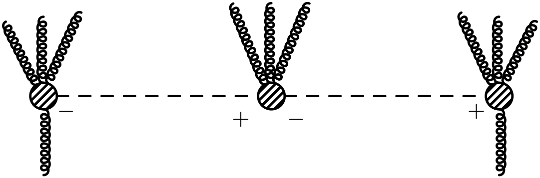
$$L_{kin} = -\partial_\mu A_+^a \partial^\mu A_-^a,$$

and the kinematical constraint:

$$\partial_- A_+ = \partial_+ A_- = 0 \Rightarrow$$

$$A_+ \text{ has } k_- = 0 \text{ and } A_- \text{ has } k_+ = 0.$$

The effective action for high energy processes in QCD.



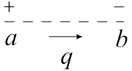
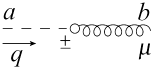
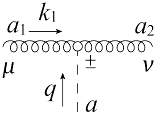
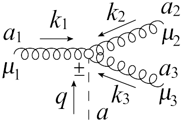
Particles and Reggeons interact via *induced interactions*:

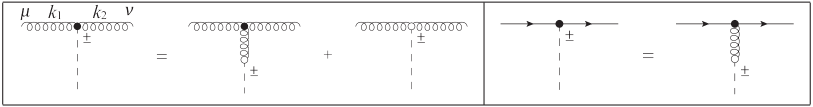
$$L_{ind} = - \operatorname{tr}\left\{ \frac{1}{g}\partial_+ \left[P\exp\left(-\frac{g}{2}\int\limits_{-\infty}^{x^-} dx'^- v_+(x') \right) \right] \cdot \partial_\sigma \partial^\sigma A_-(x) + \right.$$
$$\left. + \frac{1}{g}\partial_- \left[P\exp\left(-\frac{g}{2}\int\limits_{-\infty}^{x^+} dx'^+ v_-(x') \right) \right] \cdot \partial_\sigma \partial^\sigma A_+(x) \right\}$$

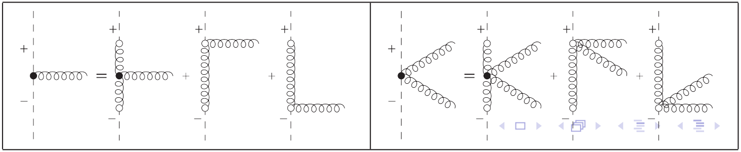
Wilson lines lead to the infinite chain of the induced vertices:

$$L_{ind} = \operatorname{tr}\left\{ \left[v_+ - g v_+ \partial_+^{-1} v_+ + g^2 v_+ \partial_+^{-1} v_+ \partial_+^{-1} v_+ - \ldots \right] \partial_\sigma \partial^\sigma A_- + \right.$$
$$\left. + \left[v_- - g v_- \partial_-^{-1} v_- + g^2 v_- \partial_-^{-1} v_- \partial_-^{-1} v_- - \ldots \right] \partial_\sigma \partial^\sigma A_+ \right\}$$

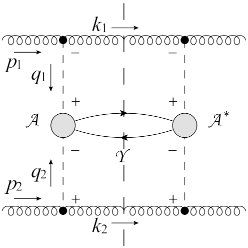
Feynman rules

 $= \frac{-i\delta_{ab}}{2q^2}$	 $= (-iq^2) n^{\mp}_{\mu} \delta_{ab}$
	$g_s f_{aa_1a_2} \left(n^{\mp}_{\mu} n^{\mp}_{\nu} \right) \frac{q^2}{k_1^{\mp}}$
	$ig_s^2 \left(n^{\mp}_{\mu_1} n^{\mp}_{\mu_2} n^{\mp}_{\mu_3} \right) \frac{q^2}{k_3^{\mp}} \left[\frac{f_{aba_1} f_{ba_2a_3}}{k_1^{\mp}} + \frac{f_{aba_2} f_{ba_1a_3}}{k_2^{\mp}} \right]$





Matrix element in the MRK limit



The **MRK asymptotics** of the squared amplitude of the process (1) has the following form:

$$\overline{|\mathcal{M}|^2}_{\rm MRK} \simeq \frac{4g_s^4}{\mathbf{k}_{T1}^2 \mathbf{k}_{T2}^2} \tilde{P}_{gg}(z_1)\tilde{P}_{gg}(z_2)\frac{\overline{|\mathcal{A}_{PRA}|^2}}{z_1 z_2}, \qquad (5)$$

where $\tilde{P}_{gg}(z) = 2C_A/z$ — the MRK gluon-gluon splitting functions, $\overline{|\mathcal{A}_{PRA}|^2}$ – **Gauge-invariant** PRA amplitude with Reggeized (**off-shell!**) initial-state partons, which explicitly depends on $t_{1,2} = \mathbf{q}_{T1,2}^2 = \mathbf{k}_{T1,2}^2$.

