Hadronic input and observables in $B_{(s)} \to \pi(K)\ell^+\ell^-$ decays at large recoil

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Motivation

- Recently, the ratio of the CKM matrix elements $|V_{td}/V_{ts}|$ has been extracted from the measured by LHCb collaboration $B \to \pi\ell^+\ell^-$ and $B \to K\ell^+\ell^-$ partial decay widths [LHCb, JHEP10(2015)034]
- Actually, the ratio of the $B \to \pi\ell^+\ell^-$ and $B \to K\ell^+\ell^-$ partial decay widths has more complicated dependence on the CKM matrix elements
- We suggest a new way to constrain the Wolfenstein parameters of the CKM matrix from the data on the observables in the flavour changing neutral current (FCNC) decays $B \to \pi\ell^+\ell^-$, $B \to K\ell^+\ell^-$ and $B_s \to K\ell^+\ell^-$

$\to$ One needs to calculate accurately the hadronic input:

- ⋆ Form factors
- ⋆ Hadronic amplitudes of nonlocal effects

Effective Hamiltonian for $B \to P\ell^+\ell^-$ decays

Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{b\to q} = \frac{4G_F}{\sqrt{2}}\left(\lambda_u^{(q)}\sum_{i=1}^{2} C_i\,\mathcal{O}_i^u + \lambda_c^{(q)}\sum_{i=1}^{2} C_i\,\mathcal{O}_i^c - \lambda_t^{(q)}\sum_{i=3}^{10} C_i\,\mathcal{O}_i\right) + h.c.$$

$$\lambda_p^{(q)} = V_{pb}V_{pq}^*, \quad p = u, c, t, \quad q = d, s$$

- $B \to K\ell^+\ell^-$: $\lambda_t^{(s)} \approx -\lambda_c^{(s)} \sim \lambda^2 \gg \lambda_u^{(s)} \sim \lambda^4$
- $B \to \pi\ell^+\ell^-$: $\lambda_t^{(d)} \sim \lambda_c^{(d)} \sim \lambda_u^{(d)} \sim \lambda^3$
- $B_s \to K\ell^+\ell^-$: $\lambda_t^{(d)} \sim \lambda_c^{(d)} \sim \lambda_u^{(d)} \sim \lambda^3$

Operators basis

$$\mathcal{O}_9 = \frac{\alpha_{\rm em}}{4\pi}\left(\bar{d}_L\gamma^\mu b_L\right)\left(\bar{\ell}\gamma_\mu\ell\right), \quad \mathcal{O}_{10} = \frac{\alpha_{\rm em}}{4\pi}\left(\bar{d}_L\gamma^\mu b_L\right)\left(\bar{\ell}\gamma_\mu\gamma_5\ell\right),$$

$$\mathcal{O}_{7\gamma} = -\frac{e\, m_b}{16\pi^2}\left(\bar{d}_L\sigma^{\mu\nu} b_R\right) F_{\mu\nu},$$

$$\mathcal{O}_1^u = \left(\bar{d}_L\gamma_\mu u_L\right)\left(\bar{u}_L\gamma^\mu b_L\right), \quad \mathcal{O}_2^u = \left(\bar{d}_L^i\gamma_\mu u_L^j\right)\left(\bar{u}_L^j\gamma^\mu b_L^i\right),$$

$$\mathcal{O}_1^c = \left(\bar{d}_L\gamma_\mu c_L\right)\left(\bar{c}_L\gamma^\mu b_L\right), \quad \mathcal{O}_2^c = \left(\bar{d}_L^i\gamma_\mu c_L^j\right)\left(\bar{c}_L^j\gamma^\mu b_L^i\right),$$

$$\mathcal{O}_3 = \left(\bar{d}_L\gamma_\mu b_L\right)\sum_q\left(\bar{q}_L\gamma^\mu q_L\right), \quad \mathcal{O}_4 = \left(\bar{d}_L^i\gamma_\mu b_L^j\right)\sum_q\left(\bar{q}_L^j\gamma^\mu q_L^i\right),$$

