# DIS Structure functions in the NLO of the Parton Reggeization Approach.

M. A. Nefedov[1], V. A. Saleev[1],

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[1]Samara National Research University, Samara, Russia

# Outline.

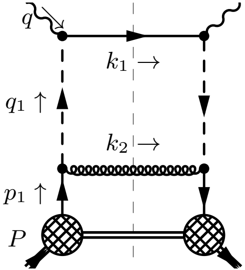
The aim of PRA is to improve description of **multi-scale** observables, sensitive to the emission of additional partons with $\mathbf{k}_T^2 \sim \mu^2$ (angular correlations, polarization observables, $p_T$-spectra of heavy quarkonia, etc.). Corresponding corrections to the **single-scale** observables (DIS, $d^2\sigma/dQ^2dy$ in DY, etc.) are **small**, so the purpose of calculating them in PRA is to ensure the consistency with the usual Collinear Parton Model (CPM) at NLO. This will be the main focus of my talk.

Unfortunately, the calculation is not complete yet. So I will consider only one NLO subprocess, which is fully understood by now, to illustrate the ideas beyond NLO calculations in PRA.

1. LO Framework
2. Real NLO corrections: $\gamma^\star + R \to q + \bar{q}$ subprocess

# DIS in the LO of PRA

## Parton Reggeization Approach at LO




$$Q^2 = -q^2, \;\; x_B = \frac{Q^2}{2qP}, \;\; \tilde{z} = \frac{q_1^+}{p_1^+}$$

$p\gamma^\star$-center-of-mass frame:

$$q^+ = -x_B P^+, \;\; q^- = \frac{Q^2}{x_B P^+}, \;\; \mathbf{q}_T = 0.$$

The mMRK approximation for the amplitude combines two kinematic limits:

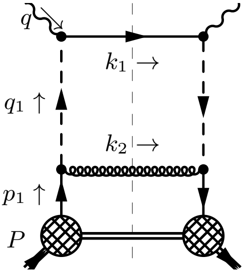
- **Collinear limit:**

$$\mathbf{q}_{T1}^2 \ll Q^2, \;\; 0 < \tilde{z} < 1$$

- **Regge limit:**

$$\mathbf{q}_{T1}^2 \sim Q^2, \;\; \tilde{z} \ll 1$$

# Parton Reggeization Approach at LO




modified-MRK approximation ($\boxed{\tilde{z} = q_1^+/p_1^+}$):

$$\overline{|\mathcal{M}|^2}_{\mathrm{mMRK}} = \frac{2g_s^2}{q_1^2} \frac{P_{qq}(\tilde{z})}{\tilde{z}} \overline{|\mathcal{A}\left(\gamma^\star Q \to q\right)|^2}$$

$\Rightarrow$ Factorization formula ($t_1 = \mathbf{q}_{T1}^2$):

$$F_{L/2}(x_B, Q^2) = \sum_j \int_0^1 \frac{dx_1}{x_1} \int dt_1 \; \Phi_j(x_1, t_1, Q^2) \times C_{2/Lj}^{(LO)}\left(z, \frac{t_1}{Q^2}\right)$$

$$q_1^+ = x_1 P_1^+, \quad \boxed{z = \frac{x_B}{x_1}}$$

# LO Hard-scattering coefficient

$\gamma Q q$-vertex [Fadin, Sherman, 1977]:

$$\Gamma^{\mu}(q_1, k) = \gamma^{\mu} + \hat{q}_1 \frac{n_-^{\mu}}{k^-},$$

$\Rightarrow$ LO Hard-scattering coefficient:

$$\boxed{C_{Lq}^{(LO)} = 0, \quad C_{2q}^{(LO)}\left(z, \frac{t_1}{Q^2}\right) = e_q^2 \cdot z\delta\left(\frac{(Q^2+t_1)z}{Q^2} - 1\right).}$$

Collinear limit $t_1 \to 0$:

$$C_{Lq}^{(LO)} = 0, \quad C_{2q}^{(LO)}\left(z, \frac{t_1}{Q^2}\right) = e_q^2 \delta\left(z-1\right).$$

# LO unintegrated PDF

$$\Phi_i(x,t,\mu^2) \quad = \quad \frac{T_i(t,\mu^2,x)}{t} \times \frac{\alpha_s(t)}{2\pi} \int\limits_x^1 dz \ \theta_z^{\mathrm{cut}} P_{ij}(z) \frac{x}{z} f_j\left(\frac{x}{z},t\right)$$

where: $\theta_z^{\mathrm{cut}} = \theta\left((1-\Delta_{KMR}(t,\mu^2)) - z\right)$, where the Kimber-Martin-Ryskin(KMR) **cut condition** [KMR, 2001]:

$$\Delta_{KMR}(t,\mu^2) = \frac{\sqrt{t}}{\sqrt{\mu^2}+\sqrt{t}},$$

follows from the **rapidity ordering** between the last emission and the hard subprocess.