Modified MRK (mMRK) approximation

Now we introduce the **modified MRK (mMRK) approximation** for the squared amplitude of the subprocess (1) as follows:

- We substitute the MRK asymptotics for the splitting fuctions $\tilde{P}_{gg}(z)$ by the full LO DGLAP expression $P_{gg}(z)$ to the MRK squared amplitude.
- Into the its denominator we substitute the factors $\mathbf{k}_{T1,2}^2$ by the exact value of $q_{1,2}^2$: $\mathbf{k}_{T1,2}^2 \to -q_{1,2}^2 = \mathbf{q}_{T1,2}^2/(1 - z_{1,2})$.
- However, the "small" light-cone components of momenta: q_1^- and q_2^+ do not propagate into the hard scattering process, so it's gauge-invariant definition is unaffected and is given by the Lipatov's EFT [L. N. Lipatov, Nucl. Phys. B 452, 369 (1995)].

After these substitutions, the mMRK approximation for the squared amplitude of the subprocess (1) takes the following form:

$$\overline{|\mathcal{M}|^2}_{\text{mMRK}} \simeq \frac{4g_s^4}{q_1^2 q_2^2} P_{gg}(z_1) P_{gg}(z_2) \frac{\overline{|\mathcal{A}_{PRA}|^2}}{z_1 z_2}. \tag{6}$$

Factorization formula for the PRA

We substitute the mMRK approximation to the factorization formula of CPM:

$$d\sigma = \int \frac{dk_1^+ d^2\mathbf{k}_{T1}}{(2\pi)^3 k_1^+} \int \frac{dk_2^- d^2\mathbf{k}_{T2}}{(2\pi)^3 k_2^-} \int d\tilde{x}_1 d\tilde{x}_2 f_g(\tilde{x}_1, \mu^2) f_g(\tilde{x}_2, \mu^2) \frac{\overline{|\mathcal{M}|^2}_{\mathrm{mMRK}}}{2S\tilde{x}_1\tilde{x}_2} d\Phi_{\mathcal{A}},$$

where $f_g(x, \mu^2)$ are the (integrated) Parton Distribution Functions (PDFs) of the CPM, $p^\mu_{1,2} = \tilde{x}_{1,2} P^\mu_{1,2}$. **Change of variables:** $(k_1^+, \tilde{x}_1) \to (z_1, x_1)$, $(k_2^-, \tilde{x}_2) \to (z_2, x_2)$, where $x_{1,2} = \tilde{x}_{1,2} z_{1,2}$.
$\Rightarrow$ k_T**-factorization formula:**

$$d\sigma = \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \tilde{\Phi}_g(x_1, t_1, \mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \tilde{\Phi}_g(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{\mathrm{PRA}},$$

where the partonic cross-section in PRA is given by:

$$d\hat{\sigma}_{\mathrm{PRA}} = \frac{\overline{|\mathcal{A}_{PRA}|^2}}{2Sx_1x_2} \cdot (2\pi)^4 \delta\left(\frac{1}{2}\left(q_1^+ n_- + q_2^- n_+\right) + q_{T1} + q_{T2} - P_{\mathcal{A}}\right) d\Phi_{\mathcal{A}}.$$

Nonintegrated Parton Distribution Functions (nPDFs)

The tree-level "unintegrated PDFs" (unPDFs) are:

$$\tilde{\Phi}_g(x,t,\mu^2) = \frac{1}{t}\frac{\alpha_s}{2\pi}\int\limits_x^1 dz\ P_{gg}(z)\frac{x}{z}f_g\left(\frac{x}{z},\mu^2\right),$$

which have the collinear divergence at $t_{1,2} \to 0$ and infrared (IR) divergence at $z_{1,2} \to 1$. It regularizes at $z_{1,2} < 1 - \Delta_{KMR}(t_{1,2},\mu^2)$, where $\Delta_{KMR}(t,\mu^2) = \sqrt{t}/(\sqrt{\mu^2}+\sqrt{t})$, and $\mu^2 \sim M_{T\mathcal{A}}^2$.
The collinear singularity is regularized by the Sudakov formfactor:

$$T_i(t,\mu^2) = \exp\left[-\int\limits_t^{\mu^2}\frac{dt'}{t'}\frac{\alpha_s(t')}{2\pi}\sum_{j=q,\bar{q},g}\int\limits_0^1 dz\ z\cdot P_{ji}(z)\theta\left(1-\Delta_{KMR}(t',\mu^2)-z\right)\right].$$

The final form of our unPDF is:

$$\Phi_i(x,t,\mu^2) = T_i(t,\mu^2)\frac{\alpha_s(t)}{2\pi}\sum_{j=q,\bar{q},g}\int\limits_x^1 dz\ P_{ij}(z)\frac{x}{z}f_j\left(\frac{x}{z},\mu^2\right)\theta\left(1-\Delta_{KMR}(t,\mu^2)-z\right).$$