$$\mathcal{O}_5 = \left(\bar{d}_L\gamma_\mu b_L\right)\sum_q\left(\bar{q}_R\gamma^\mu q_R\right), \quad \mathcal{O}_6 = \left(\bar{d}_L^i\gamma_\mu b_L^j\right)\sum_q\left(\bar{q}_R^j\gamma^\mu q_R^i\right),$$

$$\mathcal{O}_{8g} = -\frac{g_s m_b}{16\pi^2}\left(\bar{d}_L^i\sigma_{\mu\nu}(T^a)^{ij} b_R^j\right) G^{a\,\mu\nu}$$

Hadronic input

Form Factors

$$\langle P(p)|\bar{q}\gamma^\mu b|B(p+q)\rangle = f_{BP}^{+}(q^2)\,(2p^\mu + q^\mu) + \left(f_{BP}^{+}(q^2) - f_{BP}^{0}(q^2)\right)\frac{m_B^2 - m_P^2}{q^2}q^\mu$$

$$\langle P(p)|\bar{q}\sigma^{\mu\nu}q_\nu b|B(p+q)\rangle = \frac{i f_{BP}^{T}(q^2)}{m_B + m_P}\left[2q^2 p^\mu + \left(q^2 - \left(m_B^2 - m_P^2\right)\right)q^\mu\right]$$

Nonlocal effects via correlation functions

$$\mathcal{H}^{(p)}_{BP,\,\mu} = i\int d^4x\, e^{iqx}\langle P(p)|\mathrm{T}\Big\{j^{\mathrm{em}}_\mu(x), \Big[C_1\mathcal{O}_1^p(0) + C_2\mathcal{O}_2^p(0)$$
$$+\sum_{k=3-6,8g} C_k\mathcal{O}_k(0)\Big]\Big\}|B(p+q)\rangle = \left[(p\cdot q)q_\mu - q^2 p_\mu\right]\mathcal{H}^{(p)}_{BP}(q^2)$$

Amplitude

$$A(B \to P\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{\rm em}}{\pi} \lambda_t^{(q)} f_{BP}^+(q^2) \Bigg[\left(\bar{\ell}\gamma^\mu \ell\right) p_\mu \left(C_9 + \frac{2(m_b + m_q)}{m_B + m_P} C_7^{\rm eff} \frac{f_{BP}^T(q^2)}{f_{BP}^+(q^2)} \right)$$

$$+ \left(\bar{\ell}\gamma^\mu \gamma_5 \ell\right) p_\mu C_{10} - (\bar{\ell}\gamma^\mu \ell) p_\mu \frac{16\pi^2}{f_{BP}^+(q^2)} \left(\frac{\lambda_u^{(q)}}{\lambda_t^{(q)}} \mathcal{H}_{BP}^{(u)}(q^2) + \frac{\lambda_c^{(q)}}{\lambda_t^{(q)}} \mathcal{H}_{BP}^{(c)}(q^2) \right) \Bigg]$$

$f_{BP}^+(q^2)$, $f_{BP}^T(q^2)$ — $B \to P$ transition form factors

$\mathcal{H}_{BP}^{(u,c)}(q^2)$ — nonlocal hadronic amplitudes

- $B \to K\ell^+\ell^-$: only one amplitude $\mathcal{H}_{BK}^{(c)}(q^2)$ ($\lambda_u^{(s)}$ neglected)
- $B \to \pi\ell^+\ell^-$: two amplitudes $\mathcal{H}_{B\pi}^{(u)}(q^2)$ and $\mathcal{H}_{B\pi}^{(c)}(q^2)$
- $B_s \to K\ell^+\ell^-$: two amplitudes $\mathcal{H}_{B_sK}^{(u)}(q^2)$ and $\mathcal{H}_{B_sK}^{(c)}(q^2)$

Nonlocal hadronic amplitudes

- Nonlocal hadronic amplitudes are calculated combining QCD factorization and light-cone sum rules (LCSR) with the hadronic dispersion relations