# LO unintegrated PDF

$$\Phi_i(x,t,\mu^2) = \frac{T_i(t,\mu^2,x)}{t} \times \frac{\alpha_s(t)}{2\pi} \int\limits_x^1 dz\, \theta_z^{\text{cut}} P_{ij}(z) \frac{x}{z} f_j\left(\frac{x}{z},t\right)$$

$$= \boxed{\frac{\partial}{\partial t}\left[T_i(t,\mu^2,x) x f_i(x,t)\right]} \leftarrow \text{derivative form of unPDF}$$

$\Rightarrow$ **LO normalization condition:**

$$\boxed{\int_0^{\mu^2} dt\ \Phi_i(x,t,\mu^2) = x f_i(x,\mu^2)} \leftarrow \textbf{Holds exactly!}$$

Because $T(0,\mu^2,x) = 0$ and $T(\mu^2,\mu^2,x) = 1$.

## Sudakov formfactor

$$T_i(t,\mu^2,x) = \exp\left[-\int\limits_t^{\mu^2} \frac{dt'}{t'}\frac{\alpha_s(t')}{2\pi}\left(\tau_i + \Delta\tau_i\right)\right],$$

$$\tau_i = \sum_j \int\limits_0^1 dz\ \theta_z^{\rm cut} \cdot zP_{ji}(z),$$

$$\Delta\tau_i = \sum_j \int\limits_0^1 dz\ \left(1-\theta_z^{\rm cut}\right)\cdot\left[zP_{ji}(z) - \underbrace{\frac{\frac{x}{z}f_j\left(\frac{x}{z},t'\right)}{xf_i(x,t')}}_{\substack{\text{similar structure in}\\ \text{the non}-\text{emission probability}\\ \text{in ISR PS}}} P_{ij}(z)\cdot\theta(z-x)\right].$$

# Real corrections:

$$\gamma^{\star} + R \to q + \bar{q}.$$

# Physical normalization condition

- **Motivation** for PRA is the **multi-scale** correlational observables. $\Rightarrow$ NLO CPM accuracy for **single-scale** observables is enough.
- We **DO NOT** want to do our own fit of unPDFs $\Rightarrow$ using ($\overline{MS}$) PDFs of CPM as collinear input. But NLO PDFs are **scheme-dependent!**
- For the single-scale observables ($F_{2/L}(x, Q^2)$) collinear factorization **is a theorem** (up to corrections $\sim (\Lambda^2_{QCD}/Q^2)^{\#}$) ($\Rightarrow$ OPE or Feynman diagram analysis [Collins, 2011]).

$\Rightarrow$ **Physical normalization condition** at NLO:

$$f_i^{\overline{MS}}(x, \mu^2)$$

$$\swarrow \qquad\qquad \searrow$$

$$F_{2q}^{(NLO\ \mathbf{PRA})}(x, Q^2) \quad = \quad F_{2q}^{(NLO\ \mathbf{CPM})}(x, Q^2) + O(\alpha_s^2(Q^2)) + \text{Higher twist},$$

# NLO Framework in PRA

Elements of the NLO PRA framework

- Lipatov's effective action $\Rightarrow$ gauge-invariant $\mathbf{q}_T$-dependent HSC
- Physical normalization conditions
- Loop corrections $\supset$ Rapidity divergences $\Rightarrow$ Rapidity renormalization group $\Rightarrow$ resummation of terms $\sim \log(1/z)$

Last two points $\Leftrightarrow$ **NLO** unPDFs and **subtraction scheme** for HSC.

**Basic idea:** Collect all terms in PRA, contributing $O(\alpha_s)$ to the normalization conditions and **match** them to the NLO CPM result.