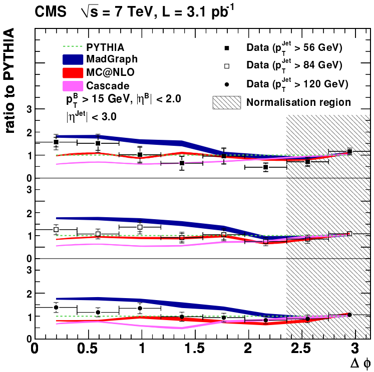
The KMR unPDF approximately satisfies the following normalization condition:

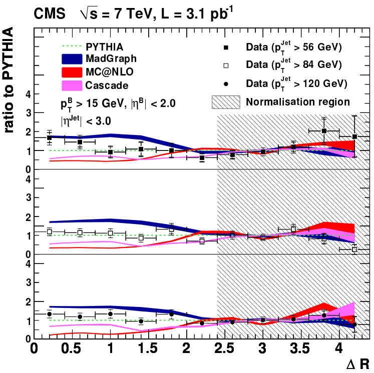
$$\int\limits_0^{\mu^2} dt\ \Phi_i(x,t,\mu^2) \simeq x f_i(x,\mu^2).$$

$B\overline{B}$ angular correlations at the LHC

Motivation

Motivation for the present study is the CMS data [CMS Collab. V. Khachatryan *et al.*, JHEP **1103**, 136 (2011)].
The $B\bar{B}$-pair is searched in **the events with at least one hard jet.**





LO contribution (b-jet is leading)

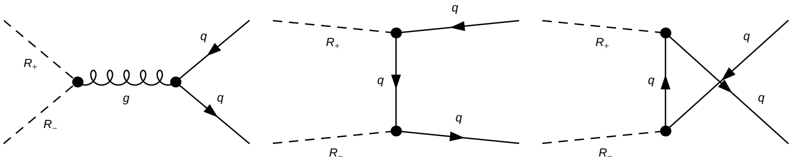
We consider the following LO PRA process:

$$\mathcal{R}_+(q_1) + \mathcal{R}_-(q_2) \to b(q_3)[\to B(p_{TB})] + \bar{b}(q_4)[\to B(p_{T\bar{B}})]$$

$$p_{TB} > p_{TB}^{min} = 15 \text{ GeV}, \ |y_B| < y_B^{max} = 2,$$
$$p_{TL}^{min} = 56, \ 84 \text{ and } 120 \text{ GeV}, \ |y_{\text{jet}}| < y_{\text{jet}}^{\max} = 3$$

Isolation condition for jet:

1) $\Delta R_{34} = \sqrt{\Delta y_{34}^2 + \Delta\phi_{34}^2} > \Delta R_{exp} = 0.5 \Rightarrow p_{TL} = \max(|\mathbf{q}_{T3}|, |\mathbf{q}_{T4}|);$
2) $\Delta R_{34} < \Delta R_{exp} \Rightarrow p_{TL} = |\mathbf{q}_{T3} + \mathbf{q}_{T4}|;$
3) $p_{TL} > p_{TL}^{min}$ and $\max(|\mathbf{q}_{T1}|, |\mathbf{q}_{T2}|) < p_{TL}.$



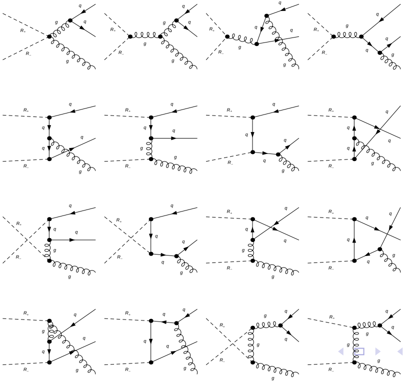
NLO* contribution (gluon jet is leading)

To take into account precise kinematics of hard gluon jet we consider NLO* PRA process:

$$\mathcal{R}_+(q_1) + \mathcal{R}_-(q_2) \to b(q_3)[\to B(p_{TB})] + \bar{b}(q_4)[\to \bar{B}(p_{T\bar{B}})] + g(q_5). \qquad (7)$$

And the corresponding kinematic constraints:

1. For B and $\bar{B}$ mesons $|y_B| < y_B^{\max}$ and $\min(p_{TB}, p_{T\bar{B}}) > p_{TB}^{\min}$.
2. Now the leading jet is the gluon one: $p_{TL} = |\mathbf{q}_{T5}|$, $\max\left(|\mathbf{q}_{T1}|, |\mathbf{q}_{T2}|, |\mathbf{q}_{T3}|, |\mathbf{q}_{T4}|\right) < p_{TL}$ and $p_{TL} > p_{TL}^{\min}$, $|y_5| < y_{\text{jet}}^{\max}$.
3. Isolation condition: $\Delta R_{35} > \Delta R_{\text{exp.}}$ and $\Delta R_{45} > \Delta R_{\text{exp.}}$.