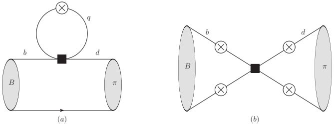
- Use the results:
  - $\mathcal{H}^{(c)}_{BK}(q^2)$ for $B \to K\ell^+\ell^-$:
    [A. Khodjamirian, Th. Mannel, Y.M. Wang (2013)]

  - $\mathcal{H}^{(c)}_{B\pi}(q^2)$ and $\mathcal{H}^{(u)}_{B\pi}(q^2)$ for $B \to \pi\ell^+\ell^-$:
    [Ch. Hambrock, A. Khodjamirian, AR (2015)]

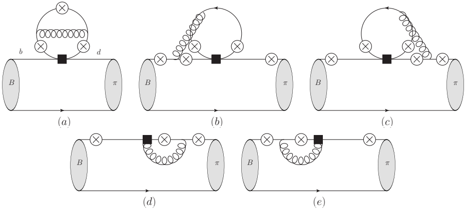
  - [New] $\mathcal{H}^{(c)}_{B_sK}(q^2)$ and $\mathcal{H}^{(u)}_{B_sK}(q^2)$ for $B_s \to K\ell^+\ell^-$

Contributions to $\mathcal{H}_{BP}^{(p)}(q^2)$

- LO, factorizable and weak annihilation
  [M. Beneke, Th. Feldmann, D. Seidel (2001)]

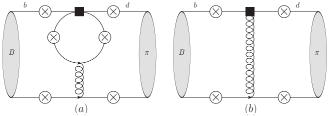


- NLO, factorizable
  [H.H.Asatryan, H.M. Asatrian, C. Greub, M. Walker (2002); H.M.Asatrian, K. Bieri, C. Greub, M. Walker (2004)]

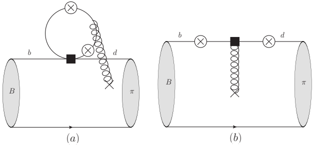


Contributions to $\mathcal{H}_{BP}^{(p)}(q^2)$

- NLO, nonfactorizable (hard gluons)
  [M. Beneke, Th. Feldmann, D. Seidel (2001)]



- Soft gluons, nonfactorizable
  [A. Khodjamirian, Th. Mannel, A.A. Pivovarov, Y.-M. Wang (2010)]
  [A. Khodjamirian, Th. Mannel, Y.-M. Wang (2013)]



Dispersion relations for nonlocal hadronic amplitudes

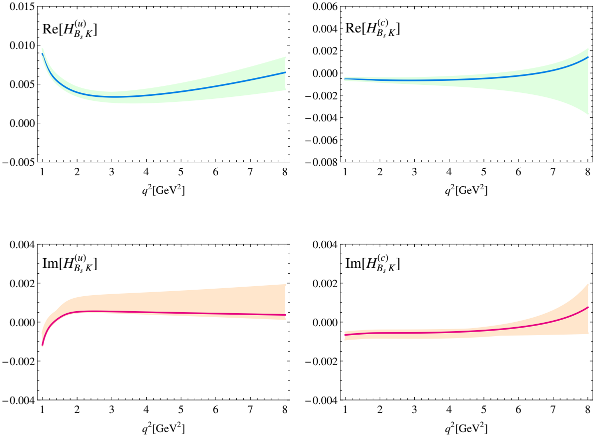
Dispersion relations (analytic continuation of $\mathcal{H}^{(u,c)}(q^2)$ to $q^2 > 0$):

$$\mathcal{H}_{BP}^{(u,c)}(q^2) = (q^2 - q_0^2)\left[\sum_{V=\rho,\omega,J/\psi,\psi(2S)} \frac{k_V f_V A_{BVP}^{u,c}}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{\rm tot})} + \int_{s_0^{u,c}}^{\infty} ds \frac{\rho^{(u,c)}(s)}{(s - q_0^2)(s - q^2 - i\epsilon)}\right] + \mathcal{H}_{BP}^{(u,c)}(q_0^2)$$