## Role of the $\mathbf{q}_T$-dependence

$$F_2^{(LO)}(x_B, Q^2) = \int\limits_0^1 \frac{dx_1}{x_1} \int\limits_0^\infty dt_1 \frac{\partial}{\partial t_1} \left[T_j(t_1, Q^2, x_1) \cdot x_1 f_j(x_1, Q^2)\right] \times C_j^{(LO)}\left(\frac{x_B}{x_1}, \frac{t_1}{Q^2}\right),$$

neglecting (power-supressed) $\int\limits_{Q^2}^{\infty} dt_1$ + some identical transformations $\Rightarrow$

$$F_2^{(LO)}(x_B, Q^2) = \underbrace{\int\limits_0^1 dx_1\, f_j(x_1, Q^2) \cdot C_j^{(LO)}\left(\frac{x_B}{x_1}, 0\right)}_{\text{LO CPM}} + \Delta F_2^{(T)}(x_B, Q^2) + \Delta F_2^{(f)}(x_B, Q^2).$$

# Role of the $\mathbf{q}_T$-dependence

$$F_2^{(LO)}(x_B, Q^2) = \underbrace{\int\limits_0^1 dx_1\ f_j(x_1, Q^2) \cdot C_j^{(LO)}\left(\frac{x_B}{x_1}, 0\right)}_{\text{LO CPM}} + \underbrace{\Delta F_2^{(T)}(x_B, Q^2) + \Delta F_2^{(f)}(x_B, Q^2)}_{O(\alpha_s)}.$$

where:

$$\Delta F_2^{(T)}(x_B, Q^2) = \int\limits_0^1 dx_1\ f_j(x_1, Q^2) \times \int\limits_0^{Q^2} dt_1 \underbrace{\left[1 - T_j(t_1, Q^2, x_1)\right]}_{O(\alpha_s)} \frac{\partial}{\partial t_1} C_j^{(LO)}\left(\frac{x_B}{x_1}, \frac{t_1}{Q^2}\right)$$

# Role of the $\mathbf{q}_T$-dependence

$$F_2^{(LO)}(x_B, Q^2) = \underbrace{\int\limits_0^1 dx_1\ f_j(x_1, Q^2) \cdot C_j^{(LO)}\left(\frac{x_B}{x_1}, 0\right)}_{\text{LO CPM}} + \underbrace{\Delta F_2^{(T)}(x_B, Q^2) + \Delta F_2^{(f)}(x_B, Q^2)}_{O(\alpha_s)}.$$

where:

$$\Delta F_2^{(f)}(x_B, Q^2) = \int\limits_0^1 dx_1 \int\limits_0^{Q^2} dt_1\ T_j(t_1, Q^2, x_1) \times \underbrace{\left[f_j(x_1, Q^2) - f_j(x_1, t_1)\right]}_{O(\alpha_s)} \frac{\partial}{\partial t_1} \bar{C}_j^{(LO)}\left(\frac{x_B}{x_1}, \frac{t_1}{Q^2}\right)$$

## $O(\alpha_s)$-term from $\mathbf{q}_T$-dependence

$$\Delta F_2^{(f)}(x_B, Q^2) = \int\limits_0^1 dx_1 \int\limits_0^{Q^2} dt_1\; T_j(t_1, Q^2, x_1) \times \underbrace{\left[f_j(x_1, Q^2) - f_j(x_1, t_1)\right]}_{O(\alpha_s)} \frac{\partial}{\partial t_1} \bar{C}_j^{(LO)}\left(\frac{x_B}{x_1}, \frac{t_1}{Q^2}\right)$$

plug-in: $C_q^{(LO)}(z, t/Q^2) = e_q^2 \cdot z\delta\left(\frac{(Q^2+t)z}{Q^2} - 1\right)$,

$$f_j(x_1, Q^2) - f_j(x_1, t_1) = -\underbrace{\frac{\partial f_j(x_1, Q^2)}{\partial \log Q^2}}_{\text{DGLAP eqns.}} \log\frac{t_1}{Q^2} + O\left(\log^2\frac{t_1}{Q^2}\right),$$

and LLA expression for the Sudakov formfactor:

$$T_q(t, \mu^2) = \exp\left[-\frac{C_F}{2}\frac{\alpha_s(\mu^2)}{2\pi}\log^2\frac{t}{\mu^2}\right]$$

## $O(\alpha_s)$-term from $\mathbf{q}_T$-dependence

One obtains:

$$\Delta F_2^{(f_g)} = \left(\frac{\alpha_s(Q^2)}{2\pi}\right) \int\limits_0^1 dx_1\ f_g(x_1, Q^2) \cdot \Delta C_{qg}^{(f_g)}\left(\frac{x_B}{x_1}\right)$$

where:

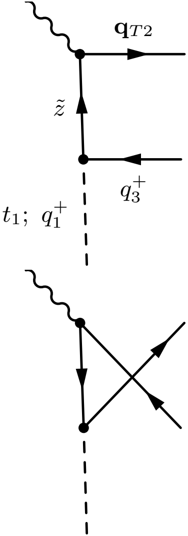
$$\Delta C_{qg}^{(f_g)}(z) = T_F z\left[\xi z\left((4+\xi)z - 2\right) + \left(1 - 2z + 2z^2\right)\log\xi\right] + \underbrace{O(\alpha_s)}_{\text{depends on SFF}}$$

where $T_F = 1/2$ and $\xi = \min\left(1, (1-z)/z\right)$. Other corrections:

$$\Delta F_2^{(f_q)},\ \Delta F_2^{(T)}$$

contain only $f_q(x_1, Q^2)$, and will be important for the $\gamma^\star + Q \to q + g$ subprocess and one-loop correction.