Fragmentation approach

In the fragmentation approach [B. Mele, P. Nason, 1991] the cross section of the inclusive pair production of B-mesons has the following form:

$$\frac{d\sigma_{\mathrm{obs.}}}{dy_B dy_{\bar{B}} d\Delta\phi} = \int\limits_{p_{TB}^{\min}}^{\infty} dp_{TB} \int\limits_{p_{TB}^{\min}}^{\infty} dp_{T\bar{B}} \int\limits_0^1 \frac{dz_1}{z_1} D_{B/b}(z_1,\mu^2) \int\limits_0^1 \frac{dz_2}{z_2} D_{B/b}(z_2,\mu^2) \times \frac{d\sigma_{b\bar{b}}}{dq_{T3}dq_{T4}dy_3dy_4d\Delta\phi}, \quad (8)$$

where $\Delta\phi = \Delta\phi_{34}$, $D_{B/b}(z,\mu^2)$ are the fragmentation functions [B. A. Kniehl, G. Kramer *et. al.*], and $q_{T3} = |\mathbf{q}_{T3}| = p_{TB}/z_1$, $q_{T4} = |\mathbf{q}_{T4}| = p_{T\bar{B}}/z_2$, $y_3 = y_B$, $y_4 = y_{\bar{B}}$.

$\Delta\phi$ cross sections, $\sqrt{S} = 7$ TeV.

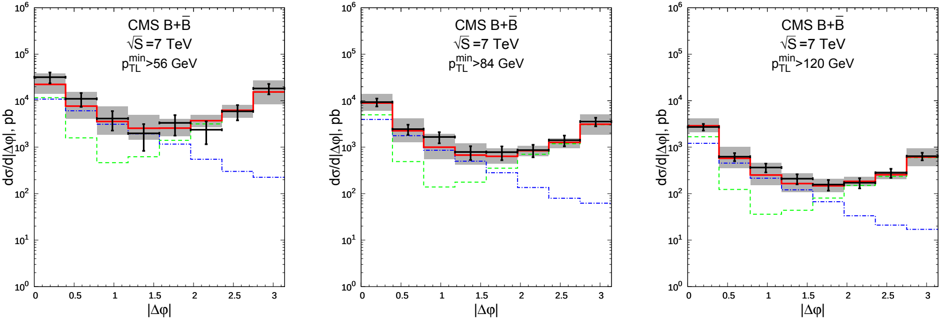


Figure 1: $\Delta\phi$ cross sections of $B\overline{B}$ pair production. Green dashed line is the LO contribution, blue dash-dotted line – NLO* contribution, red solid line is their sum. Shaded bands show theoretical uncertainties. CMS data from the [CMS Collab. V. Khachatryan *et al.*, JHEP **1103**, 136 (2011)].

ΔR cross sections, $\sqrt{S} = 7$ TeV.

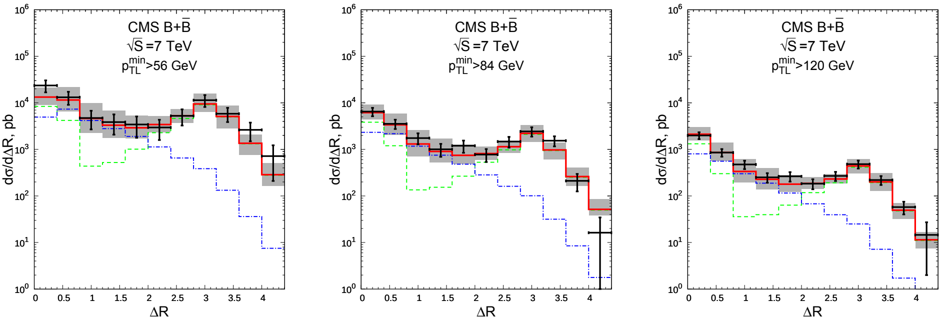


Figure 2: ΔR cross sections of $B\overline{B}$ pair production. Green dashed line is the LO contribution, blue dash-dotted line – NLO* contribution, red solid line is their sum. Shaded bands show theoretical uncertainties. CMS data from the [CMS Collab. V. Khachatryan *et al.*, JHEP **1103**, 136 (2011)].

Conclusions

- In the framework of the PRA we have got a good description of the anglular correlations of $B\bar{B}$ production within uncertainties and without any additional parameters.
- As we could see, careful calculation of kinematics of the associated hard jet in the LO approximation of the PRA consistently merged with the NLO* correction from the emission of additional hard gluon leads us to right description of the experimental data.

Thank you for your attention!