- $A_{BVP}^{u,c} = |A_{BVP}^{u,c}| e^{i\delta_{BVP}^{u,c}}$
- $|A_{BVP}^{u,c}|$ are extracted from nonleptonic $B \to V\pi$ decays
- $\delta_{BVP}^{u,c}$ are extracted from the fit of the dispersion relation to $\mathcal{H}_{BP}^{(u,c)}(q^2)$ at $q^2 < 0$
- For $\rho^{(u,c)}(s)$ we apply quark-hadron duality

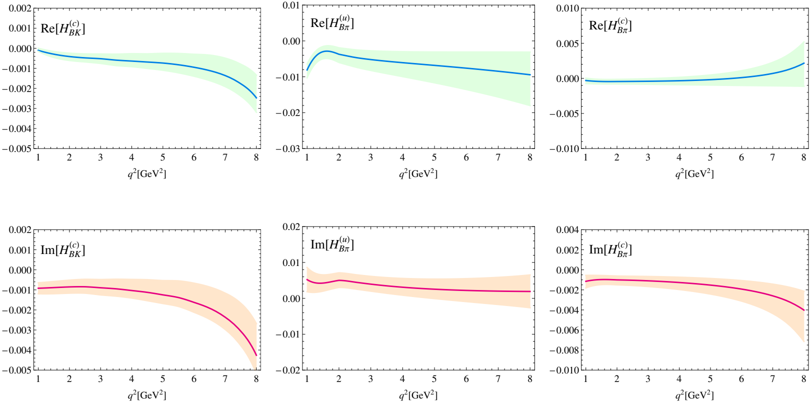
Nonlocal hadronic amplitudes

Results for $\mathcal{H}^{(u)}_{B_s K}, \mathcal{H}^{(c)}_{B_s K}$



Nonlocal hadronic amplitudes

Results for $\mathcal{H}^{(c)}_{BK}, \mathcal{H}^{(u)}_{B\pi}, \mathcal{H}^{(c)}_{B\pi}$



Form factors from LCSR

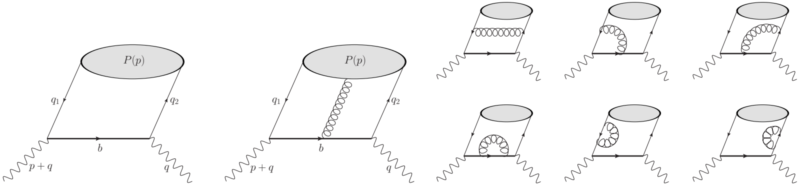
Underlying correlation functions:

$$F^{\mu}_{BP}(p,q) = i\int d^4x\, e^{iqx}\langle P(p)|\,T\{\bar{q}_1(x)\Gamma^{\mu}b(x), m_b\bar{b}(0)i\gamma_5 q_2(0)\}|0\rangle$$

$$= \begin{cases} F_{BP}(q^2,(p+q)^2)p^{\mu} + \tilde{F}_{BP}(q^2,(p+q)^2)q^{\mu}, & \Gamma^{\mu} = \gamma^{\mu} \\ F^{T}_{BP}(q^2,(p+q)^2)\left[q^2p^{\mu} - (q\cdot p)q^{\mu}\right], & \Gamma^{\mu} = -i\sigma^{\mu\nu}q_{\nu} \end{cases}$$

$$B^+ \to K^+: \quad q_1 = u,\ q_2 = s, \qquad B^+ \to \pi^+: \quad q_1 = u,\ q_2 = d$$

$$B^0_s \to K^+: \quad q_1 = s,\ q_2 = u, \qquad B^0_s \to \bar{K}^0: \quad q_1 = s,\ q_2 = d$$