# NLO subprocess $\gamma^\star + R \to q + \bar{q}$.




Kinematics ($\tilde{z} = (q_1^+ - q_3^+)/q_1^+$, $\psi$ – azimuthal angle of $\mathbf{k}_T$):

$$\mathbf{q}_{T2} = \mathbf{k}_T + \mathbf{q}_{T1} \frac{\tilde{z} - z}{1 - z},$$

$$\mathbf{k}_T^2 = \frac{(\tilde{z} - z)(1 - \tilde{z})}{1 - z} \left[ \frac{Q^2}{z} - \frac{t_1}{1 - z} \right],$$

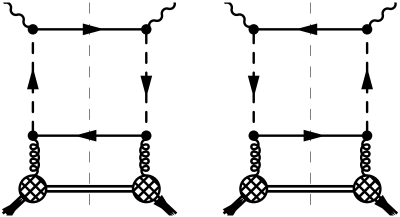
$$F_2^{(NLO,g)} = e_q^2 \cdot \frac{\alpha_s}{2\pi} \int \frac{dx_1}{x_1} \int dt_1\ \Phi_g(x_1, t_1, \mu^2) \times \underbrace{\int\limits_z^1 \frac{z \cdot d\tilde{z}}{(1 - z)} \int\limits_0^{2\pi} \frac{d\psi}{2\pi}\, \mathcal{C}_2^{(NLO,g)}}_{C_2^{(NLO,g)}\left(z = \frac{x_B}{x_1}, \frac{t_1}{Q^2}\right)}$$

# NLO subprocess $\gamma^\star + R \to q + \bar{q}$.

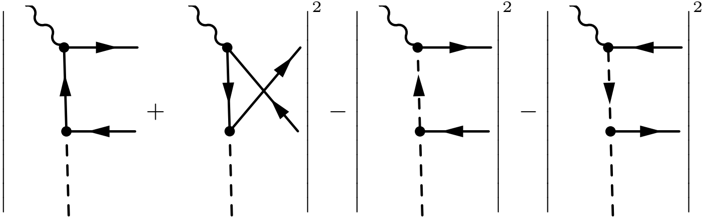
$$
\begin{aligned}
\mathcal{C}_2^{(NLO,g)} &= \frac{T_F}{t_1\left(Q^2(\tilde{z}-1)(z-\tilde{z})+\mathbf{q}_{T2}^2 z^2\right)^2\left(Q^2\tilde{z}(z-\tilde{z})+\mathbf{q}_{T2}^2 z^2\right)^2} \\
&\times \left\{Q^{10}(\tilde{z}-1)^2(z-\tilde{z})^4+Q^8(\tilde{z}-1)(z-\tilde{z})^3(2\mathbf{q}_{T2}^2 z(z+3\tilde{z}-4)+t_1\tilde{z}(z-\tilde{z}))\right. \\
&+ Q^6\mathbf{q}_{T2}^2 z(z-\tilde{z})^2\left(\mathbf{q}_{T2}^2 z\left(z^2+2z(6\tilde{z}-7)+6(\tilde{z}-4)\tilde{z}+19\right)+t_1(z-\tilde{z})(z(2\tilde{z}-1)+2(\tilde{z}-1)\tilde{z})\right) \\
&+ Q^4\mathbf{q}_{T2}^4 z^2(z-\tilde{z})\left(6\mathbf{q}_{T2}^2(z-1)z(z+2\tilde{z}-3)+t_1(z-\tilde{z})\left(z^2+z(4\tilde{z}-2)+2(\tilde{z}-1)\tilde{z}\right)\right) \\
&+ \left.2Q^2\mathbf{q}_{T2}^6 z^4\left(3\mathbf{q}_{T2}^2(z-1)^2+t_1(z-\tilde{z})(z+2\tilde{z}-1)\right)+2\mathbf{q}_{T2}^8 t_1 z^6\right\}
\end{aligned}
$$

# Double-counting subtraction

LO contains:



$\Rightarrow$



## Collinear limit

In the CL, **both** NLO contribution and subtraction terms contain **collinear divergences.**

- Collinear limit for $C_2^{(NLO,g)}$ ($g+\gamma^\star \to q+\bar{q}$, **textbook result!**):

$$C_2^{(NLO,g)}\Big|_{CL} = \frac{2T_F}{1-\epsilon}\left(\frac{\bar{\alpha}_s}{2\pi}\right)\left\{\left(\frac{\mu^2}{Q^2}\right)^\epsilon \left(\frac{z}{1-z}\right)^\epsilon \right.$$
$$\left. \times \left(-\frac{1}{\epsilon}\right)\left[z^2+(1-z)^2-\epsilon\right]+\left[6z(1-z)-1\right]+O(\epsilon)\right\}.$$

- Collinear limit for the **subtraction term**:

$$\Delta C_2^{(t+u,g)}\Big|_{CL} = \frac{2T_F}{(1-\epsilon)^2(2-\epsilon)}\left(\frac{\bar{\alpha}_s}{2\pi}\right)\left(\frac{\mu^2}{Q^2}\right)^\epsilon\left(\frac{2z}{1-z}\right)^\epsilon$$
$$\times\left(-\frac{1}{\epsilon}\right)\left[2\left(z^2+(1-z)^2\right)-\frac{\epsilon}{2}(5z^2-2z+7)+\frac{\epsilon^2}{2}(z^2+7)-\epsilon^3\right],$$

where $\bar{\alpha}_s = \alpha_s(4\pi)^\epsilon/\Gamma(1-\epsilon)$ – $\overline{MS}$-coupling. Collinear divergence cancels:

$$\boxed{C_2^{(NLO,g)}\Big|_{CL} - \Delta C_2^{(t+u,g)}\Big|_{CL} = \text{finite}}$$

## Physical normalization in the CL

Physical normalization condition in the CL ($O(\alpha_s)$ gluon contribution only):

$$F_{2q}^{NLO\ \textbf{CPM}}(x_B,Q^2) = e_q^2\left[x_B f_q^{\overline{MS}}(x_B,Q^2) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right) f_g^{\overline{MS}} \otimes C_g^{\overline{MS}} + \text{c.c.}\right]$$

$$\|$$

$$F_{2q}^{NLO\ \textbf{PRA}}(x_B,Q^2)\Big|_{CL} = e_q^2\Bigg[x_B f_q^{PRA}(x_B,Q^2) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right) f_g^{PRA} \otimes \left(C_2^{(NLO,g)} - \Delta C_2^{(t+u,g)}\right)_{CL} + \text{c.c.}\Bigg].$$

PDFs in the **PRA scheme**:

$$f_g^{PRA} = f_g^{\overline{MS}} + O(\alpha_s^2),\quad f_q^{PRA} = f_q^{\overline{MS}} + \left(\frac{\alpha_s}{2\pi}\right) f_g^{\overline{MS}} \otimes \Delta C_{qg} + O(\alpha_s^2),$$

**Matching coefficient:**

$$\Delta C_{qg}(z) = T_F\left\{\left[z^2 + (1-z)^2\right]\log\left(\frac{1-z}{2z}\right) - \frac{3}{4}(1-z)(1-5z)\right\}.$$

## unPDFs in the PRA scheme

We want our unPDFs to satisfy:

$$\int_0^{\mu^2} dt\ \Phi_i^{PRA}(x,t,\mu^2) = x f_i^{PRA}(x,\mu^2),$$

to this end, one have to introduce $O(\alpha_s)$ correction to the splittings in the definition of unPDFs (**PRA scheme is just a part of definition of NLO unPDFs!**):

$$\Delta P_{ij}(z) = \left(\frac{\alpha_s}{2\pi}\right)\left\{-\left(\frac{\beta_0}{2}+\Delta p_{ij}(z)\right)\Delta C_{ij}(z) + \int_z^1 \frac{dz_1}{z_1}\left[\Delta C_{ik}\left(\frac{z}{z_1}\right)P_{kj}^{(0,+)}(z_1) - P_{ik}^{(0,+)}(z_1)\Delta C_{kj}\left(\frac{z}{z_1}\right)\right]\right\},$$

where $\beta_0 = 11C_A/3 - 4n_F T_F/3$, $P_{ij}^{(0,+)}$ – LO DGLAP splittings with $1/(1-z)_+$-prescription, and($p_i = \lim_{z\to 1}(1-z)P_{ii}^{(0)}(z)$):

$$\Delta p_{ij}(z) = (p_i - p_j)\log(1-z) + \sum_k \int_0^1 dz_1\ z_1\left[P_{kj}^{(0,+)}(z_1) - P_{ki}^{(0,+)}(z_1)\right],$$