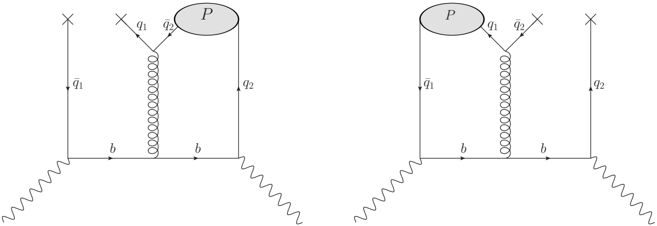
The current accuracy of the OPE result

$$\{F^{(T)}_{B\pi}(q^2), F^{(T)}_{BK}(q^2), F^{(T)}_{B_sK}(q^2)\} \Rightarrow \mathrm{OPE} \Rightarrow \{f^{+(T)}_{B\pi}(q^2), f^{+(T)}_{BK}(q^2), f^{+(T)}_{B_sK}(q^2)\}$$

$$\mathrm{OPE} = \left(T_0^{(2)} + (\alpha_s/\pi)T_1^{(2)} + (\alpha_s/\pi)^2 T_2^{(2)}\right) \otimes \varphi_P^{(2)}$$
$$+\frac{\mu_P}{m_b}\left(T_0^{(3)} + (\alpha_s/\pi)T_1^{(3)}\right) \otimes \varphi_P^{(3)} + \frac{\delta_\pi^2}{m_b\chi} T_0^{(4)} \otimes \varphi_P^{(4)}$$
$$+\langle\bar{q}q\rangle\left(T_0^{(5)} \otimes \varphi_P^{(2)} + T_0^{(6)} \otimes \varphi_P^{(3)}\right)$$

- LO twist 2, 3, 4 $q\bar{q}$ and $\bar{q}qG$ terms
  [V.Belyaev, A.Khodjamirian, R.Rückl (1993); V.Braun, V.Belyaev, A.Khodjamirian, R.Rückl (1996)]
- NLO $O(\alpha_s)$ twist 2 (collinear factorization)
  [A.Khodjamirian, R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997)]
- NLO $O(\alpha_s)$ twist 3 (coll. factorization for asympt. DA)
  [P. Ball, R. Zwicky (2001); G.Duplancic, A.Khodjamirian, B.Melic, Th.Mannel, N.Offen (2007)]
- Part of NNLO $O(\alpha_s^2\beta_0)$ twist 2 [A. Bharucha (2012)]
- [New] LO twist 5 and twist 6 in factorization approximation
  [AR (2017)]

Higher twist effects in $B \to P$ form factors



- In the framework of the factorization approximation

$$F_{BP}^{\mathrm{tw5,6}}(q^2) \sim \langle \bar{q}q \rangle \left(T_0^{(5)} \otimes \varphi_P^{(2)} + T_0^{(6)} \otimes \varphi_P^{(3)} \right)$$

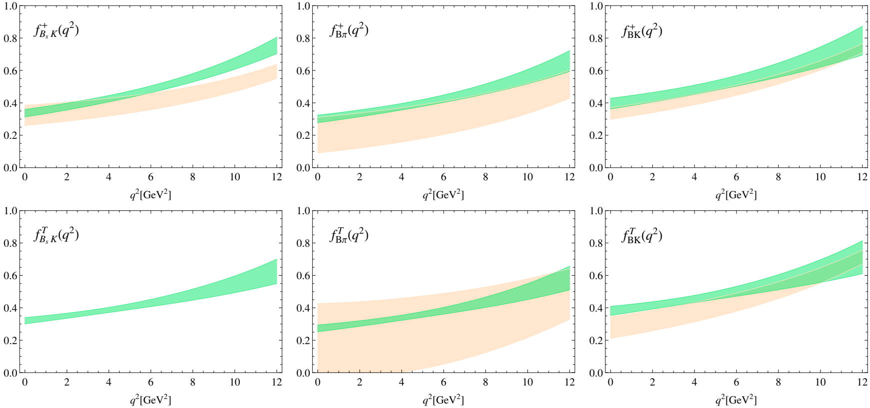
- Calculation reveals [AR, ArXiv:1705.01929]:

$$[f_{BP}^{+}(0)]_{\mathrm{tw5,6}} / f_{BP}^{+}(0) < 0.1\%$$

- $\Rightarrow$ Truncation up to twist-4 contributions is reliable

$B \to P$ form factors from LCSR: results

LCSR results are fitted to the z-expansion (BCL parametrization)



Observables in $B \to P\ell^+\ell^-$ decays

q^2-binned branching fraction

$$\mathcal{B}(\bar{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) = \frac{G_F^2 \alpha_{\rm em}^2 |\lambda_t^{(q)}|^2}{192\pi^5} \Big\{ \mathcal{F}_{BP}[q_1^2, q_2^2] + \kappa_q^2 \, \mathcal{H}_{BP}[q_1^2, q_2^2]$$
$$+ 2\kappa_q \Big(\cos\xi_q \, \mathcal{C}_{BP}[q_1^2, q_2^2] - \sin\xi_q \, \mathcal{S}_{BP}[q_1^2, q_2^2] \Big) \Big\} \tau_B$$

q^2-binned direct CP-asymmetry

$$\mathcal{A}_{BP}[q_1^2, q_2^2] = \frac{\mathcal{B}(\bar{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) - \mathcal{B}(B \to \bar{P}\ell^+\ell^-[q_1^2, q_2^2])}{\mathcal{B}(\bar{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) + \mathcal{B}(B \to \bar{P}\ell^+\ell^-[q_1^2, q_2^2])}$$
$$= \frac{-2\kappa_q \sin\xi_q \, \mathcal{S}_{BP}[q_1^2, q_2^2]}{\mathcal{F}_{BP}[q_1^2, q_2^2] + \kappa_q^2 \, \mathcal{H}_{BP}[q_1^2, q_2^2] + 2\kappa_q \cos\xi_q \, \mathcal{C}_{BP}[q_1^2, q_2^2]}$$

$$\frac{\lambda_u^{(q)}}{\lambda_t^{(q)}} = \frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*} \equiv \kappa_q \, e^{i\xi_q}, \quad q = d, s$$

Binned parts of the decay width

$$\mathcal{F}_{BP}[q_1^2, q_2^2] = \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 \, p_{BP}^3 |f_{BP}^+(q^2)|^2 \left(\left|c_{BP}(q^2)\right|^2 + |C_{10}|^2 \right)$$

$$\mathcal{H}_{BP}[q_1^2, q_2^2] = \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 \, p_{BP}^3 \left|h_{BP}(q^2)\right|^2$$

$$\begin{pmatrix} \mathcal{C}_{BP}[q_1^2, q_2^2] \\ \mathcal{S}_{BP}[q_1^2, q_2^2] \end{pmatrix} = \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 \, p_{BP}^3 \left|f_{BP}^+(q^2) c_{BP}(q^2) h_{BP}(q^2)\right| \begin{pmatrix} \cos \delta_{BP}(q^2) \\ \sin \delta_{BP}(q^2) \end{pmatrix}$$

$$c_{BP}(q^2) = C_9 + \frac{2(m_b + m_q)}{m_B + m_P} C_7^{\mathrm{eff}} \frac{f_{BP}^T(q^2)}{f_{BP}^+(q^2)} + 16\pi^2 \frac{\mathcal{H}_{BP}^{(c)}(q^2)}{f_{BP}^+(q^2)}$$

$$h_{BP}(q^2) = 16\pi^2 \left(\mathcal{H}_{BP}^{(c)}(q^2) - \mathcal{H}_{BP}^{(u)}(q^2) \right), \quad \delta_{BP}(q^2) = \mathrm{Arg}(h_{BP}(q^2)) - \mathrm{Arg}(c_{BP}(q^2))$$