## Subtraction term in $k_T$-factorization

**$\hat{t}$-channel subtraction term:**

$$\Delta C_{2t}^{(PRA,g)}\left(z, \frac{t_1}{Q^2}\right) = \frac{\alpha_s}{2\pi}\int\limits_z^1 \frac{z\cdot d\tilde{z}}{1-\tilde{z}} \int\limits_0^{2\pi}\frac{d\phi_2}{2\pi}\cdot \Delta\mathcal{C}_{2t}^{(PRA,g)}\cdot \underbrace{\theta\left(\tilde{z}_{\max}-\tilde{z}\right)}_{\text{rapidity ordering}},$$

where $\tilde{z}_{\max} = (1+zY)/(1+Y)$, $Y = |\mathbf{q}_{T1}-\mathbf{q}_{T2}|/|\mathbf{q}_{T2}|$,

$$\Delta\mathcal{C}_{2t}^{(PRA,g)} = \frac{Q^2}{(-\hat{t})}\frac{P_{QR}(\tilde{z},\mathbf{q}_{T1},\mathbf{q}_{T2})}{z},$$

$$\hat{t} = -\frac{\mathbf{q}_{T2}^2}{1-\tilde{z}} - \frac{\tilde{z}}{1-\tilde{z}}\left(\mathbf{q}_{T1}^2 + 2\mathbf{q}_{T1}\mathbf{q}_{T2}\right), \quad \mathbf{q}_{T2}^2 = Q^2\,\frac{\tilde{z}-z}{z}.$$

TMD splitting function, not averaged over $\phi_2$ (comp. with [F. Hautmann, *et. al.*, 2012]):

$$P_{QR}(z,\mathbf{q}_{T1},\mathbf{q}_{T2}) = T_F\,\frac{z^2(\mathbf{q}_{T1}^2-2\mathbf{q}_{T1}\mathbf{q}_{T2})^2 + 2z(\mathbf{q}_{T1}\mathbf{q}_{T2})(\mathbf{q}_{T1}^2-2\mathbf{q}_{T1}\mathbf{q}_{T2}) + \mathbf{q}_{T1}^2\mathbf{q}_{T2}^2}{\mathbf{q}_{T1}^2\left(\mathbf{q}_{T2}^2 + z(\mathbf{q}_{T1}^2-2\mathbf{q}_{T1}\mathbf{q}_{T2})\right)}.$$

**$\hat{u}$-channel subtraction term is obtained via:**

$$\tilde{z}\to 1-(\tilde{z}-z).$$

## Taking $O(\alpha_s)$-contributions together.

We will compare:

$$F_{2q}^{NLO\ \mathbf{CPM}}(x_B, Q^2) = e_q^2 \left[ x_B f_q^{\overline{MS}}(x_B, Q^2) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right) f_g^{\overline{MS}} \otimes C_g^{\overline{MS}} + \text{c.c.} \right],$$

with:

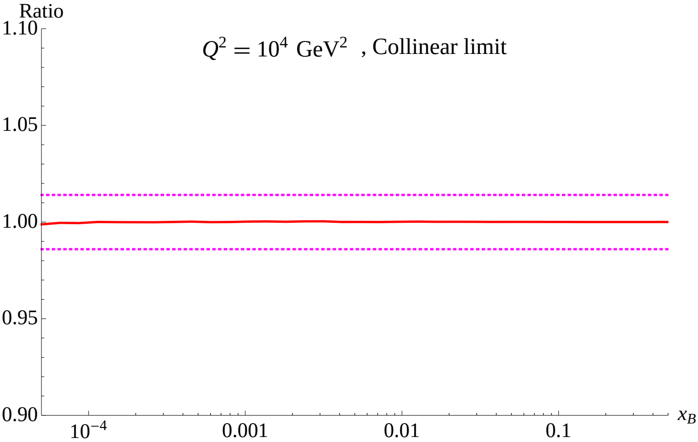
$$\downarrow \text{LO CL} \qquad \downarrow \text{ from } \mathbf{q}_{T1} \text{ dependence of LO HSC}$$

$$\begin{aligned} F_{2q(\text{test})}^{NLO\ \mathbf{PRA}}(x_B, Q^2) &= e_q^2 \left\{ x_B f_q^{PRA}(x_B, Q^2) + \left(\frac{\alpha_s}{2\pi}\right) x_B f_g^{PRA}(x_B, Q^2) \otimes \Delta C_{qg}^{(f)} \right. \\ &+ \left. \left(\frac{\alpha_s}{2\pi}\right) \Phi_g^{PRA} \otimes \left[ C_2^{(NLO,g)} - \Delta C_2^{(t+u,g)} \right] + \text{c.c.} \right\} \end{aligned}$$