Numerical results on observables

- Binned parts of the decay width

Decay mode	$\mathcal{F}_{BP}[\mathrm{GeV}^3]$	$\mathcal{D}_{BP}[\mathrm{GeV}^3]$	$\mathcal{C}_{BP}[\mathrm{GeV}^3]$	$\mathcal{S}_{BP}[\mathrm{GeV}^3]$
$B^- \to K^-\ell^+\ell^-$	$75.0^{+10.5}_{-9.7}$	—	—	—
$B^- \to \pi^-\ell^+\ell^-$	$47.7^{+6.4}_{-5.9}$	$16.1^{+2.8}_{-10.1}$	$14.3^{+7.8}_{-5.8}$	$-9.8^{+7.1}_{-7.2}$
$\bar{B}_s \to K^0\ell^+\ell^-$	$61.0^{+7.0}_{-6.8}$	$7.8^{+3.4}_{-2.5}$	$-12.9^{+2.4}_{-2.2}$	$-3.4^{+1.1}_{-2.6}$

- Our predictions (CKM matrix elements taken from global fit)

Process	$\mathcal{B}_{BP}[\mathrm{GeV}^{-2}] \times 10^{-8}$	$\mathcal{A}_{BP}$
$B^- \to K^-\ell^+\ell^-$	$4.38^{+0.62}_{-0.57} \pm 0.28$	0
$B^- \to \pi^-\ell^+\ell^-$	$0.131^{+0.023}_{-0.022} \pm 0.010$	$-0.15^{+0.11}_{-0.11}$
$\bar{B}_s \to K^0\ell^+\ell^-$	$0.154^{+0.018}_{-0.017} \pm 0.011$	$-0.04^{+0.01}_{-0.03}$

$$[q_1^2, q_2^2] = [1.0, 6.0]\,\mathrm{GeV}^2$$

CKM factors: way of extraction

- Switch to Wolfenstein parameters A, λ, ρ, η
- $\lambda = 0.22506 \pm 0.00050$ is fixed from the global CKM fit
- $A = \dfrac{(192\pi^5)^{1/2}}{G_F \alpha_{em} \lambda^2} \left(\dfrac{1}{\mathcal{F}_{BK}[q_1^2, q_2^2]} \right)^{1/2} \left(\dfrac{\mathcal{B}_{BK}[q_1^2, q_2^2]}{\tau_B} \right)^{1/2}$
- $\eta = \dfrac{1}{2\lambda^2(1-\lambda^2/2)} \left(\dfrac{\mathcal{F}_{BK}[q_1^2, q_2^2]}{\mathcal{S}_{B\pi}[q_1^2, q_2^2]} \right) \left(\mathcal{A}_{B\pi}[q_1^2, q_2^2] \dfrac{\mathcal{B}_{B\pi}[q_1^2, q_2^2]}{\mathcal{B}_{BK}[q_1^2, q_2^2]} \right)$
- $$\frac{\mathcal{B}_{B\pi}[q_1^2, q_2^2]}{\mathcal{B}_{BK}[q_1^2, q_2^2]} = \frac{\lambda^2}{\mathcal{F}_{BK}[q_1^2, q_2^2]} \Bigg(\left[(1-\rho)^2 + \eta^2\right] \mathcal{F}_{B\pi}[q_1^2, q_2^2] + \frac{\left[\rho(1-\rho) - \eta^2\right]^2 + \eta^2}{(1-\rho)^2 + \eta^2} \left(1 - \frac{\lambda^2}{2}\right)^2 \mathcal{D}_{B\pi}[q_1^2, q_2^2] + 2\left[\rho(1-\rho) - \eta^2\right] \left(1 - \frac{\lambda^2}{2}\right) \mathcal{C}_{B\pi}[q_1^2, q_2^2] \Bigg)$$
- $B_s \to K \ell^+ \ell^+$ provides additional constraints on these parameters

Conclusion

- ⋆ LCSR for $B \to \pi$, $B \to K$ and $B_s \to K$ form factors updated
- ⋆ Nonlocal hadronic amplitudes in $B \to P\ell^+\ell^-$ calculated
- ⋆ Predictions for the binned values of observables in $B \to P\ell^+\ell^-$ updated (including the new result for $B_s \to K\ell^+\ell^-$)
- ⋆ A systematic way to extract the CKM matrix elements (in terms of Wolfenstein parameters) suggested