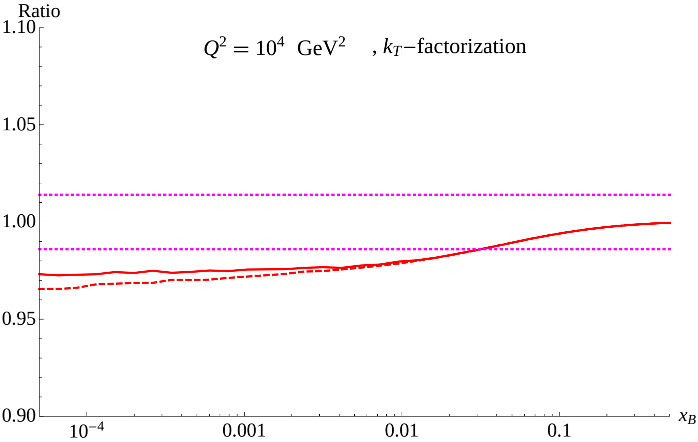
$$\uparrow \text{NLO term in } \mathbf{k}_T\text{-factorization}$$

Equality should hold up to:

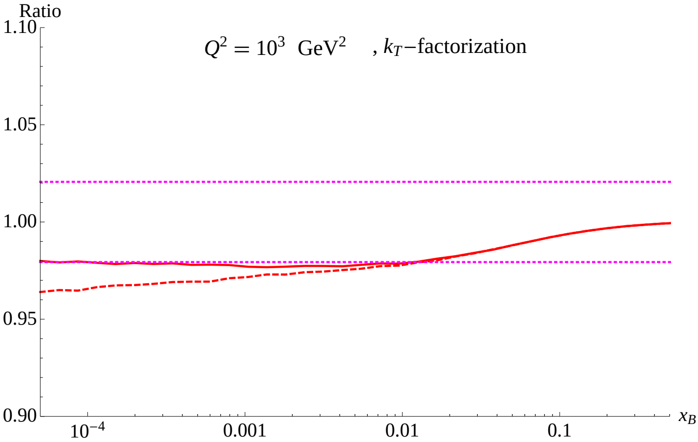
- $O(\alpha_s^2)$-corrections from $\mathbf{q}_{T1}$-dependence of NLO term
- Higher-twist corrections from $\mathbf{q}_{T1}^2 > Q^2$.




Magenta lines $= 1 \pm \alpha_s^2(Q^2)$




Magenta lines $= 1 \pm \alpha_s^2(Q^2)$, dashed curve – no $t_1 > Q^2$.

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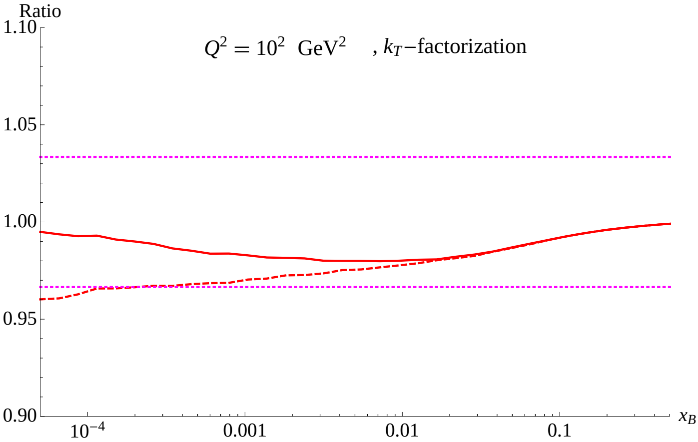
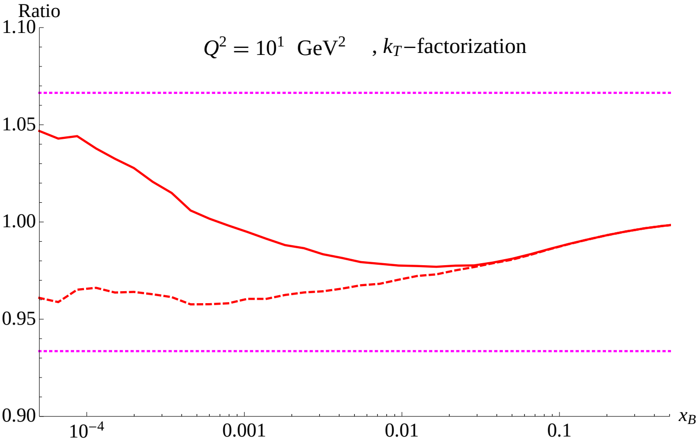
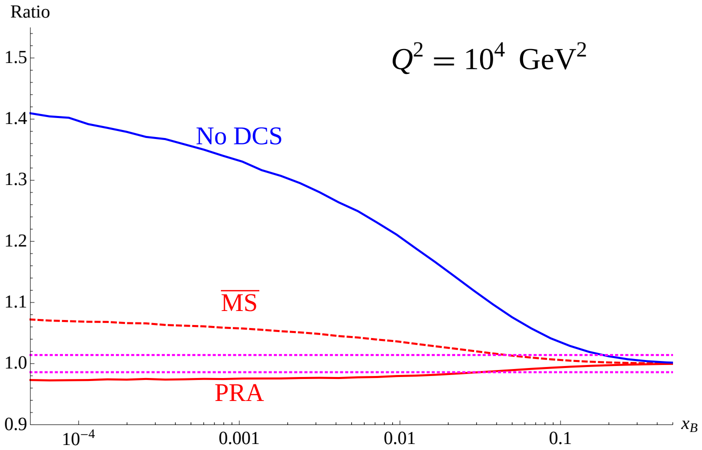



Magenta lines $= 1 \pm \alpha_s^2(Q^2)$, dashed curve – no $t_1 > Q^2$.




Magenta lines $= 1 \pm \alpha_s^2(Q^2)$, dashed curve – no $t_1 > Q^2$.

## Role of different contributions



# Conclusions

- We have identified all $O(\alpha_s)$ contributions to $F_{2q}$ in the NLO of PRA, relevant for the $\gamma^\star + R \to q + \bar{q}$-subprocess
- We successfully matched them to the NLO CPM result
- Significant part of the framework of NLO calculations in PRA is constructed. It will be applied to the **multiscale** processes (polarization observables in Drell-Yan, angular decorrelations of jets and vector bosons), where NLO PRA will be competetive with NNLO of CPM or NLO+MC calculations.

**Thank you for your attention!**

# Backup slides

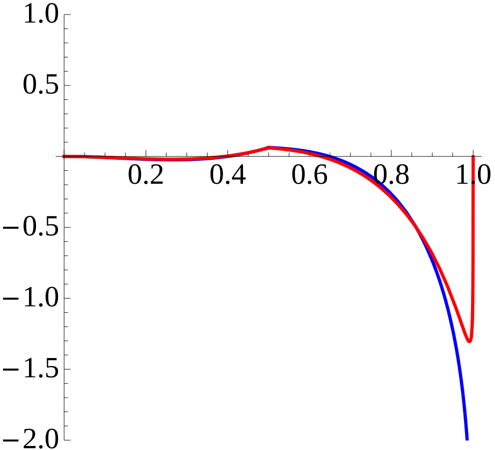
# Full expression for $\Delta C_{qg}^{(f_g)}$

The following expression has been used in the numerical calculations:

$$\begin{aligned}
\Delta C_{qg}^{(f_g)} &= \frac{T_F z}{2\kappa^{3/2}} \left\{ z\sqrt{\pi} \left[ -(1-2z)\mathrm{erfc}\left( \frac{1-2\kappa\log\xi}{2\sqrt{\kappa}} \right) e^{\frac{1}{4\kappa}} \right.\right. \\
&+ \left. 4z\ \mathrm{erfc}\left( \frac{1-\kappa\log\xi}{\sqrt{\kappa}} \right) e^{\frac{1}{\kappa}} \right] \\
&+ 2\sqrt{\kappa} e^{-\kappa\log^2\xi} \left[ z\xi\left(1-2z(1+\xi)\right) \right. \\
&- \left.\left. \kappa\left(1-2z(1+\xi)(1-z(1+\xi))\right)\log\xi \right] \right\},
\end{aligned}$$

where $\kappa = C_F \alpha_s(Q^2)/(2\pi)$, and $\xi = \min(1, (1-z)/z)$.
Besides it's suspicious structure, the usual perturbative expansion of this expression in positive powers of $\alpha_s$ **exists**. The first term of it's expansion is presented on a slide # 17.

## The plot of $\Delta C_{qg}^{(f_g)}(z)$




Blue curve – LO term, red curve – full expression.