Backup

Theory & Experiment

Decay mode	$B^- \to K^- \ell^+ \ell^-$	$B^- \to \pi^- \ell^+ \ell^-$	$\bar{B}_s \to K^0 \ell^+ \ell^-$
Measurement or calculation	$\mathcal{B}_{BK}[1.0\,,6.0]$	$\mathcal{B}_{B\pi}[1.0\,,6.0]$	$\mathcal{B}_{B_s K}[1.0, 6.0]$
Belle	$2.72\,^{+0.46}_{-0.42} \pm 0.16$	—	—
CDF	$2.58 \pm 0.36 \pm 0.16$	—	—
BaBar	$2.72\,^{+0.54}_{-0.48} \pm 0.06$	—	—
LHCb	$2.42 \pm 0.7 \pm 0.12$	$0.091^{+0.021}_{-0.020} \pm 0.003$	—
HPQCD	3.62 ± 1.22	—	—
Fermilab/MILC	3.49 ± 0.62	0.096 ± 0.013	—
This work	$4.38^{+0.62}_{-0.57} \pm 0.28$	$0.131^{+0.023}_{-0.022} \pm 0.010$	$0.154^{+0.018}_{-0.017} \pm 0.011$

$$\mathcal{A}_{B\pi}[1.0, 6.0] = -0.15^{+0.11}_{-0.11}\,, \qquad \mathcal{A}_{B_s K}[1.0, 6.0] = -0.04^{+0.01}_{-0.03}$$

$B_s \to K\ell\nu_\ell$ at large hadronic recoil

- The integrated over an interval $0 \leq q^2 \leq q_0^2$ differential decay width

$$\Delta\zeta_{B_s K}\,[0, q_0^2] \equiv \frac{1}{|V_{ub}|^2} \int_0^{q_0^2} dq^2 \frac{d\Gamma(\bar{B}_s \to K^+ \ell \bar{\nu}_\ell)}{dq^2} = \frac{G_F^2}{24\pi^3} \int_0^{q_0^2} dq^2 p_{B_s K}^3 |f_{B_s K}^+(q^2)|^2$$

- [New] Our prediction

$$\Delta\zeta_{B_s K}\,[0, 12\,\mathrm{GeV}^2] = 7.03^{+0.73}_{-0.69}\,\mathrm{ps}^{-1}$$

- For comparison

$$\Delta\zeta_{B\pi}\,[0, 12\,\mathrm{GeV}^2] = 5.30^{+0.67}_{-0.63}\,\mathrm{ps}^{-1}$$

- The ratio $\mathcal{B}(B_s \to K\ell\nu_\ell)/\mathcal{B}(B \to \pi\ell\nu_\ell) = \Delta\zeta_{B\pi}/\Delta\zeta_{B_s K}$ is independent of V_{ub}

LCSR: a general scheme

- Transform the OPE result to the dispersion form (in $(p+q)^2$ variable):

$$F_{BP}^{(T)(\mathrm{OPE})}(q^2,(p+q)^2) = \frac{1}{\pi}\int_{m_b^2}^{\infty} ds \frac{\mathrm{Im}F_{BP}^{(T)(\mathrm{OPE})}(q^2,s)}{s-(p+q)^2}$$

- Matching to hadronic dispersion relation (isolating ground B-meson state)
- Applying the quark hadron duality
- Applying the Borel transform
- Finally

$$f_{BP}^{+}(q^2) = \frac{e^{m_B^2/M^2}}{2m_B^2 f_B}\frac{1}{\pi}\int_{m_b^2}^{s_0^B} ds\, \mathrm{Im}F_{BP}^{(\mathrm{OPE})}(q^2,s)e^{-s/M^2}$$

$$f_{BP}^{T}(q^2) = \frac{(m_B+m_P)e^{m_B^2/M^2}}{2m_B^2 f_B}\frac{1}{\pi}\int_{m_b^2}^{s_0^B} ds\, \mathrm{Im}F_{BP}^{T(\mathrm{OPE})}(q^2,s)e^{-s/M^2}$$